Turing Machine Example

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Given below is the formal description of a Turing Machine $M$ that decides the language

$$L = \{ x \in \Sigma^* : x \text{ is an even-length palindrome} \}.$$

(A palindrome is a string reads the same forwards and backwards; i.e., it is equal to its reverse.)

(1) The states of $M$ are:

- $q_0$, the initial state;
- $q_A$, the accept state;
- $q_R$, the reject state;
- $q_a^1$, for each $a \in \Sigma$. Informally this state means that we are in the stage of moving right, remembering that $a$ was the first symbol of the string that we want to match with the last one.
- $q_a^2$, for each $a \in \Sigma$. Informally this state means that we just got to the end of the string, and $a$ was the first symbol that we want to match with the last one (the one we just saw before we realized we got to the end of the string).
- $q_3$. This state means that we are moving left to the beginning of the (remaining) string.

(2) The input alphabet of $M$ is (of course) $\Sigma$.

(3) The tape alphabet of $M$ is $\Gamma = \Sigma \cup \{\sqcup\}$.

(4) The state transition function $\delta$ of $M$ is defined as follows, for every $a \in \Gamma$ and $b \in \Sigma$:

$$\delta(q_0, a) = \begin{cases} 
(q_A, \sqcup, R), & \text{if } a = \sqcup \\
(q_a^1, \sqcup, R), & \text{if } a \neq \sqcup 
\end{cases}$$

$$\delta(q_1^b, a) = \begin{cases} 
(q_1^b, a, R), & \text{if } a \neq \sqcup \\
(q_2^b, \sqcup, L), & \text{if } a = \sqcup 
\end{cases}$$

$$\delta(q_2^b, a) = \begin{cases} 
(q_R, \sqcup, L), & \text{if } a \neq b \\
(q_3, \sqcup, L), & \text{if } a = b 
\end{cases}$$

$$\delta(q_3, a) = \begin{cases} 
(q_3, a, L), & \text{if } a \neq \sqcup \\
(q_0, \sqcup, R), & \text{if } a = \sqcup 
\end{cases}$$

Here is the computation of $M$ on input 0110:

$$q_0 0110 \vdash \sqcup q_1^0 110 \vdash \sqcup 1 q_1^0 10 \vdash \sqcup 1 1 q_1^0 0 \vdash \sqcup 1 1 1 q_1^0 0 \vdash \sqcup 1 q_1 1 \vdash q_3 \sqcup 11 \vdash \sqcup q_0 11 \vdash \sqcup \sqcup q_1 1 \vdash \sqcup \sqcup q_1^1 1 \vdash \sqcup \sqcup q_2^1 1 \vdash \sqcup q_3 \vdash \sqcup \sqcup q_0 \vdash \sqcup \sqcup \sqcup q_A$$

1In the lecture I used the notation $[q_i, a]$ for this state, which perhaps conveys more clearly the fact that the symbol $a$ is a component of the state — i.e., that the state “remembers” $a$ — but $q_i^a$ is shorter and typographically better looking!