

# CSCC63

## Hamiltonian cycle problem

Instance:  $\langle G \rangle$ , where  $G$  is a directed graph  
Question: Does  $G$  have a Hamiltonian cycle

Hamiltonian cycle: Visits every node exactly once.

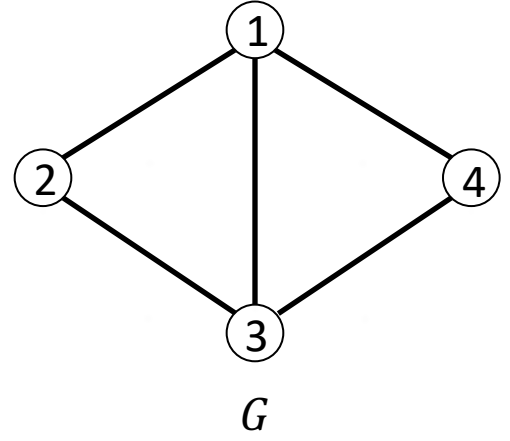
Prove:

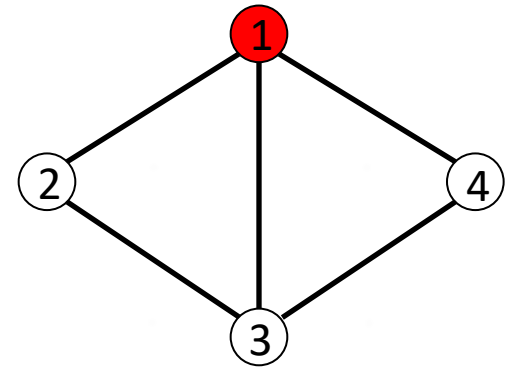
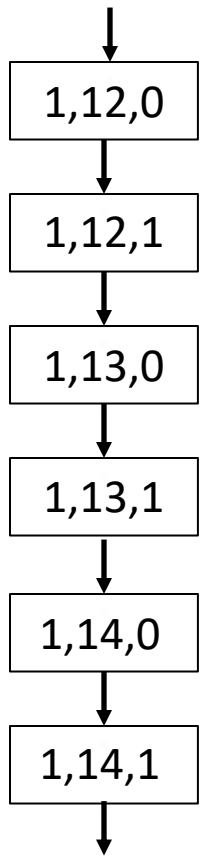
Vertex Cover  $\leq_m^p$  Directed Hamiltonian Cycle

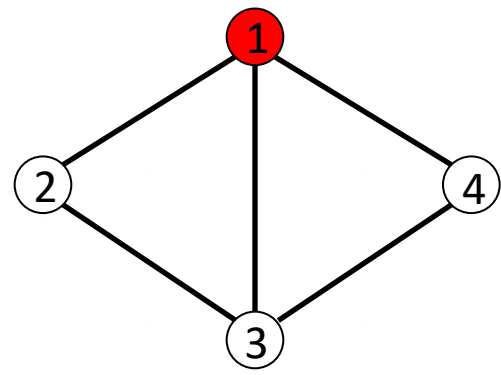
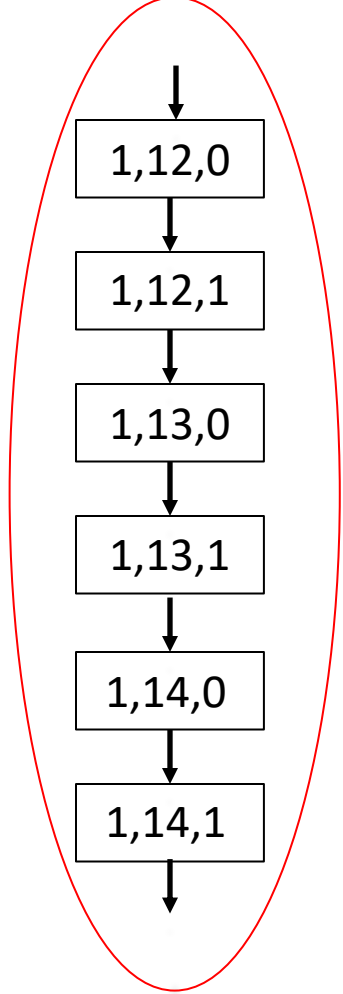
Given  $\langle G, b \rangle$  where  $G$  is a directed graph and  $b$  is a positive integer, construct directed graph  $G_D$  s.t.

$G$  has a vertex cover of size  $b$

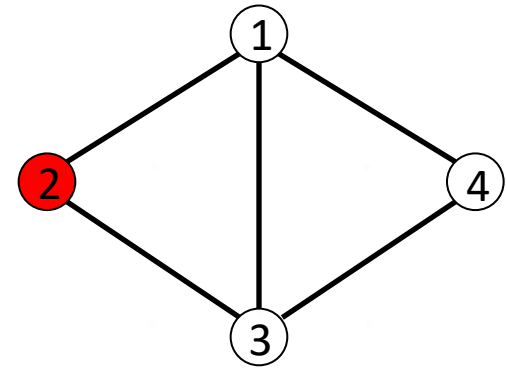
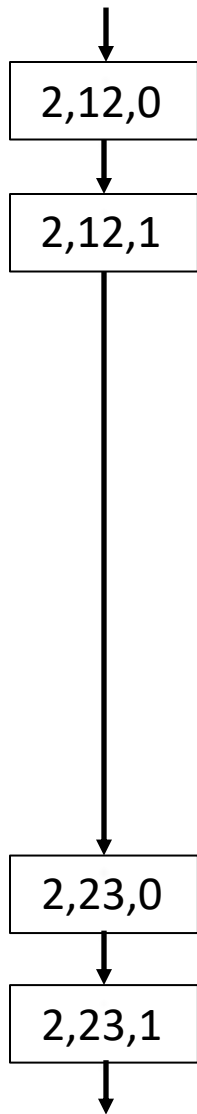
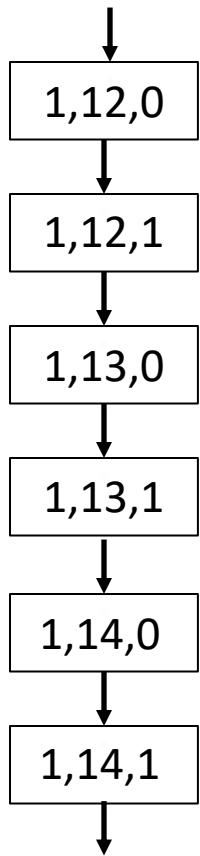
$\iff G_D$  has a directed Hamiltonian cycle

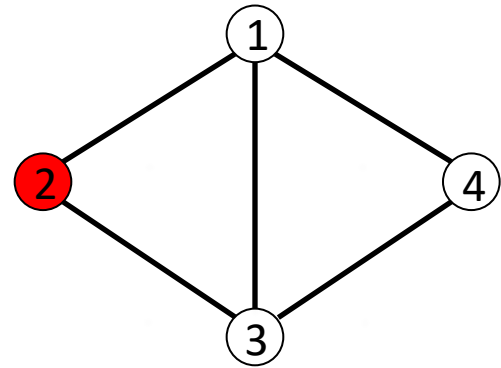
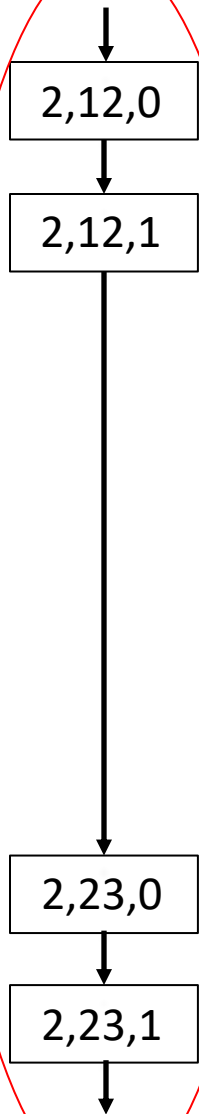
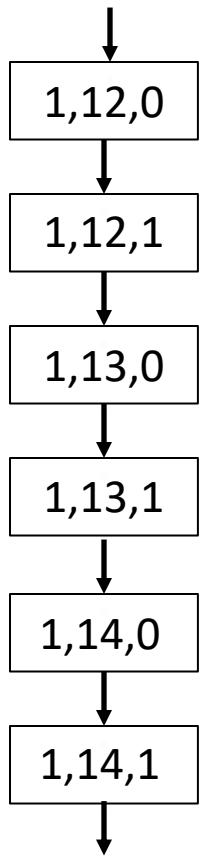




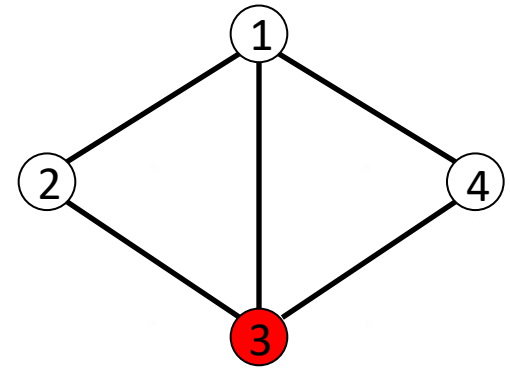
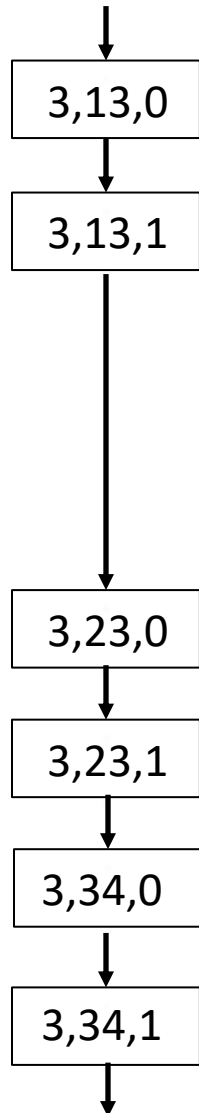
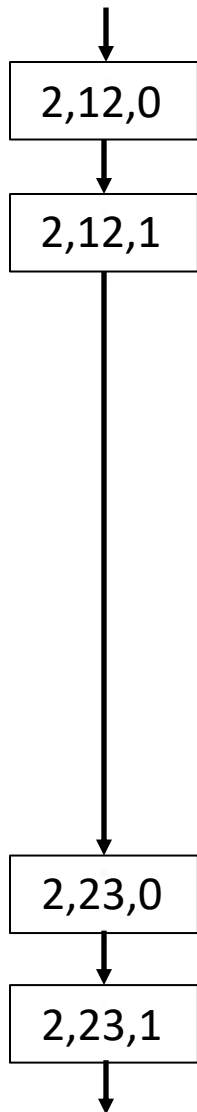
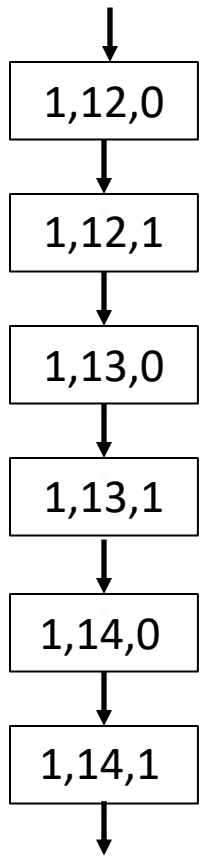


nodes of  $G_D$   
corresponding  
to node 1 of  $G$

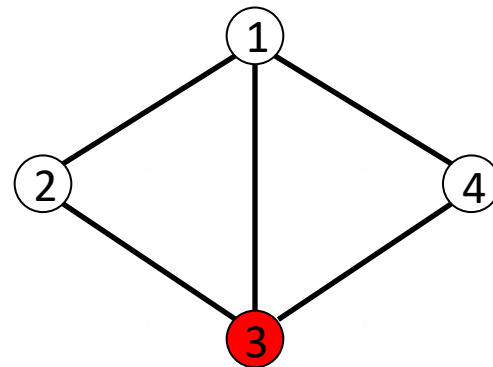
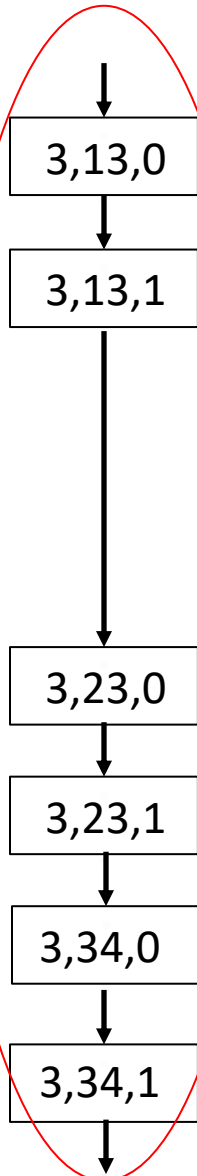
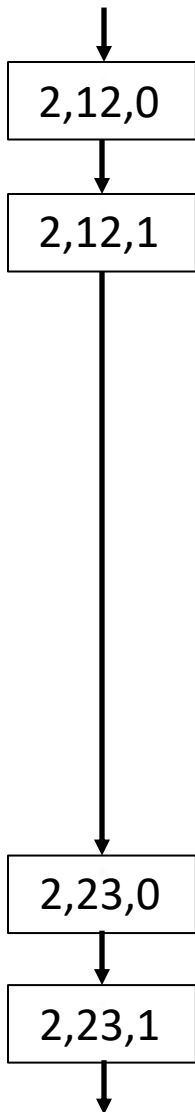
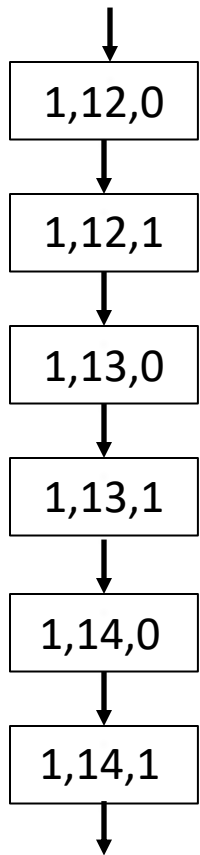




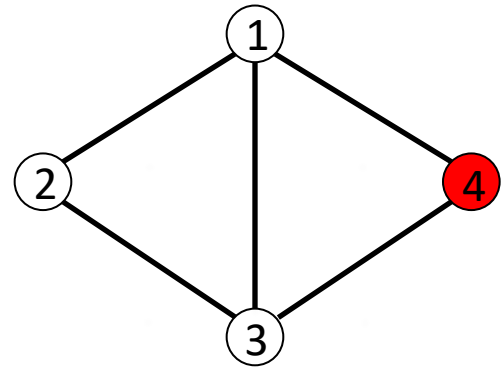
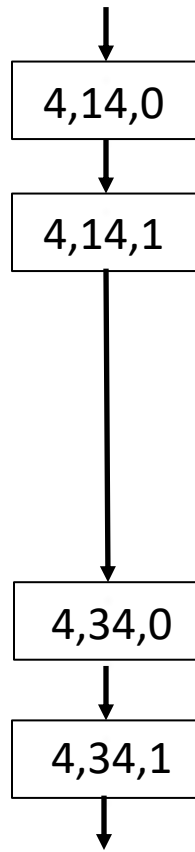
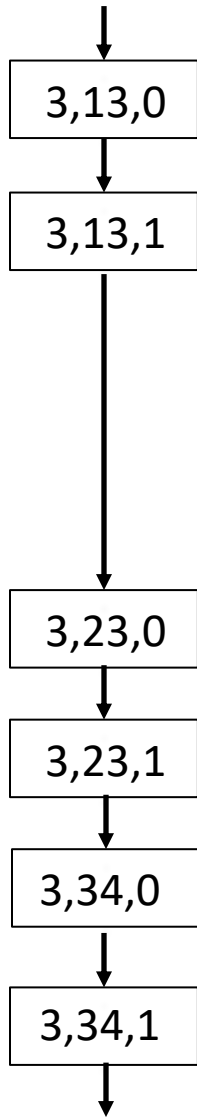
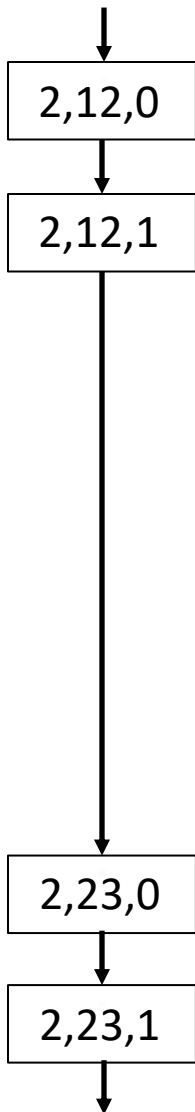
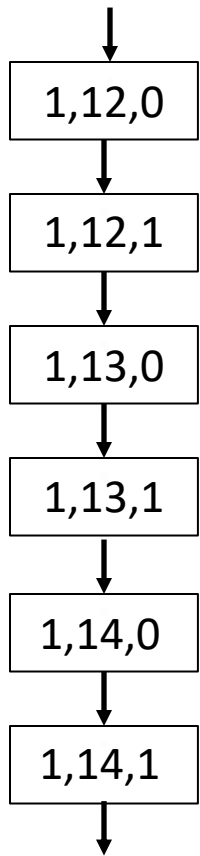
nodes of  $G_D$   
corresponding  
to node 2 of  $G$

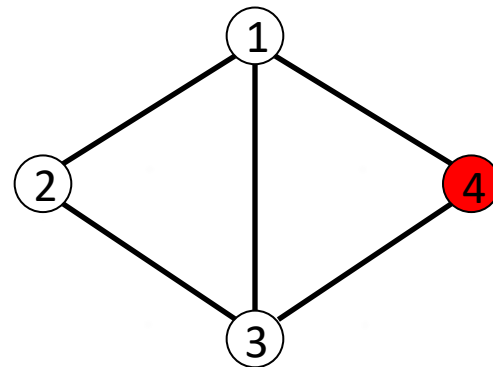
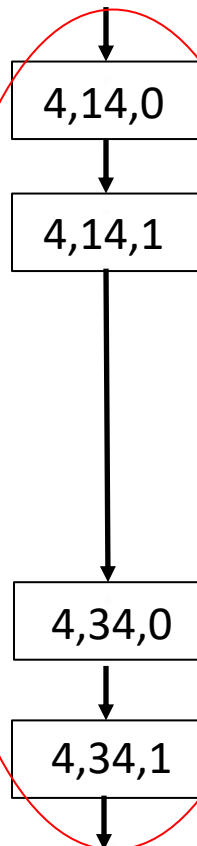
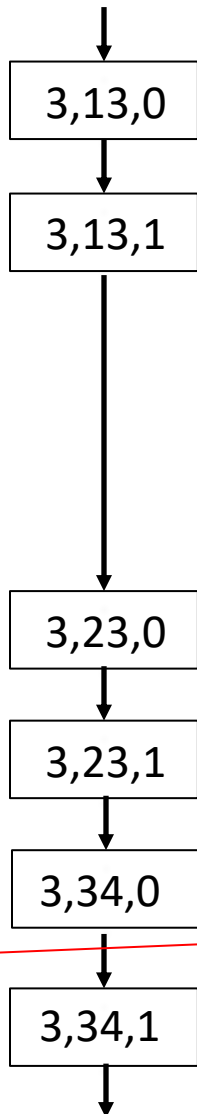
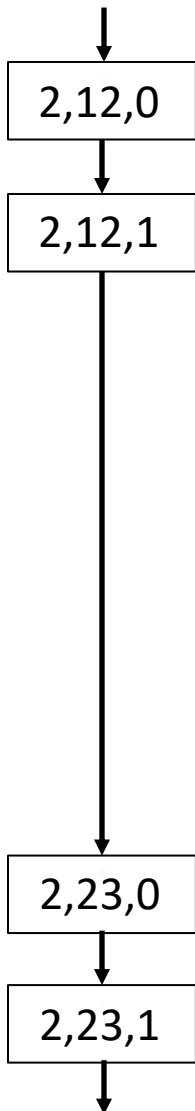
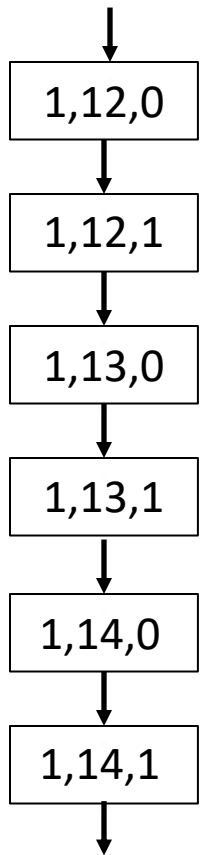




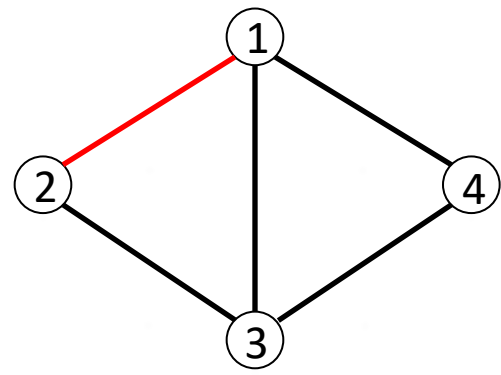
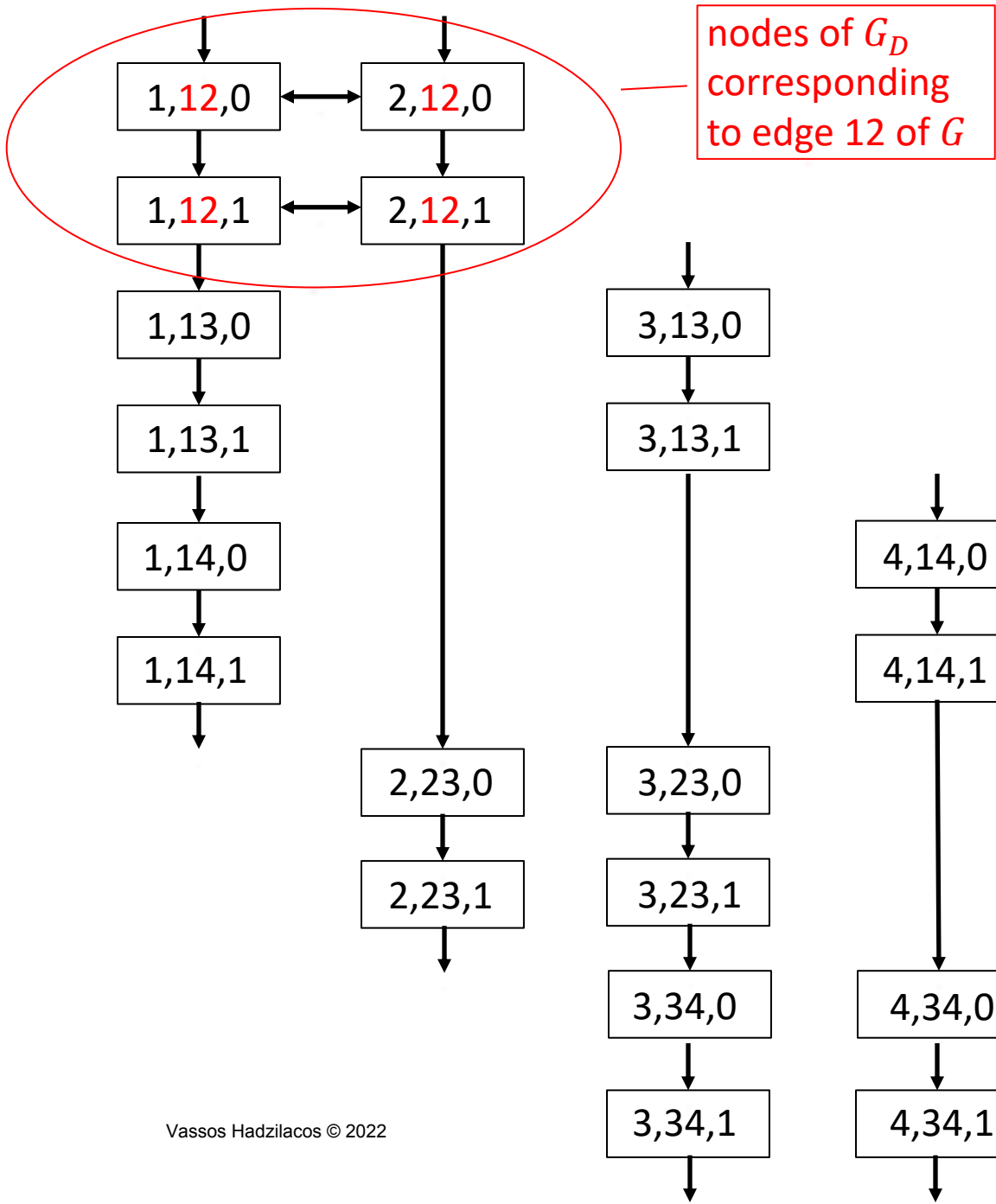


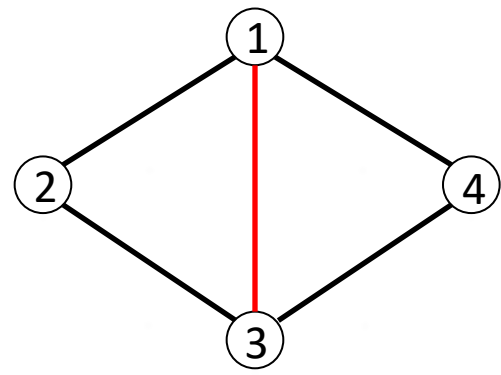
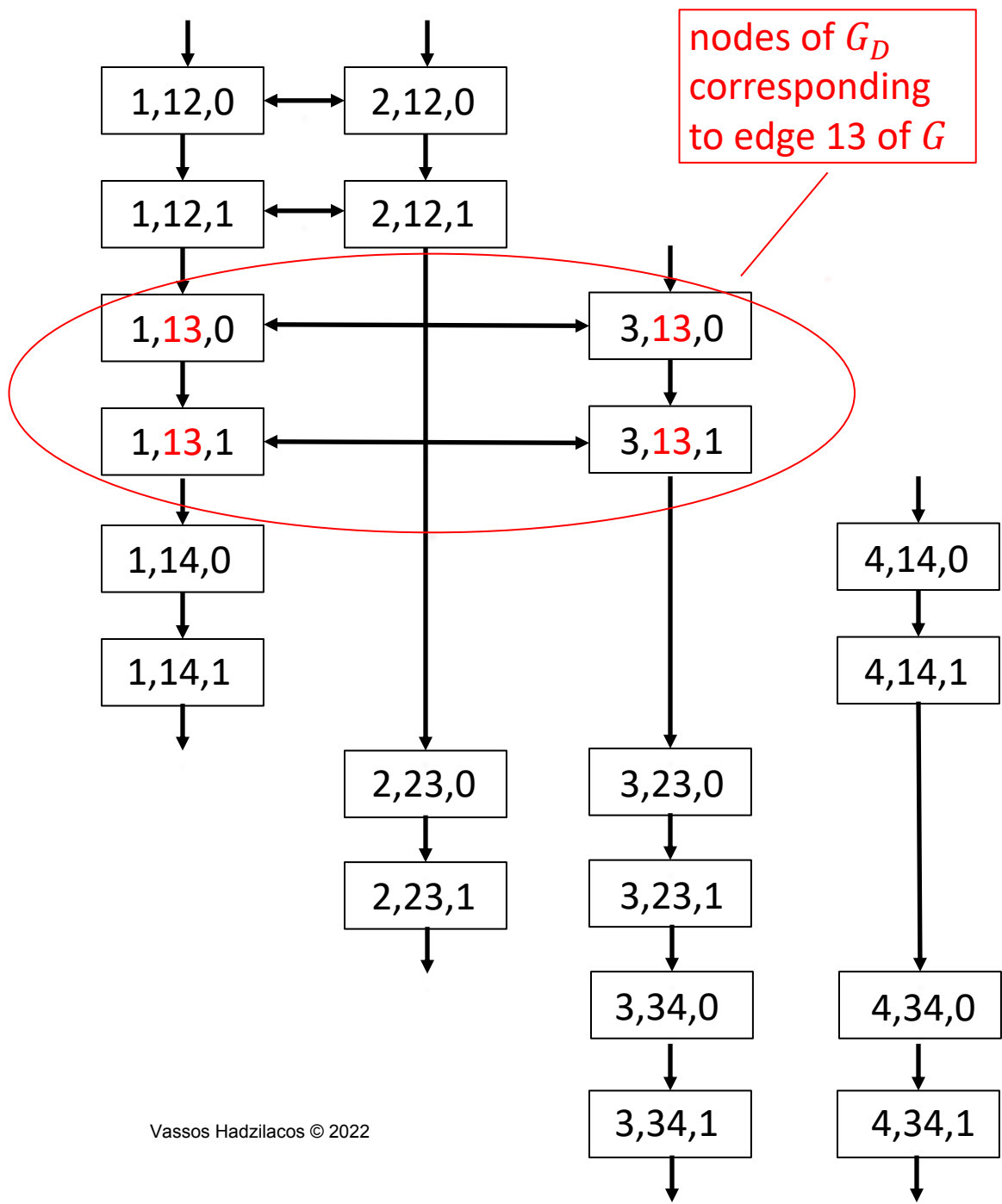
nodes of  $G_D$   
corresponding  
to node 3 of  $G$

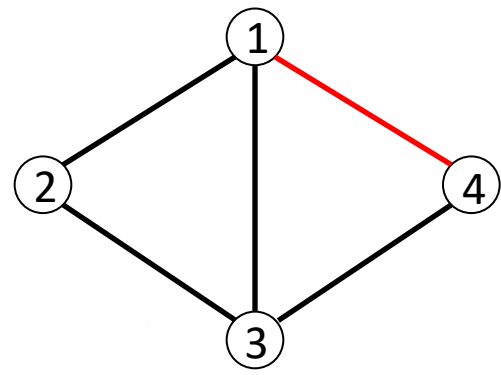
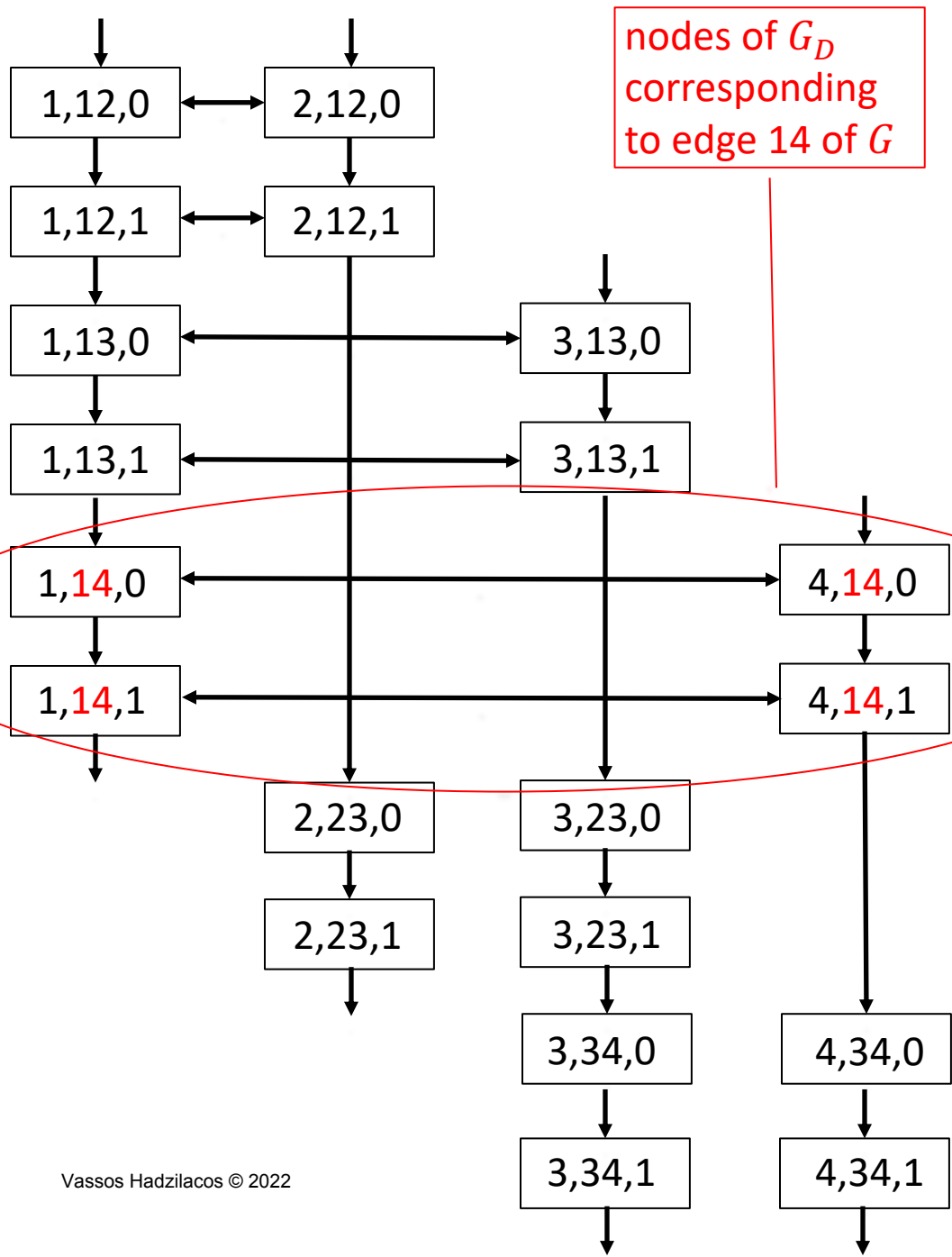


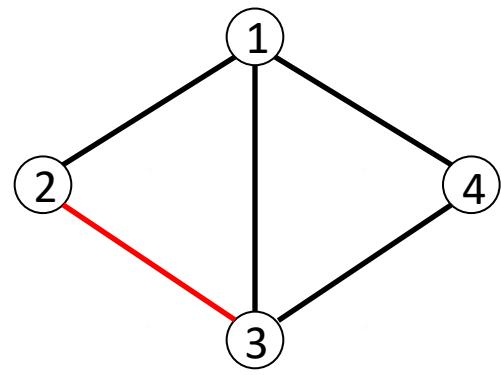
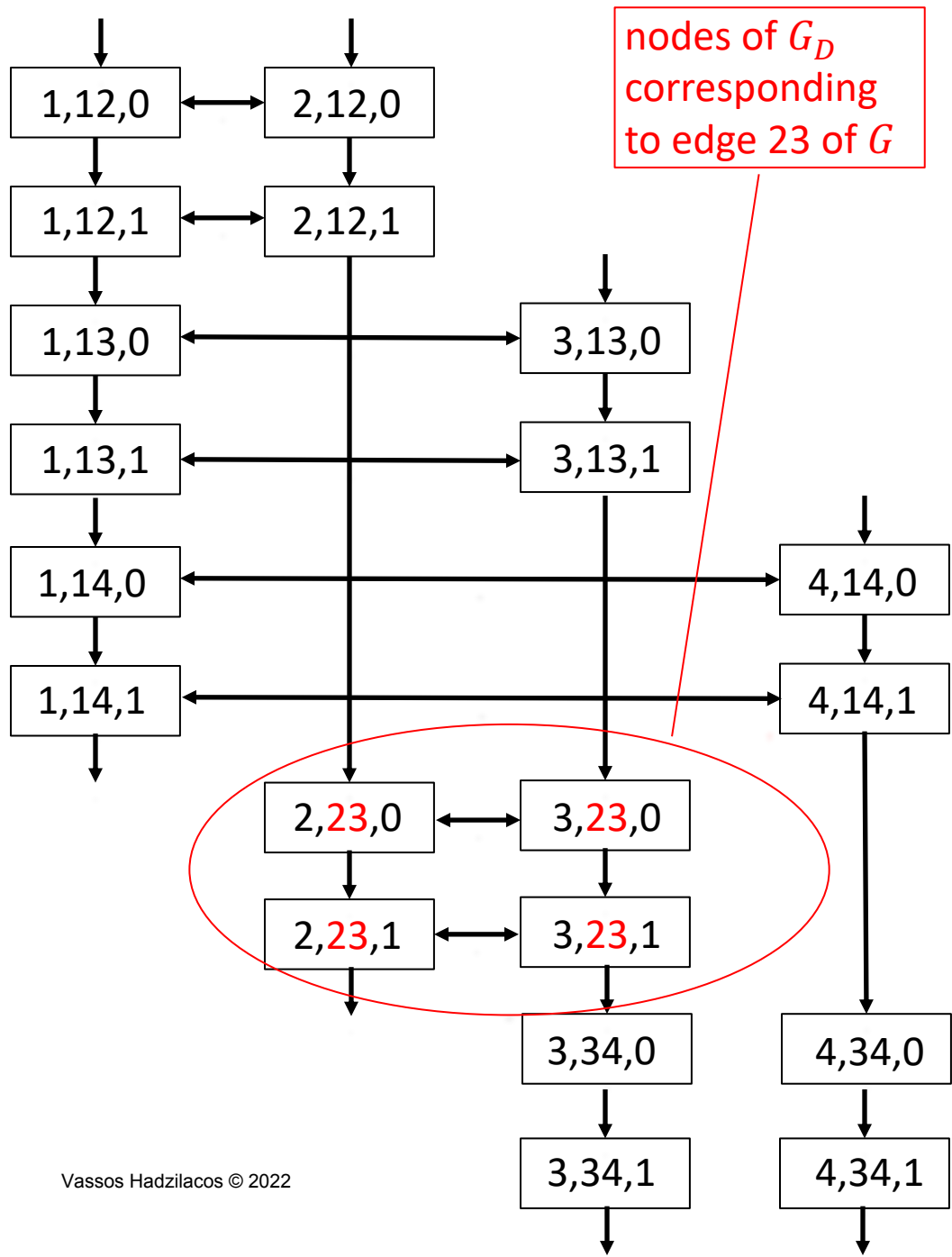


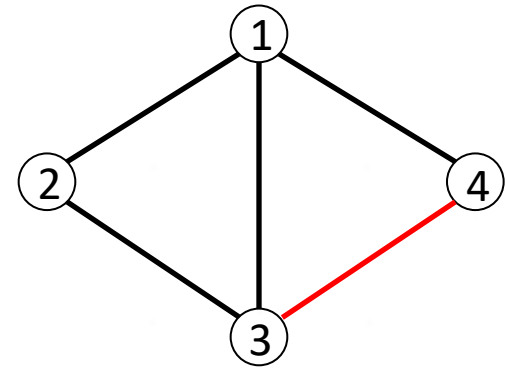
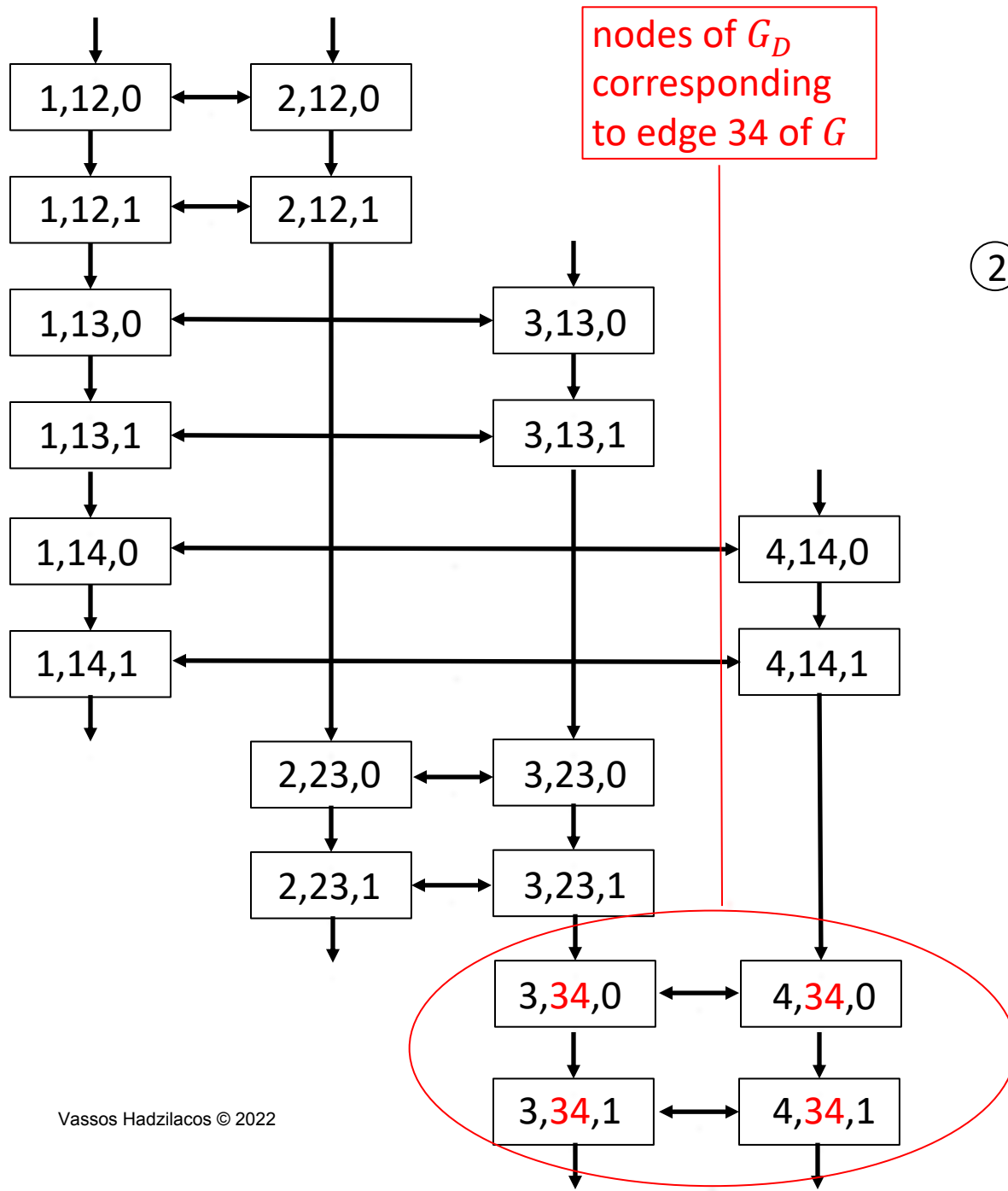
nodes of  $G_D$   
corresponding  
to node 4 of  $G$



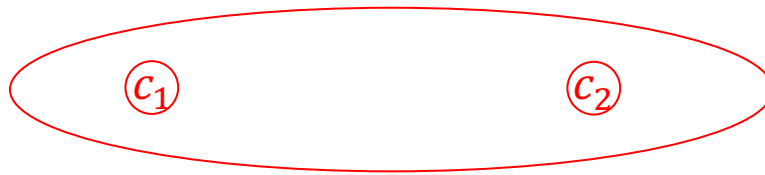




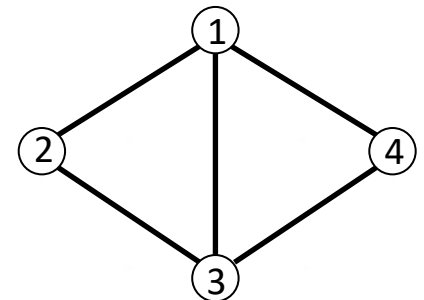
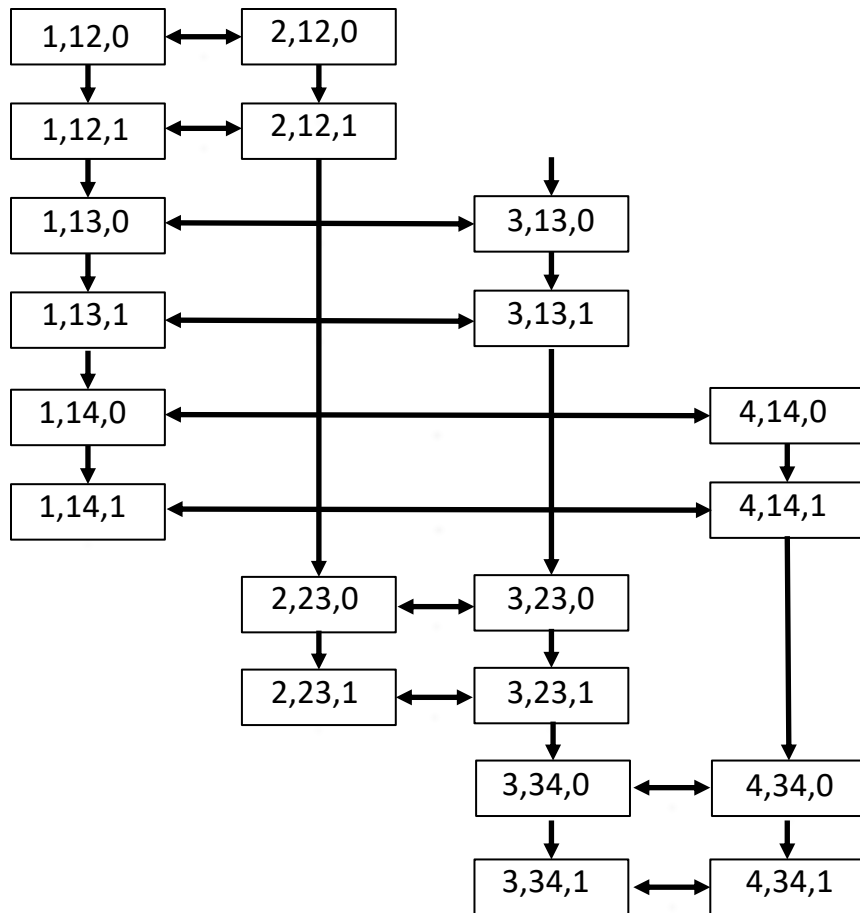


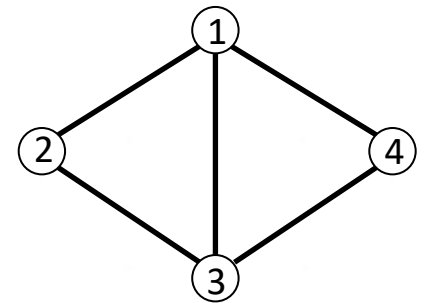
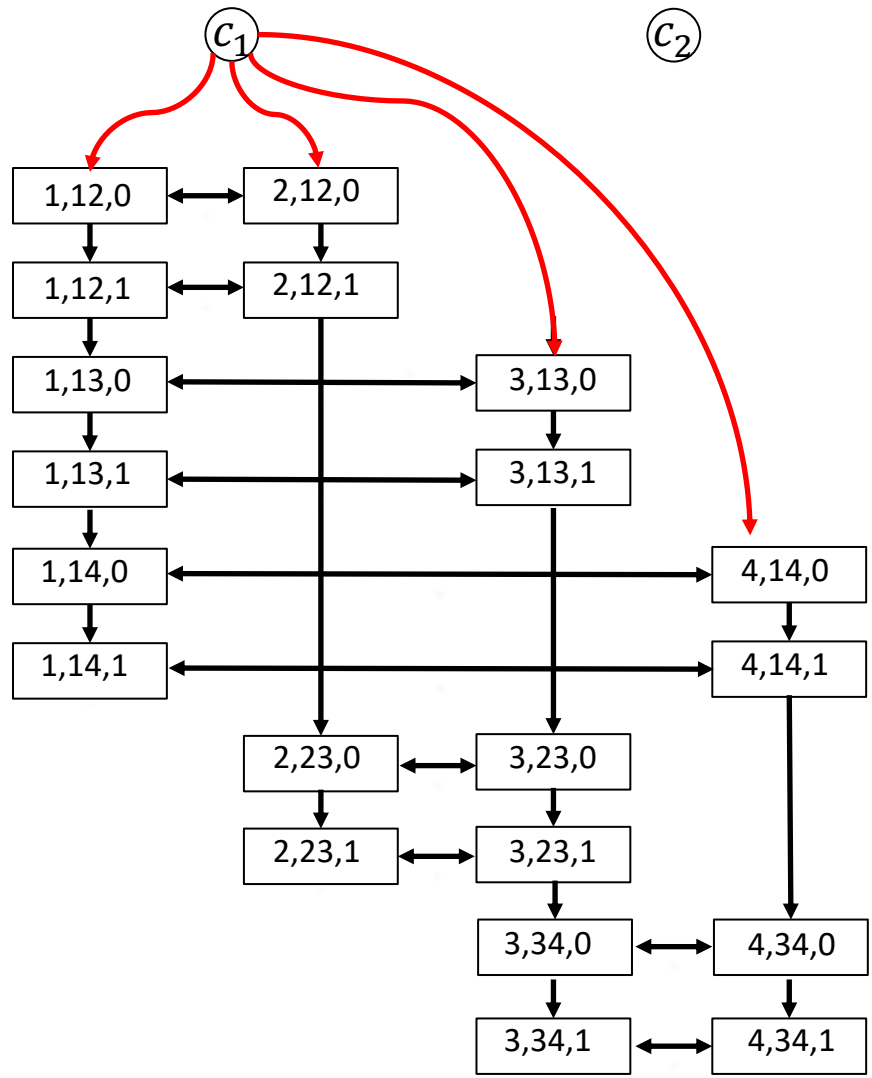


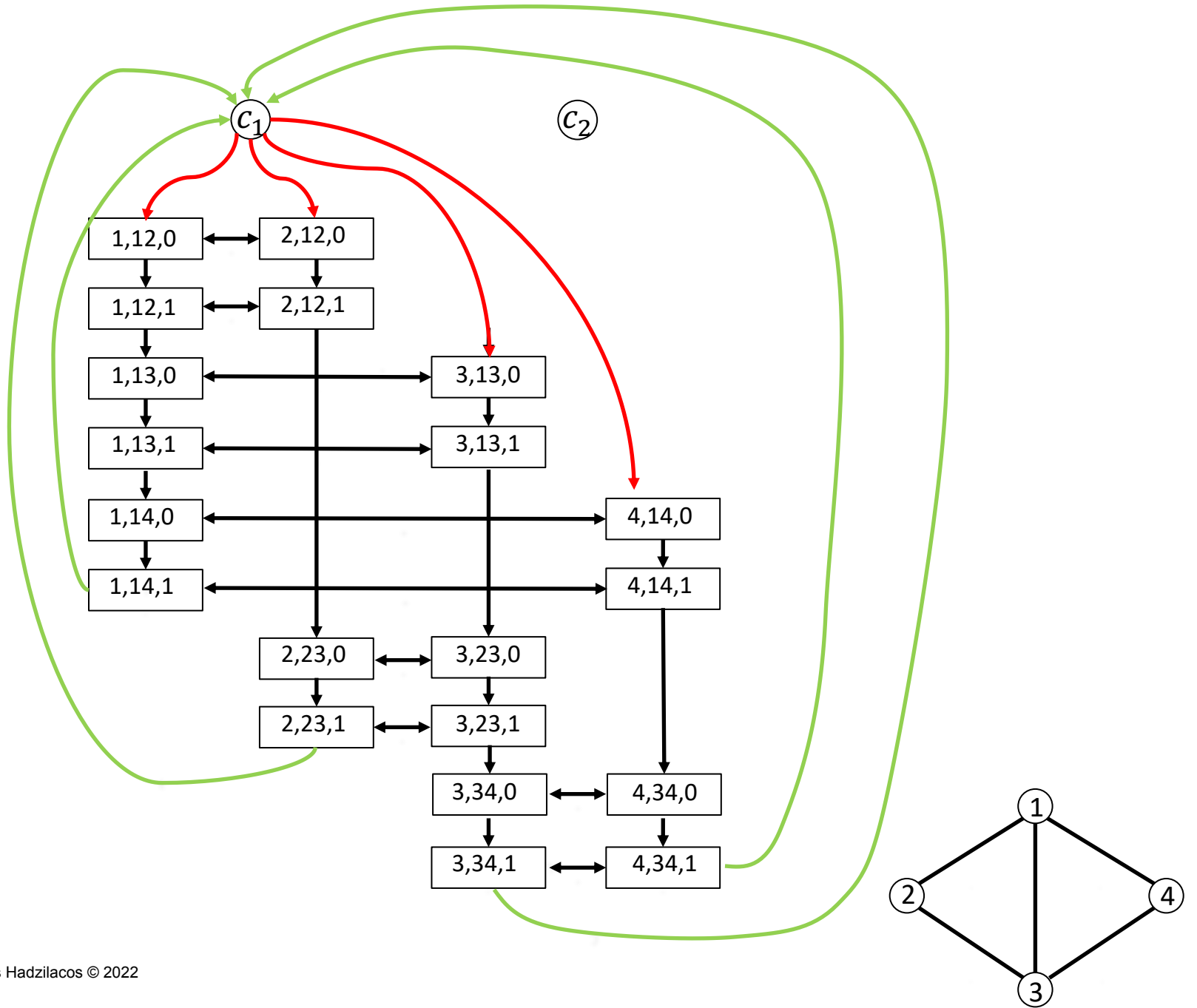


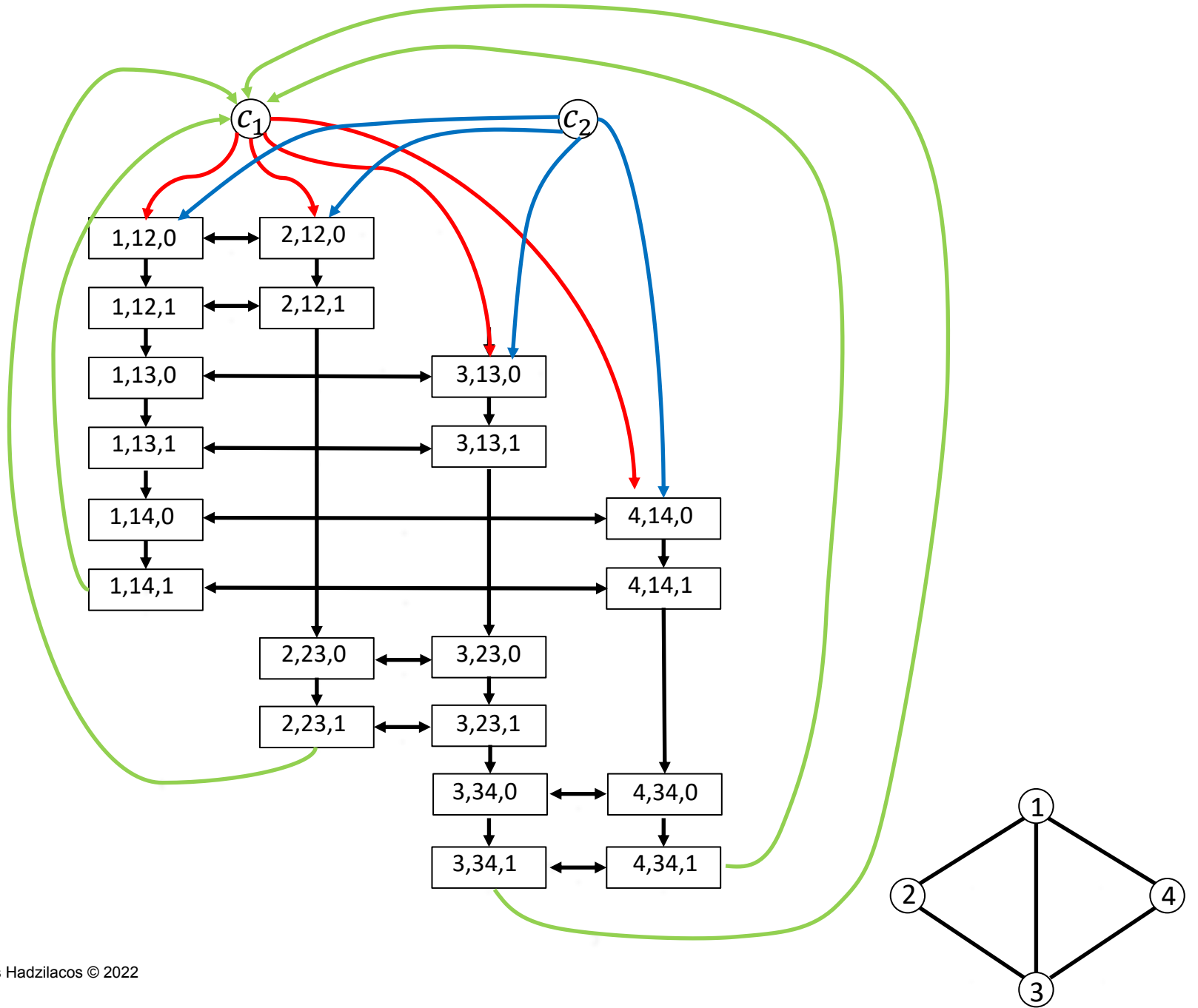


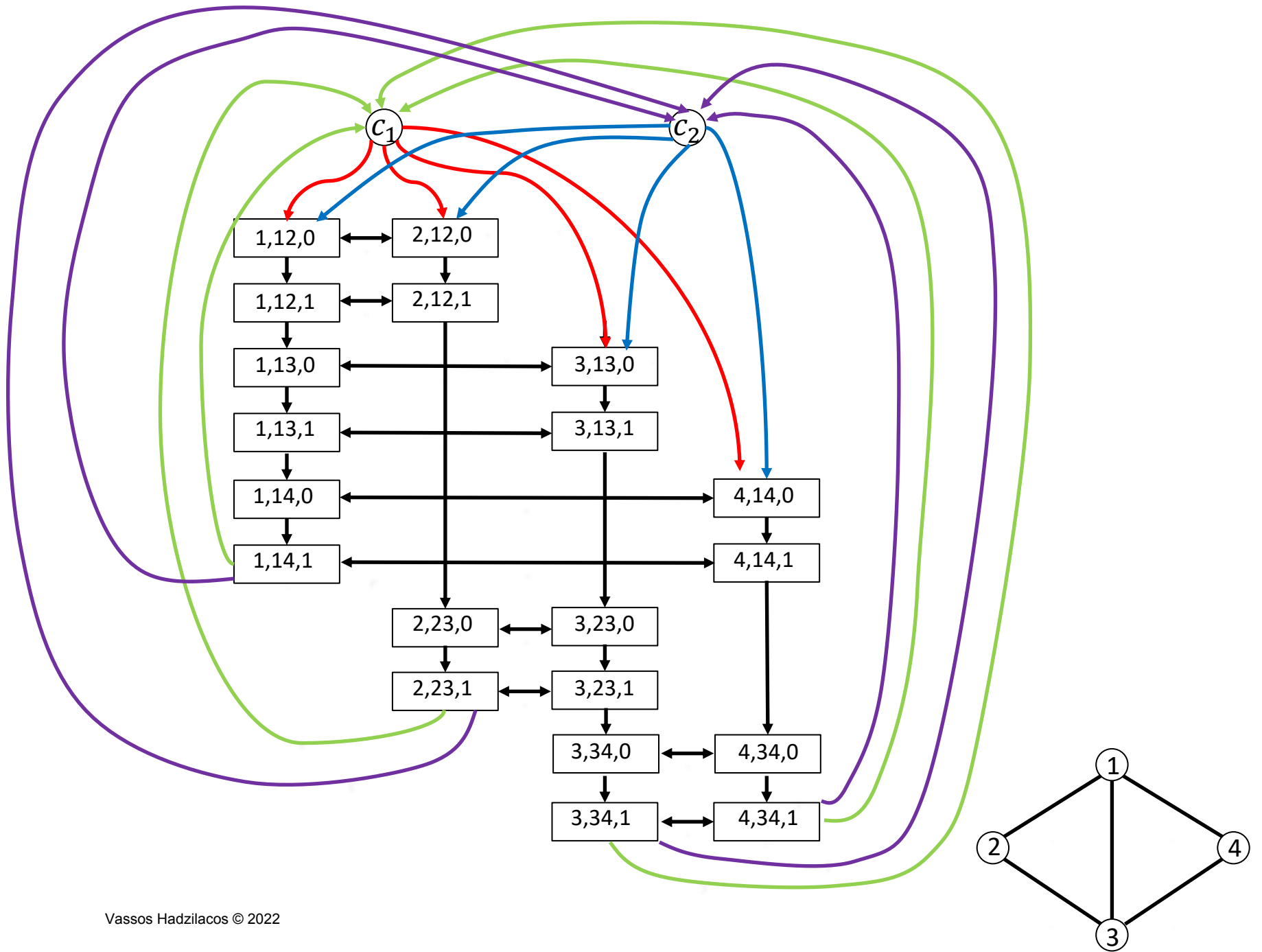
nodes of  $G_D$   
corresponding to  
potential cover of  
size 2

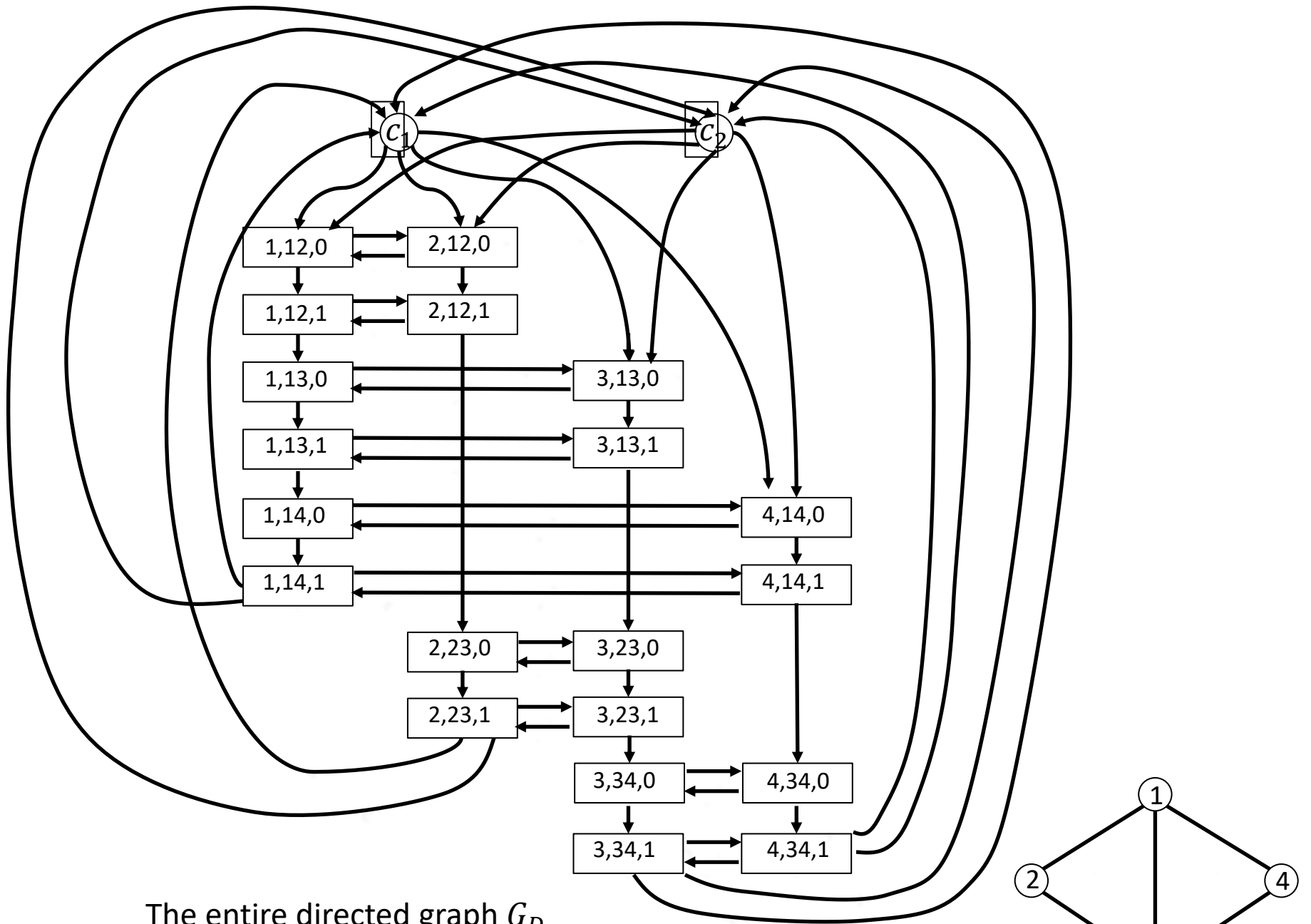






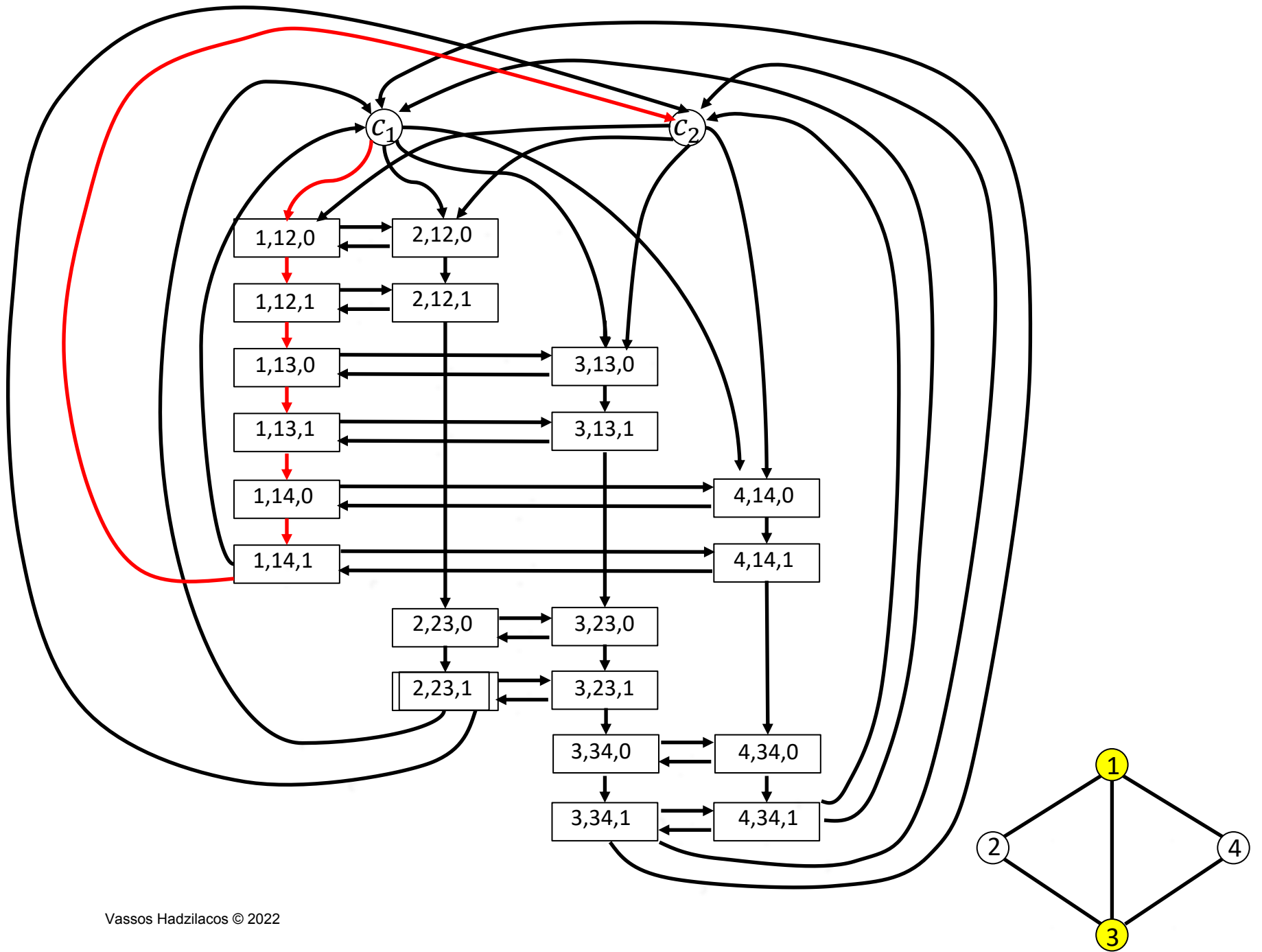


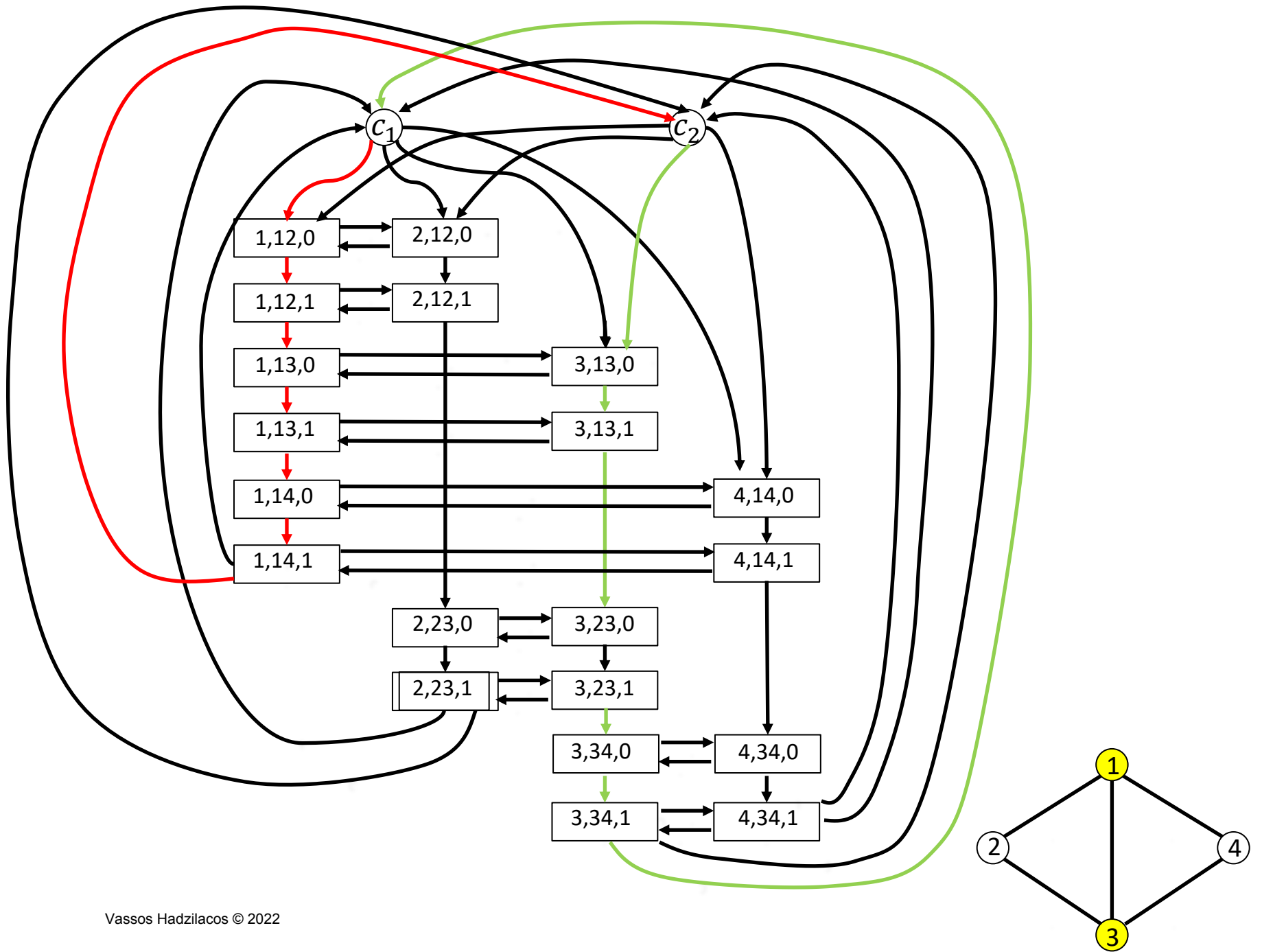




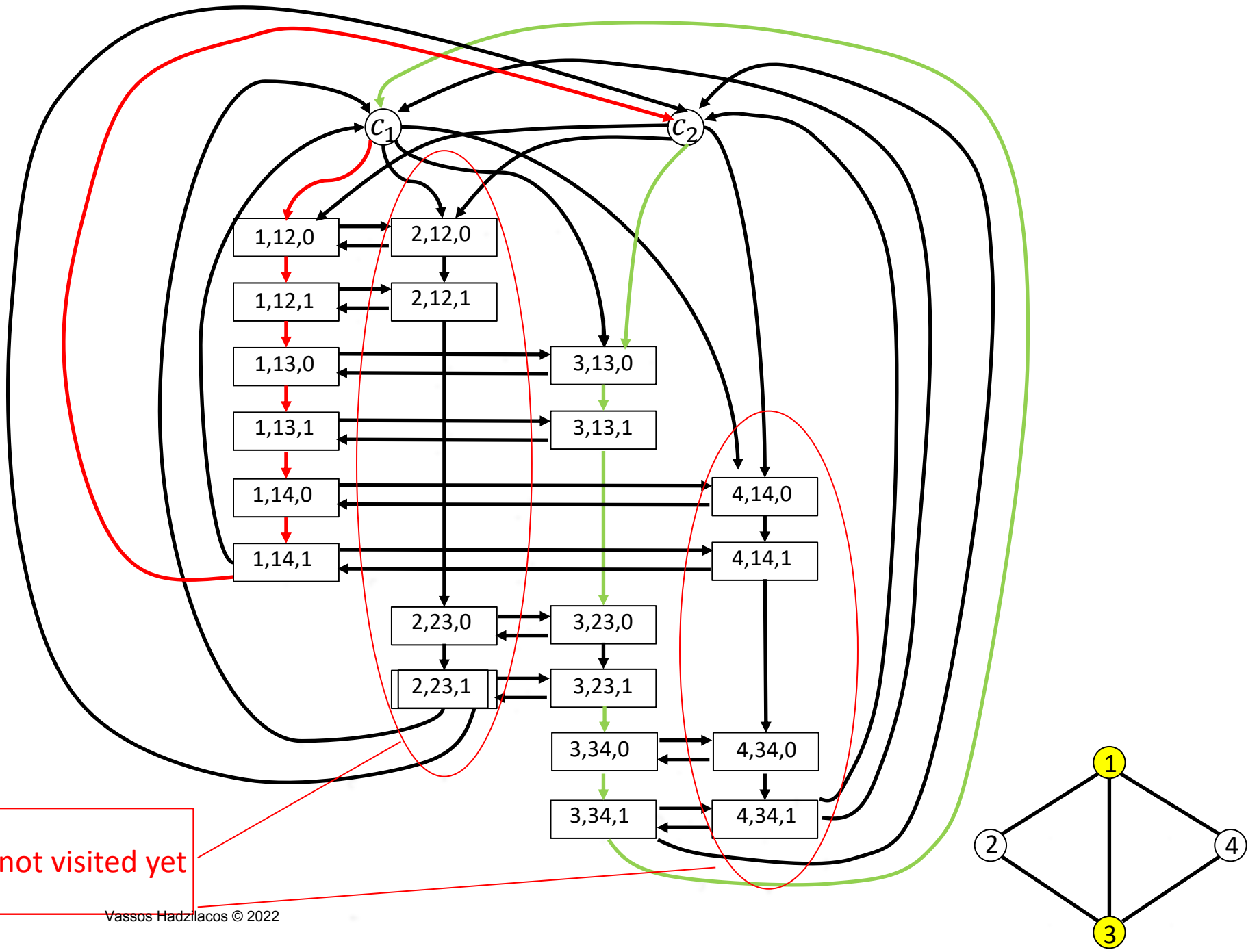
The entire directed graph  $G_D$  constructed from  $G$ .

The undirected graph  $G$ .

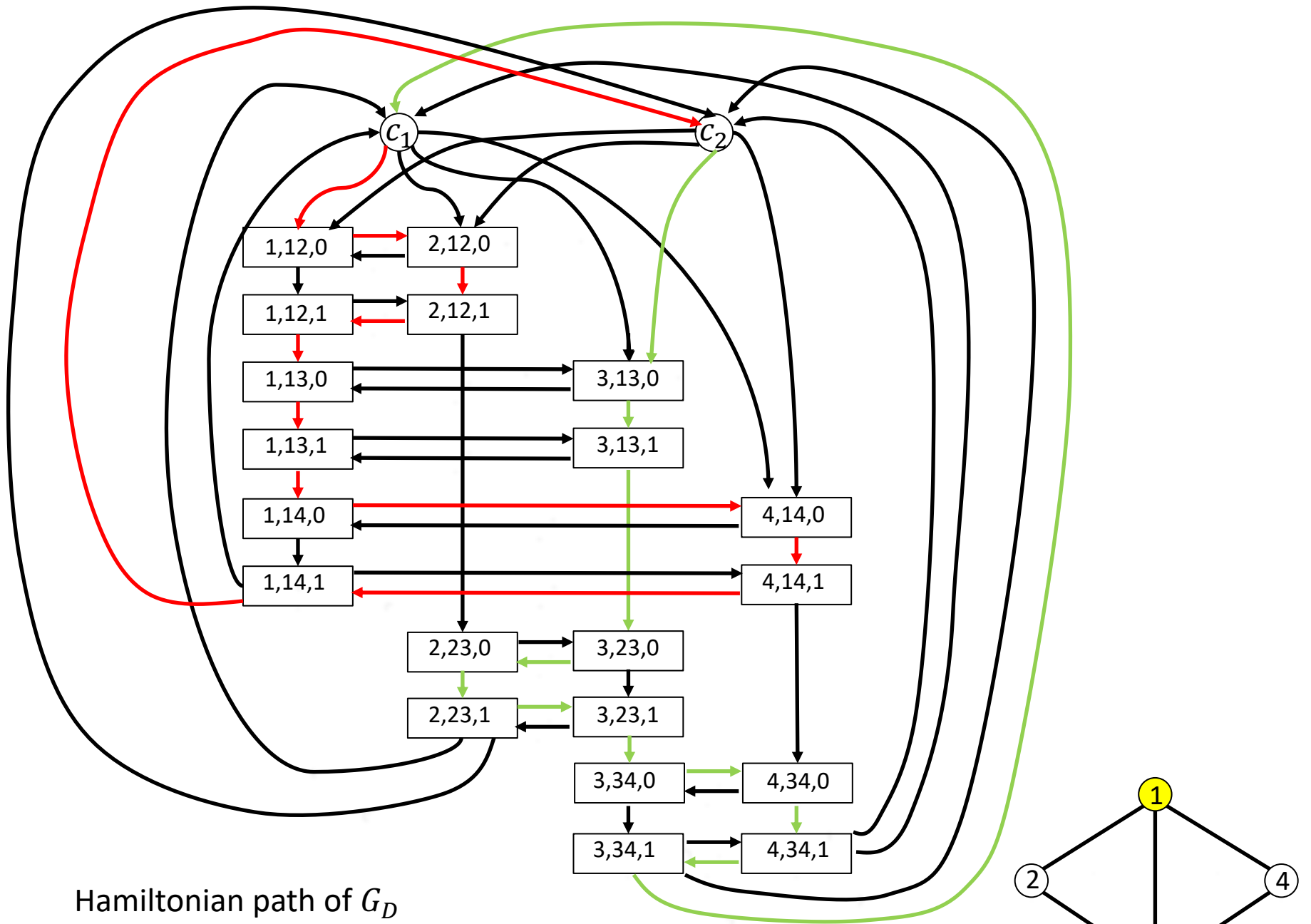








not visited yet



Hamiltonian path of  $G_D$   
 corresponding to  
 vertex cover  $\{1,3\}$  of  $G$ .

Vassos Hadzilacos © 2022

