Computer Science C63 — March 12, 2025
Scarborough Campus — University of Toronto

Homework Assignment #6
Due: March 19, 2025, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (20 marks) Suppose you are building a computer system whose components are a CPU, a memory card, a hard drive, a network interface card, a keyboard, a monitor, a mouse, and so on. For each of these components there are different manufacturers whose devices you could use to build your computer. Unfortunately, some components are incompatible; for example, an Intel processor might not work with an NVIDIA network interface card; or a Logitech keyboard might not work with an Apple monitor. You want to determine if there is a possible selection of devices, one for each component, so that no two are incompatible.

This decision problem can be formulated as follows:

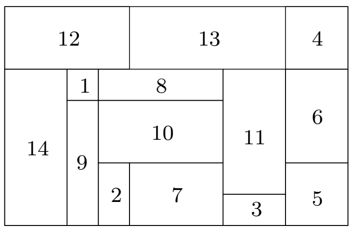
Instance: $\langle D_1, D_2, \ldots, D_k, X\rangle$, where each D_i, $1 \le i \le k$, is a non-empty set of devices for component i, so that $D_i \cap D_j = \varnothing$ for all $i \neq j$ (so no device can be used for multiple components); and X is a set of pairs $\{d, d'\}$ where $d \in D_i$ and $d' \in D_j$ for some $i \neq j$ (so X is the set of incompatible devices).

Question: Is there some $(d_1, d_2, \ldots, d_k) \in D_1 \times D_2 \times \cdots \times D_k$, so that for all $i \neq j$, $\{d_i, d_j\} \notin X$? That is, is there a selection of devices, one for each component, no two of which are incompatible?

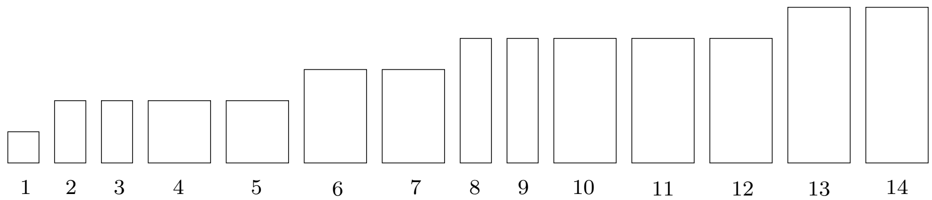
Prove that this problem is **NP**-complete.

[Continued on the next page]

Question 2. (25 marks) You are given a number of rectangular ***tiles***, and a rectangular ***frame***. A ***tiling*** of the frame is a placement of the given tiles inside the frame so as to cover precisely the entire surface of the frame — with no gaps, no tile overlaps, and no spillover — using ***all*** the given tiles and nothing else. Note that the only way to do this is by placing each tile so that its sides are parallel to the sides of the frame; a tile can be placed with its long side horizontally or vertically. An example of an instance where a tiling is possible is shown below.



FRAME



TILES

The ***frame tiling problem***, FRAMETILING, is defined as follows:

Instance: $\langle T, F \rangle$, where $T = (a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$ is a sequence of pairs of positive integers (the dimensions of the tiles) and $F = (A, B)$ is a pair of positive integers (the dimensions of the frame).

Question: Is there a tiling of an $A \times B$ frame using all of (and only) the n tiles whose dimensions are $a_1 \times b_1, a_2 \times b_2, \ldots, a_n \times b_n$.

Prove that this problem is **NP**-complete.

Hints: (1) The position on the Cartesian plane of a rectangle whose edges are parallel to the axes can be fully specified by giving the coordinates of its lower-left and upper-right corners.
(2) For the reduction you may find it useful to first consider the somewhat easier version of the problem where the tiling can use a subset of the given tiles, not necessarily all of them. If you cannot solve the problem as specified above, for partial credit you can present a solution to this simpler version.