Computer Science C63 — January 19, 2022
Scarborough Campus — University of Toronto

Homework Assignment #2
Due: January 26, 2022, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *It is your responsibility to ensure that the PDF files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

**Question 1.** (15 marks) Describe a TM that accepts the following language $L$ over the alphabet $\{0, 1, \#\}$:

$$L = \{\#x : \ x \in \{0,1\}^* \text{ and } x \text{ contains exactly twice as many 1s as 0s}\}.$$[1]

For example, #, #101, #010111 are strings in $L$, while #001, #01, #1001111, 100111 are strings not in $L$.

First describe your TM in point-form English, explaining how it works informally. Then describe all the components of the TM, giving the transition function in graphical form as in Example 3.9 of your textbook, and relate the states of your TM to your point-form English description.

A nice TM simulator, courtesy of Mustafa Qureish (former student here, now a PhD student, whom some of you may have known as a TA), can be found in https://mustafaquraish.github.io/TMSim/. Feel free to use this to "test" your TM, but don't submit its description in the format required by the simulator. The grader will rely only on your description to be convinced of its correctness. It is therefore important that your TM be as simple as possible, and your description of it be very clear. Overly complicated TMs and descriptions that the grader is unable to follow will receive few (possibly zero) marks. This is as much an exercise in TM design as it is in clear communication.

[**Question 2 on the next page**]

[1] The initial # only helps mark the beginning of the input, which simplifies the task a little.

**Question 2.** (25 marks)

**a.** Vassos has been observing the operation of a Turing Machine with five states and four tape symbols on some input. After 53 million moves he notices that the TM never moved its tape head past the ~~20th~~ 10th (tenth) cell, and exclaims in exasperation: "I've seen enough, this thing will never halt!" Is he right? Justify your answer.

**b.** Let $M$ be a Turing machine with the following property: There is a constant $c$ such that for every input string $x$, $M$ does not move its tape head to the right of cell $c$. Prove that $\mathcal{L}(M)$ is a regular language.

**c.** Prove that a language $L$ is regular if and only if it is recognized by a Turing machine that always moves its tape head to the right.