

University of Toronto
Scarborough Campus
April 19, 2022

CSC C63 Final Exam
Instructor: Vassos Hadzilacos

Aids allowed: One 8.5×11 'cheat sheet' (may be written on both sides)

Duration: Three hours

READ THE INSTRUCTIONS CAREFULLY

- There should be ?? pages in this exam booklet, including this cover page.
- Answer all questions.
- Put all answers in this booklet, in the spaces provided.
- For rough work, use the backs of the pages; *these will not be graded*.
- The last three pages are blank and can be used for rough work or for overflow. If you use them for overflow, you must *clearly indicate this* on the page(s) containing the question(s) whose answer(s) you are providing in the overflow pages, and you *must not detach them* from the booklet.
- In your answers you may use any result discussed in this course or its prerequisites merely by naming or describing it. In particular, you may use any of the facts regarding undecidable, unrecognizable, and NP-complete problems listed on page 2.
- Good luck!

LIST OF UNDECIDABLE OR UNRECOGNIZABLE SETS:

- $U = \{\langle M, x \rangle : \text{TM } M \text{ accepts input } x\}$ is undecidable but recognizable; thus \overline{U} is unrecognizable.
- $H = \{\langle M, x \rangle : \text{TM } M \text{ halts on input } x\}$ is undecidable but recognizable; thus \overline{H} is unrecognizable.
- $\overline{E} = \{\langle M \rangle : \text{TM } M \text{ accepts some input}\}$ is undecidable but recognizable; thus E is unrecognizable.
- $FIN = \{\langle M \rangle : \text{TM } M \text{ recognizes a finite language}\}$ and \overline{FIN} are both unrecognizable.
- $REG = \{\langle M \rangle : \text{TM } M \text{ recognizes a regular language}\}$ and \overline{REG} are both unrecognizable.

LIST OF NP-COMPLETE PROBLEMS:

- SAT/CNF-SAT/3SAT
Instance: $\langle F \rangle$, where F is a propositional formula/CNF formula/CNF formula where each clause has at most three literals.
Question: Is F satisfiable?
- CLIQUE/INDEPENDENT SET/VERTEX COVER
Instance: $\langle G, k \rangle$, where G is an undirected graph and k is a positive integer.
Question: Does G have a clique/independent set/vertex cover of size k ?
- EXACT COVER
Instance: $\langle U, \mathcal{C} \rangle$, where U is a set and \mathcal{C} is a set of subsets of U .
Question: Is there a subset of \mathcal{C} that contains pairwise disjoint sets whose union is U ?
- 3-DIMENSIONAL MATCHING
Instance: $\langle A, B, C, M \rangle$, where A, B, C are disjoint sets such that $|A| = |B| = |C| = n$, and $M \subseteq A \times B \times C$.
Question: Is there a subset of M that is a matching of size n ?
- DIRECTED/UNDIRECTED HAMILTONIAN CYCLE
Instance: $\langle G \rangle$, where G is a directed/undirected graph.
Question: Does G have a Hamiltonian cycle?
- DIRECTED/UNDIRECTED (s, t) HAMILTONIAN PATH
Instance: $\langle G, s, t \rangle$, where G is a directed/undirected graph, and s, t are nodes of G .
Question: Does G have a Hamiltonian path that starts at s and ends in t ?
- SUBSET SUM
Instance: $\langle A, k \rangle$, where A is a sequence of positive integers and k is a positive integer.
Question: Is there a subsequence of A whose elements add up to k ?
- PARTITION
Instance: $\langle A \rangle$, where A is a sequence of positive integers.
Question: Does A have a subsequence whose elements add up to half of the sum of the elements of A ?
- ZERO-ONE LINEAR PROGRAMMING
Instance: $\langle A, \mathbf{b} \rangle$, where A is an $m \times n$ matrix of integers, and $\mathbf{b} \in \mathbb{Z}^m$ is an m -dimensional vector of integers.
Question: Is there a vector $\mathbf{x} \in \{0, 1\}^n$ such that $A\mathbf{x} \leq \mathbf{b}$?