University of Toronto
Scarborough Campus
October 22, 2007

CSC C73 Midterm Examination

Aids allowed: One 8.5×11 'cheat sheet' (may be written on both sides)

Duration: One hour and fifty minutes

- There should be 7 pages in this exam booklet, including this cover page.
- Answer all questions.
- Put all answers in this booklet, in the spaces provided.
- For rough work, use the backs of the pages; ***these will not be marked***.
- Good luck!

Family Name _____ Given Name _____

Student Number _____

Problem	Marks Rec'ved	Marks Worth
1.		15
2.		20
3.		15
4.		15
5.		35
TOTAL		100

QUESTION 1. (15 marks)

Recall the minimum delay scheduling problem: We are given the length and deadline of each of n jobs, and a release time. We wish to schedule all the jobs in disjoint time intervals after the release time so as to minimize the maximum lateness experienced by any job. (The lateness of a job in a schedule is the amount of time by which the job missed its deadline in that schedule.)

Consider now the following instance of this problem: The release time is 0; there are five jobs A, B, C, D, and E, whose length and duration is given by the following table:

Job	Length	Deadline
A	2	11
B	4	5
C	6	7
D	8	15
E	4	20

Suppose we schedule the jobs in consecutive non-overlapping intervals in the order A, B, C, D, E, starting at time 0. Is this an optimal schedule?

Justify your answer and show your work.

ANSWER:

QUESTION 2. (20 marks)

Following are four statements about minimum spanning trees (MSTs) of an undirected, connected, weighted graph G. For each statement, state whether it is true or false by circling the appropriate response. Do not justify your answers.

Do not guess: *5 for each correct answer, 0 for no answer, −3 for wrong answer.*

(a) If G is not a tree and it has a unique edge e of maximum weight, then e cannot be in any MST of G. **True / False**

(b) If edge e has minimum weight, then e is in some MST of G. **True / False**

(c) If e is in some MST of G, then it is a minimum weight edge that crosses some cut of G. **True / False**

(d) The edges of a shortest path between any two nodes must be in some MST of G. **True / False**

QUESTION 3. (15 marks)

Given below are three encodings of the symbols a, b, and c. For each encoding either give frequencies $f(a)$, $f(b)$, and $f(c)$ such that Huffman's algorithm can produce this encoding, or explain why Huffman's algorithm cannot possibly produce the encoding for *any* frequencies.

(a)

Symbol:	a	b	c
Code:	0	10	11

ANSWER:

(b)

Symbol:	a	b	c
Code:	0	1	00

ANSWER:

(c)

Symbol:	a	b	c
Code:	00	01	10

ANSWER:

QUESTION 4. (15 marks)

You are choosing among three algorithms for the same problem.

- Algorithm A solves a problem of size n by dividing it into eight subproblems each of size $n/2$, and combining the solutions of the subproblems in $\Theta(n)$ time.
- Algorithm B solves a problem of size n by dividing it into nine subproblems each of size $n/3$, and combining the solutions of the subproblems in $\Theta(n^2)$ time.
- Algorithm C solves a problem of size n by recursively solving two subproblems each of size $n-1$, and combining the solutions of the subproblems in constant time.

Rank the algorithms in terms of their asymptotic time efficiency. Explain how you obtained your ranking. (For partial credit you may derive the time complexity of only some of these algorithms.)

ANSWER:

QUESTION 5. (35 marks)

You are given two ***sorted*** arrays A and B, each containing n numbers, and you want to find the median of the $2n$ numbers in the two arrays. Give an algorithm that does this in $O(\log n)$ time. For convenience, you may assume that $n = 2^k + 1$, where k is a non-negative integer. Explain why your algorithm is correct, and why it achieves the required time complexity.

ANSWER:

[More space, if needed, is provided on the next page]

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THE END