CSCC73
Algorithm design & analysis

Week 12 Tutorial
Question 1

- Shown above is a flow network.
- Write down a linear program to find the maximum flow in this network.
- Find an optimal solution for this LP.
In LPs, if an optimal solution exists then there is an optimal solution that is a vertex of the feasible region.

Give an example of an optimization problem in two variables, where

• the objective function is quadratic,
• every constraint is linear,
• the feasible region is non-empty and bounded, but
• no vertex of the feasible region is an optimal solution.

Hint: Think geometrically!
Question 3b

The library has had its budget cut and can no longer lend books for free. It charges $r_j$ dollars for each copy of title $b_j$ that it lends. The library wants to assign books to subscribers consistent with their preferences and its rules so as to maximize the total revenue from lending books.

Express this optimization problem as a 0-1 linear program.
A public library has $m$ titles (book names) $b_1, ..., b_m$. It owns $c_j$ copies of title $b_j$, for each $j$, $1 \leq j \leq m$. There are $n$ subscribers to the library $s_1, ..., s_n$. Subscriber $s_i$ is interested in a subset $B_i$ of the library's titles, for each $i$, $1 \leq i \leq n$. Each subscriber can borrow a maximum of five copies, and cannot borrow two copies of the same title.

Describe a polynomial time algorithm that assigns copies of books that the library owns to subscribers interested in them so as to maximize the total number of books borrowed. Analyze the running time of your algorithm. (Hint: Reduce the problem to maximum flow.)
Question 4 (time permitting)

We are given a set of points \((x_1,y_1), ..., (x_m,y_m)\) on the Cartesian plane, and we want to find a linear function \(f(x) = ax+b\) that minimizes the maximum error over all the points.

The error of \(f(x) = ax+b\) for a point \((x_i,y_i)\) is \(|ax_i+b−y_i|\). So the maximum error of \(f(x) = ax+b\) for the set of points \((x_1,y_1), ..., (x_m,y_m)\) is \(\max_{1 \leq i \leq m} |ax_i+b−y_i|\)

Write a LP to find a linear function that minimizes the maximum error over the points \((-2,-1), (3,1), (5,1), (7,2)\).

NB: In class we did something similar, with the error defined as the sum of the errors of the given points.