Computer Science C63 | March 9, 2022
Scarborough Campus | University of Toronto

Homework Assignment #5
Due: March 16, 2022, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (10 marks) Prove that the universal language

$$U = \{\langle M, x\rangle : \text{ Turing machine } M \text{ accepts input } x\}$$

is NP-hard. For full credit prove this from first principles, based on the definition of NP-hard and polynomial-time mapping reduction — in particular, without using the fact that any specific problem, such as SAT, is NP-complete. For partial credit you may prove this assuming that SAT is NP-complete.

Question 2. (25 marks) Let $G = (V, E)$ be an undirected graph and $\mathbf{wt} : V \to \mathbb{Z}^+$ be a node weight function. The ***weight*** of a set of nodes $V' \subseteq V$, denoted $\mathbf{wt}(V')$, is the sum of their weights: $\mathbf{wt}(V') = \sum_{u\in V'} \mathbf{wt}(u)$. MAXWEIGHTIS is the following optimization problem:

INPUT: $\langle G, \mathbf{wt}\rangle$, where $G = (V, E)$ is an undirected graph and $\mathbf{wt} : V \to \mathbb{Z}^+$ is a node weight function.

OUTPUT: $\langle V'\rangle$, where V' is a maximum weight independent set of nodes of G; i.e., a set $S \subseteq V$ such that (a) S is an independent set of nodes of G (i.e., for all $u, v \in S$, $\{u, v\} \notin E$); and (b) for any independent set S' of nodes of G, $\mathbf{wt}(S) \geq \mathbf{wt}(S')$.

a. Formulate a decision problem $\Pi \in \mathbf{NP}$ such that MAXWEIGHTIS $\leq_T^p \Pi$. Carefully prove that your problem Π has this property. ("Carefully" here means that you should give convincing arguments why your reduction algorithm is (a) correct and (b) polynomial time. In particular you should state clearly the invariant(s) of the loop(s) used by your reduction algorithm.)

b. Prove that your problem Π from part (a) is in $\mathbf{NP}$.