TM for even-length palindromes

1. If the symbol under the head is ⊣, accept; else "remember" that symbol, replace it by ⊣ and move R
2. While scanning a symbol ≠ ⊣ move R
3. Move L from the first ⊣ found
4. If the symbol under the head is different from the one "remembered", reject else replace it by ⊣ and move L
5. While scanning a symbol ≠ ⊣ move L
6. Move R and go to step 1
TM for even-length palindromes

\[ M = (Q, \Sigma, \Gamma, \delta, q_0, h_A, h_R) \]

\[ Q = \{ q_0, q_0^0, q_1^0, q_2^0, q_1^1, q_2^1, q_3, h_A, h_R \} \]

- \( q_0 \) - initial state
- \( q_0^0 \) - scanning right, first symbol was 0
- \( q_1^0 \) - scanning right, first symbol was 1
- \( q_2^0 \) - reached right end, first symbol was 0
- \( q_2^1 \) - reached right end, first symbol was 1
- \( q_3 \) - scanning left ("rewinding")
- \( h_A \) - accept state
- \( h_R \) - reject state
TM for even-length palindromes

The transition function:

- \(\delta(q_0, a) = \begin{cases} (h_A, \sqcup, R), & \text{if } a = \sqcup \\ (q_1^a, \sqcup, R), & \text{if } a \neq \sqcup \end{cases} \forall a \in \{0,1,\sqcup\}\)

- \(\delta(q_1^b, a) = \begin{cases} (q_1^b, a, R), & \text{if } a \neq \sqcup \\ (q_2^b, \sqcup, L), & \text{if } a = \sqcup \end{cases} \forall a \in \{0,1,\sqcup\}, b \in \{0,1\}\)

- \(\delta(q_2^b, a) = \begin{cases} (h_R, \sqcup, L), & \text{if } a \neq b \\ (q_3, \sqcup, L), & \text{if } a = b \end{cases} \forall a \in \{0,1,\sqcup\}, b \in \{0,1\}\)

- \(\delta(q_3, a) = \begin{cases} (q_3, a, L), & \text{if } a \neq \sqcup \\ (q_0, \sqcup, R), & \text{if } a = \sqcup \end{cases} \forall a \in \{0,1,\sqcup\}\)
# TM for even-length palindromes

The transition function in tabular form:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>⊥</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$(q_1^0,\sqcup, R)$</td>
<td>$(q_1^0,\sqcup, R)$</td>
<td>$(h_A,\sqcup, R)$</td>
</tr>
<tr>
<td>$q_1^0$</td>
<td>$(q_1^0, 0, R)$</td>
<td>$(q_1^0, 1, R)$</td>
<td>$(q_2^0,\sqcup, L)$</td>
</tr>
<tr>
<td>$q_1^1$</td>
<td>$(q_1^1, 0, R)$</td>
<td>$(q_1^1, 1, R)$</td>
<td>$(q_2^1,\sqcup, L)$</td>
</tr>
<tr>
<td>$q_2^0$</td>
<td>$(q_3,\sqcup, L)$</td>
<td>$h_R$</td>
<td>X</td>
</tr>
<tr>
<td>$q_2^1$</td>
<td>$h_R$</td>
<td>$(q_3,\sqcup, L)$</td>
<td>X</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$(q_3, 0, L)$</td>
<td>$(q_3, 1, L)$</td>
<td>$(q_0,\sqcup, R)$</td>
</tr>
</tbody>
</table>
TM for even-length palindromes

The transition function in graphical form:

- $\delta(q, a) = (p, b, D)$, for $b \neq a$

- $\delta(q, a) = (p, b, D)$, for $b = a$

- Missing transitions: implicitly going to $h_R$ (reject)
TM for even-length palindromes

Diagram for transition function of this TM:
TM for even-length palindromes

Computation on input 0110:

Exercise: Trace the computation on inputs 0111 and 010.