A universe of 16 elements

Five sets over this universe:
purple, orange, red, green, blue

Set cover

A minimum cover
Optimal cover = \{A, B\}

Greedy cover = \{C, D, E, F\}

Generalize:
Universe of \(n = 2^m - 2\) elements
Optimal cover: 2 sets
Greedy cover: \(m-1\) sets
Greedy/Optimal = \(\Theta(\log n)\)
Not a constant approximation factor

Universe of 30 = \(2^5 - 2\) elements
Example for tight analysis of greedy makespan algorithm

- \( m(m - 1) \) jobs, each of length 1
- 1 job of length \( m \)

\[
\frac{\text{Greedy makespan}}{\text{Optimal makespan}} = \frac{2m - 1}{m} = 2 - \frac{1}{m}
\]
Example for tight analysis of longest-first greedy algorithm

- odd $m$
- 2 jobs of length $2m - 1$
- 2 jobs of length $2m - 2$
- 2 jobs of length $2m - 3$
- ...
- 2 jobs of length $m + 1$
- 3 jobs of length $m$

Makespan = $4m - 1$
Example for tight analysis of longest-first greedy algorithm

- odd $m$
- 2 jobs of length $2m - 1$
- 2 jobs of length $2m - 2$
- 2 jobs of length $2m - 3$
- ...
- 2 jobs of length $m + 1$
- 3 jobs of length $m$

Makespan = $3m$

\[
\frac{\text{Longest first}}{\text{Optimal}} = \frac{4m - 1}{3m} = \frac{4}{3} - \frac{1}{3m}
\]