CSCC63

Post’s Correspondence Problem (PCP)
Example:

An instance of PCP

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>$y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>111</td>
</tr>
<tr>
<td>2</td>
<td>101</td>
<td>0</td>
</tr>
<tr>
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<td>01</td>
</tr>
</tbody>
</table>

$x_1$

$y_1$
Example:

An instance of PCP

<table>
<thead>
<tr>
<th>$i$</th>
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<td>01</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>01</td>
</tr>
</tbody>
</table>

SUCCESS!  
(a “yes” instance of PCP)
Example:

Another instance of PCP

<table>
<thead>
<tr>
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<th>$y_i$</th>
</tr>
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<tbody>
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<td>01</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>01</td>
</tr>
</tbody>
</table>

- Pair 2 cannot be used (otherwise top string will always have more 0s than the bottom string).
- Given that pair 2 cannot be used, pair 1 cannot be used (otherwise top string will always have fewer 1s than the bottom string).
- For the same reason, pair 4 cannot be used.
- Only pair 3 can be used, but there is already a mismatch on the first symbol.

FAILURE!
(a “no” instance of PCP)
MPCP $\leq_m$ PCP

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</tbody>
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<table>
<thead>
<tr>
<th>$i$</th>
<th>$v_i$</th>
<th>$w_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1@1@</td>
<td>@1@1@1</td>
</tr>
<tr>
<td>2</td>
<td>1@0@1@</td>
<td>@0</td>
</tr>
<tr>
<td>3</td>
<td>1@0@</td>
<td>@0@1</td>
</tr>
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<td>0@</td>
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</table>
MPCP \leq_m PCP

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</table>

$x_1$

$1$

$1$

$1$

$1$

$y_1$

$1$

$1$

$1$

$1$
\( \text{MPCP} \leq_m \text{PCP} \)

<table>
<thead>
<tr>
<th>( i )</th>
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<th>( y_i )</th>
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<tr>
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<td>0</td>
<td>01</td>
</tr>
</tbody>
</table>

\[ \begin{array}{ccc}
\hline
i & v_i & w_i \\
\hline
1 & 1@1@ & @1@1@1 \\
2 & 1@0@1@ & @0 \\
3 & 1@0@ & @0@1 \\
4 & 0@ & @0@1 \\
\hline
\end{array} \]

\[ \begin{tabular}{ccc}
\hline
\( x_1 \) & \( x_2 \) \\
\hline
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
\hline
\end{tabular} \]

\[ \begin{tabular}{ccc}
\hline
\( y_1 \) & \( y_2 \) \\
\hline
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
\hline
\end{tabular} \]
MPCP $\leq_m$ PCP

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</tr>
<tr>
<td>4</td>
<td>0@</td>
<td>@0@1</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|c|}
\hline
x_1 & x_2 & x_1 \\
\hline
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\hline
y_1 & y_2 & y_1 \\
\hline
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
\hline
\end{array}
\]
MPCP $\leq_m$ PCP
\[ \text{MPCP} \leq_m \text{PCP} \]

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(x_i)</th>
<th>(y_i)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
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<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>3</td>
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<td>0@</td>
<td>0@1</td>
</tr>
</tbody>
</table>

\[ x_1 \quad x_2 \quad x_1 \]

<table>
<thead>
<tr>
<th></th>
<th>(y_1)</th>
<th>(y_2)</th>
<th>(y_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ v_1 \quad v_2 \]

<table>
<thead>
<tr>
<th></th>
<th>(w_1)</th>
<th>(w_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1@1@</td>
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</tr>
<tr>
<td>1</td>
<td>0@</td>
<td>1@</td>
</tr>
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MPCP $\leq_m$ PCP

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<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_1$</th>
</tr>
</thead>
</table>
| 1 1 1 0 1 1 1 1
| 1 1 1 0 1 1 1 1
<table>
<thead>
<tr>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_1$</th>
</tr>
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</table>
| 1 0 1 1 1 1 1
| 0 1 1 1 1 1 1

<table>
<thead>
<tr>
<th>$v_1$</th>
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</tr>
</thead>
</table>
| 1 @ 1 @ 1 @ 0 @ 1 @ 1 @ 1 @ 1 @
| @ 1 @ 1 @ 1 @ 0 @ 1 @ 1 @ 1 @
<table>
<thead>
<tr>
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MPCP $\leq_m$ PCP
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<td>0</td>
<td>@1@1@</td>
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</tr>
<tr>
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<td>4</td>
<td>0@</td>
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</table>

Same as $v_1$ with @ at beginning

Same as $w_1$
### MPCP \( \leq_m \text{PCP} \)

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<tbody>
<tr>
<td>0</td>
<td>@1@1@</td>
<td>@1@1@1</td>
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</tr>
<tr>
<td>4</td>
<td>0@</td>
<td>@0@1</td>
</tr>
</tbody>
</table>

### Diagram:

- **Original Table:**
  - \( x_1 \) / \( x_2 \) / \( x_1 \)
  - \( \begin{array}{lll} 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \)
  - \( \begin{array}{lll} 0 & 1 & 1 \\ 0 & 1 & 1 \end{array} \)

- **Transformed Table:**
  - \( v_0 \) / \( v_1 \)
  - \( \begin{array}{llll} @ & 1 & @ & 1 \\ @ & 1 & @ & 1 \end{array} \)
  - \( \begin{array}{llll} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{array} \)

- **Weighted Tables:**
  - \( w_0 \) / \( w_1 \)
  - \( \begin{array}{llll} @ & 1 & @ & 1 \\ @ & 1 & @ & 1 \end{array} \)
  - \( \begin{array}{llll} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{array} \)
\[ \text{MPCP} \leq_m \text{PCP} \]

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</tr>
</tbody>
</table>

\[
\begin{array}{ccc}
\hline
x_1 & x_2 & x_1 \\
\hline
1 & 1 & 1 \\
1 & 1 & 1 \\
\hline
y_1 & y_2 & y_1 \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
\hline
v_0 & v_1 & w_0 \\
\hline
@ & 1 & @ \\
@ & 1 & @ \\
\hline
w_1 & w_2 & w_1 \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
\hline
v_0 & v_1 & w_0 \\
\hline
@ & 1 & @ \\
@ & 1 & @ \\
\hline
w_1 & w_2 & w_1 \\
\hline
\end{array}
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MPCP $\leq_m$ PCP

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<tbody>
<tr>
<td>0</td>
<td>@1@1@</td>
<td>@1@1@1</td>
</tr>
<tr>
<td>1</td>
<td>1@1@</td>
<td>@1@1@1</td>
</tr>
<tr>
<td>2</td>
<td>1@0@1@</td>
<td>@0</td>
</tr>
<tr>
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<td>1@0@</td>
<td>@0@1</td>
</tr>
<tr>
<td>4</td>
<td>0@</td>
<td>@0@1</td>
</tr>
<tr>
<td>5</td>
<td>$$</td>
<td>@$$</td>
</tr>
</tbody>
</table>

$x_1$ $x_2$ $x_1$

$y_1$ $y_2$ $y_1$

$v_0$ $v_2$ $v_1$ $v_5$

$w_0$ $w_2$ $w_1$ $w_5$