Computer Science C63 — February 9, 2022
Scarborough Campus — University of Toronto

Homework Assignment #4
Due: Friday, February 18, 2022, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

In your answers for this assignment you may appeal to Church's thesis when arguing that a language is decidable or recognizable, or that a function is computable. That is, you may replace an argument that a Turing machine for a certain task exists by a description, in high-level pseudocode, of an algorithm that achieves this task.

Question 1. (20 marks) Consider the following sets:

$$A = \{\langle M, x\rangle : \text{ Turing machine } M \text{ on input } x \text{ never attempts to move left from the leftmost cell}\}$$
$$B = \{\langle M, x\rangle : \text{ Turing machine } M \text{ on input } x \text{ never moves left}\}.$$

For each of the sets A and B, state whether it is decidable and prove your answer.

Question 2. (20 marks) Consider the following sets:

$$C = \{\langle M_1, M_2, k\rangle : \text{ There are at least } k \text{ inputs } y \text{ such that Turing machines } M_1 \text{ and } M_2 \text{ both halt on } y\}$$
$$D = \{\langle M_1, M_2\rangle : M_1 \text{ and } M_2 \text{ are Turing machines such that } \mathcal{L}(M_1) \neq \mathcal{L}(M_2) \text{ and } \mathcal{L}(M_1) \cap \mathcal{L}(M_2) = \varnothing\}.$$

For each of the sets C and D, state whether it is decidable, and if not whether it is recognizable. If the set not recognizable, state whether its complement is recognizable. Prove all your claims. (The $\mathcal{L}(M_1) \neq \mathcal{L}(M_2)$ condition for D is to rule out a very easy solution.)