CSC 2203 - Packet Switch and Network Architectures

Department of Computer Science, University of Toronto, Fall 2008

Midterm Exam – Handout # 6 – 90 Minutes **Date:** Thursday, October 9
Name: **Student ID:**

1. **(2 point)** Four users are downloading files through a shared link of capacity 2 Mbps. Alice and Bob are downloading large files and each are connected to the shared link through slow access links of capacity 0.1 Mbps. Carl is connected via a 10 Mbps link, but his application is using just 0.8 Mbps bandwidth. David is connected via a 1 Gbps link and is downloading a movie that can take up as much capacity as it can get.

What is the max-min fair allocation of the capacity of the common link to these flows?

a. Alice:
b. Bob:
c. Carl:
d. David:

2. (**2 points**) Let $\boldsymbol{X_1} \sim \text{Geom}(\boldsymbol{p_1})$, $\boldsymbol{X_2} \sim \text{Geom}(\boldsymbol{p_2})$ be two independent geometric random variables. What is the distribution of $\boldsymbol{Y} = \min(\boldsymbol{X_1}, \boldsymbol{X_2})$?

Hint. From the definition of geometric random variable, we know $\Pr(\boldsymbol{X_1} > \boldsymbol{k}) = (1-\boldsymbol{p_1})^{\boldsymbol{k}}$. Find what is the probability $\Pr(\min(\boldsymbol{X_1}, \boldsymbol{X_2}) > \boldsymbol{k})$? Note that $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ are independent.

3. **(3 points)** There are two packets in a queue, namely p_a and p_b, waiting to be served by a Generalized Processor Sharing (GPS) server. If there are no more arrivals then p_a will depart before p_b. Can a future packet arrival in the GPS system change the departure order of p_a and p_b? Justify your answer – if yes, give a simple example and if no, explain why.

4. **(4 points)** In this problem, we will investigate what the motivation for packetization is in data networks. Assume we wish to transmit a S = 1 Mbit file over a network. The average bit error probability of the network is $P_b = 10^{-5}$. We need to decide the number of packets to split the file into for transmission. Assume the cost associated with the overhead and added complexity of packetization is given by c_pN where N is the number of packets we split the file to, and c_p = 1 dollar/packet. Also each time some bit in a packet is transmitted with an error, the whole packet has to be retransmitted. Assume the cost of transmitting data over the network is given by c_r = 1 cent/bit.

(a) Assume N = 1, i.e., we decide to transmit the file as a whole. What is the expected total cost of transmitting the file?

(b) What is the optimal number of packets the file should be split into to minimize the expected cost of transmitting it over this network?

(c) How low does the bit error probability of the network needs to be so that the optimal scheme is to just send the file as a whole?

5. **(2 points)** In the lectures, we have seen several ways of using parallelism to scale packet switches. Give two problems that must be solved in almost any architecture employing parallelism.

6. **(5 points)** Let N flows share a link of capacity C. These flows want to send at rate $f_1 \leq \ldots \leq f_N$. Due to capacity constraint of C, these flows need to be assigned rates $r_1, \ldots, r_N$ such that it is feasible, that is, $\sum_{i=1}^{N} r_i \leq C, \quad f_i \geq r_i \geq 0, \quad \forall i.$

(a) **Definition of Max-Min Fairness**: A vector of rates $r = (r_1, \ldots, r_N)$ is said to be max-min fair if it is feasible; and for any i, r_i cannot be increased while maintaining feasibility without decreasing r_j for some flow $j \neq i$, such that $r_j \leq r_i$.

(b) **Definition of Max-Min Fairness in terms of allocation policy**: As discussed in the class, a given set of flows can be allocated rates in the max-min fair way in the following manner:

(i) Initially: S = {1, …, N}, $m = N$ and $R = C$. Here S denotes the remaining flows, m denotes the remaining number of flows and R denotes the remaining capacity.
(ii) Pick the smallest flow $i \in S$. If $f_i < R/m$, set $r_i = f_i$ else set $r_i = R/m$.
(iii) Set $m = m - 1$, $R = R - r_i$. Delete i from S.
(iv) If $m > 0$ repeat from (ii).

6.1. Show that any assignment generated by (b) satisfies conditions of (a).

6.2. Show that there is a unique assignment that satisfies (a).

6.3. Using 6.1 and 6.2 show that (a) and (b) are equivalent.

7. **(2 points)** Let us consider an output link serving ***N*** flows based on Deficit Round Robin (DRR) with a quantum size Q. For Q = 1 bits, is the order of packet departures the same as Fair Queueing? If so, explain why, and if not show a counter-example.