CSCC63

Hamiltonian cycle problem
Instance: \( \langle G \rangle \), where \( G \) is a directed graph

Question: Does \( G \) have a Hamiltonian cycle

Hamiltonian cycle: Visits every node exactly once.

Prove:

Vertex Cover \( \leq_{p}^{m} \) Directed Hamiltonian Cycle

Given \( \langle G, b \rangle \) where \( G \) is a directed graph and \( b \) is a positive integer, construct directed graph \( G_D \) s.t.

\[
\text{\( G \) has a vertex cover of size } b
\]

\[\iff\]

\[G_D\text{ has a directed Hamiltonian cycle}\]
A graph $G$ with vertices 1, 2, 3, and 4.
nodes of $G_D$ corresponding to node 1 of $G$. 
nodes of $G_D$ corresponding to node 2 of $G$.
nodes of $G_D$ corresponding to node 3 of $G$.
nodes of $G_D$ corresponding to node 4 of $G$
nodes of $G_D$ corresponding to edge 12 of $G$
nodes of $G_D$ corresponding to edge 13 of $G$
nodes of $G_D$ corresponding to edge 14 of $G$
nodes of $G_D$ corresponding to edge 23 of $G$
nodes of $G_D$ corresponding to edge 34 of $G$
nodes of $G_D$ corresponding to potential cover of size 2
The entire directed graph $G_D$ constructed from $G$. 

The undirected graph $G$. 

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not visited yet
Hamiltonian path of $G_D$ corresponding to vertex cover $\{1,3\}$ of $G$. 