CSCC63

Geography is PSPACE-complete

Given a directed graph G and a node s two players, Alice and Bob, alternate choosing edges that always form a simple path starting at s.

The first player unable to extend the path loses.

Instance: $\langle G, s \rangle$, where $G = (V, E)$ is a directed graph, $s \in V$.

Question: Does Alice have a winning strategy?

Winning strategy for Alice: Alice can choose the odd edges in the path so that she can always extend the path no matter how Bob choses the even edges.

$$F = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 ((\overline{x}_1 \vee \overline{x}_2 \vee x_3 \vee x_4) \wedge (\overline{x}_2 \vee \overline{x}_4) \wedge (x_2 \vee x_4 \vee \overline{x}_5))$$

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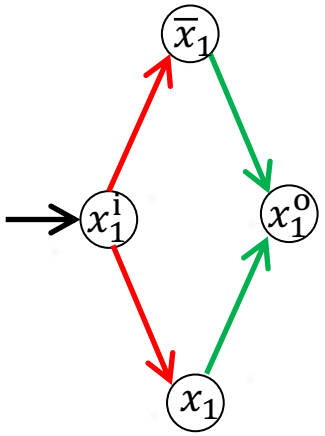


Gadget
for x_1

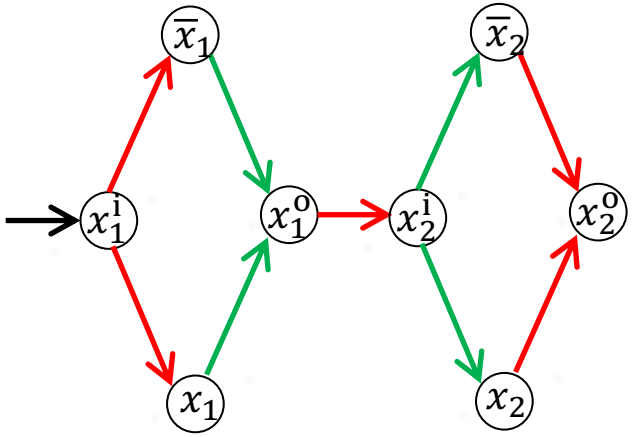
Edge chosen by Alice

Edge chosen by Bob

Path goes through x_1: x_1 set to 1 (true)

Path goes through $\overline{x}_1$: x_1 set to 0 (false)

$$F = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 ((\overline{x}_1 \vee \overline{x}_2 \vee x_3 \vee x_4) \wedge (\overline{x}_2 \vee \overline{x}_4) \wedge (x_2 \vee x_4 \vee \overline{x}_5))$$



Gadget for x_1

Gadget for x_2

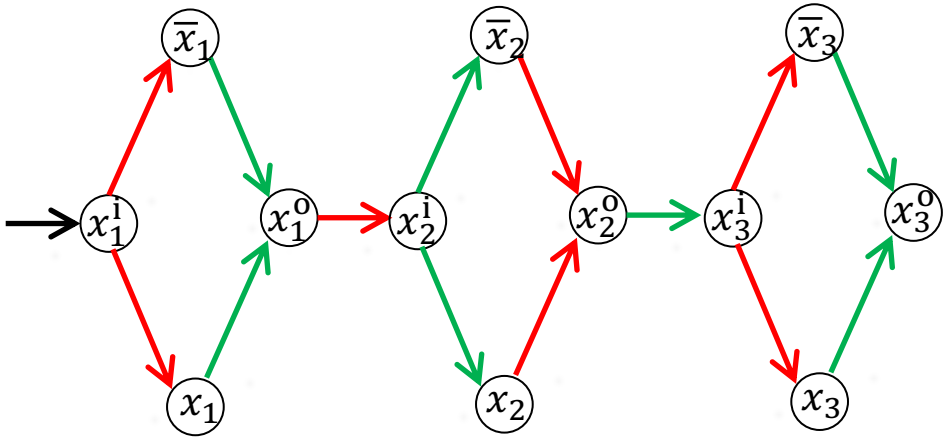
Edge chosen by Alice

Edge chosen by Bob

Path goes through x_i: x_i set to 1 (true)

Path goes through $\overline{x}_i$: x_i set to 0 (false)

$$F = \exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 ((\overline{x}_1 \vee \overline{x}_2 \vee x_3 \vee x_4) \wedge (\overline{x}_2 \vee \overline{x}_4) \wedge (x_2 \vee x_4 \vee \overline{x}_5))$$



Gadget for x_1 | Gadget for x_2 | Gadget for x_3

Edge chosen by Alice

Edge chosen by Bob

Path goes through x_i: x_i set to 1 (true)

Path goes through $\overline{x}_i$: x_i set to 0 (false)

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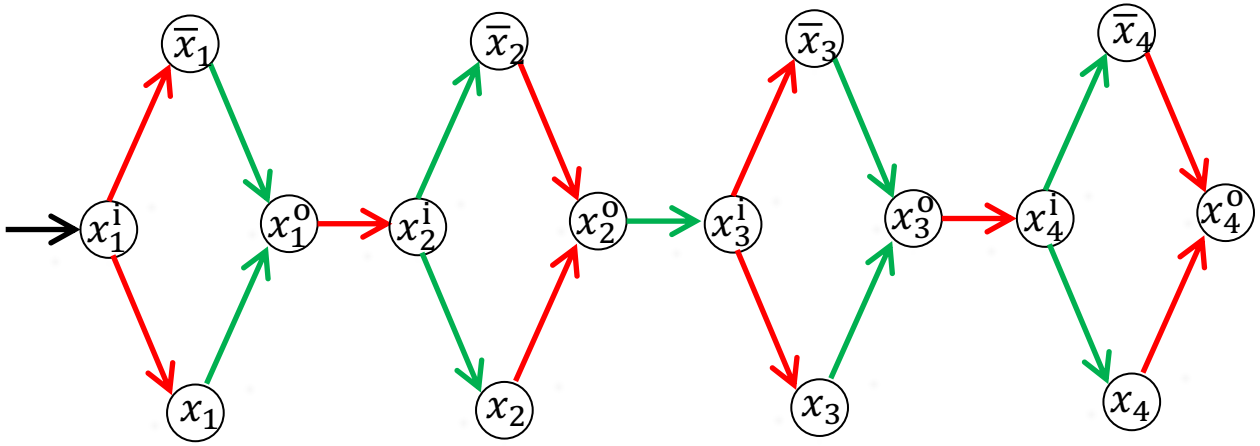

Gadget for x_1 | Gadget for x_2 | Gadget for x_3 | Gadget for x_4

Edge chosen by Player I

Edge chosen by Player II

Path goes through x_i: x_i set to 1 (true)

Path goes through $\overline{x}_i$: x_i set to 0 (false)

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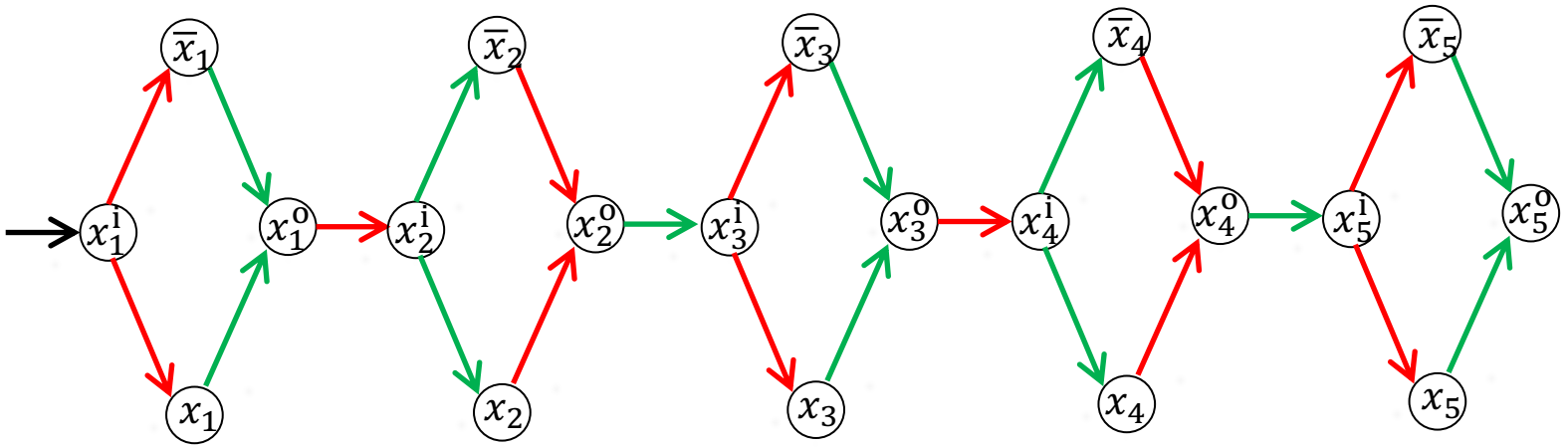

Gadget for x_1 | Gadget for x_2 | Gadget for x_3 | Gadget for x_4 | Gadget for x_5

Edge chosen by Player I

Edge chosen by Player II

Path goes through x_i: x_i set to 1 (true)

Path goes through $\overline{x}_i$: x_i set to 0 (false)

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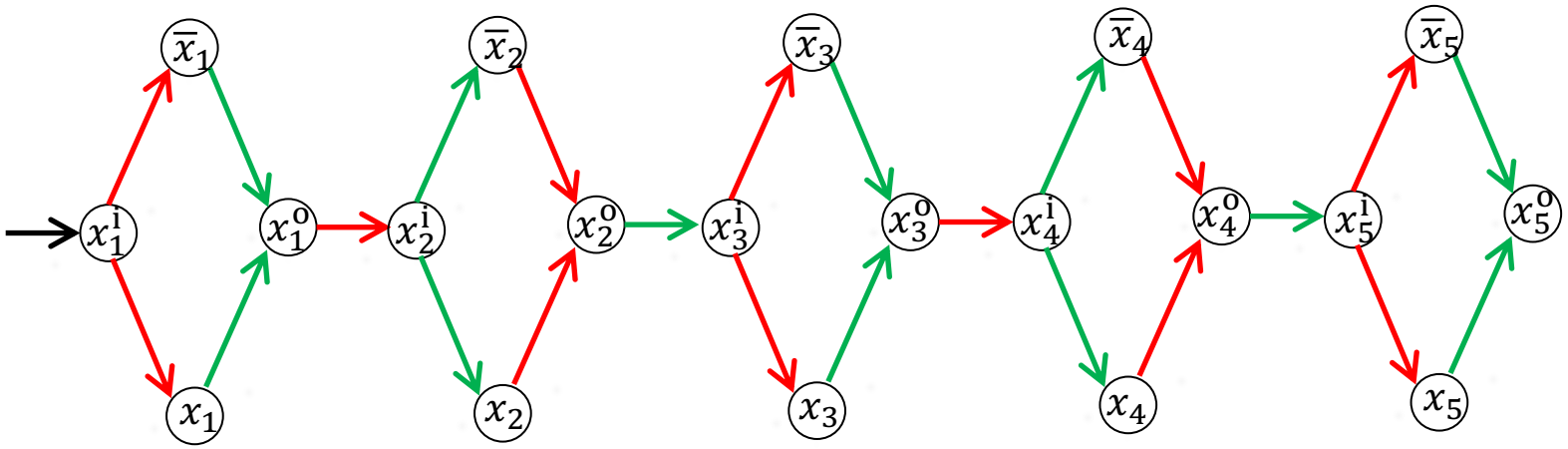

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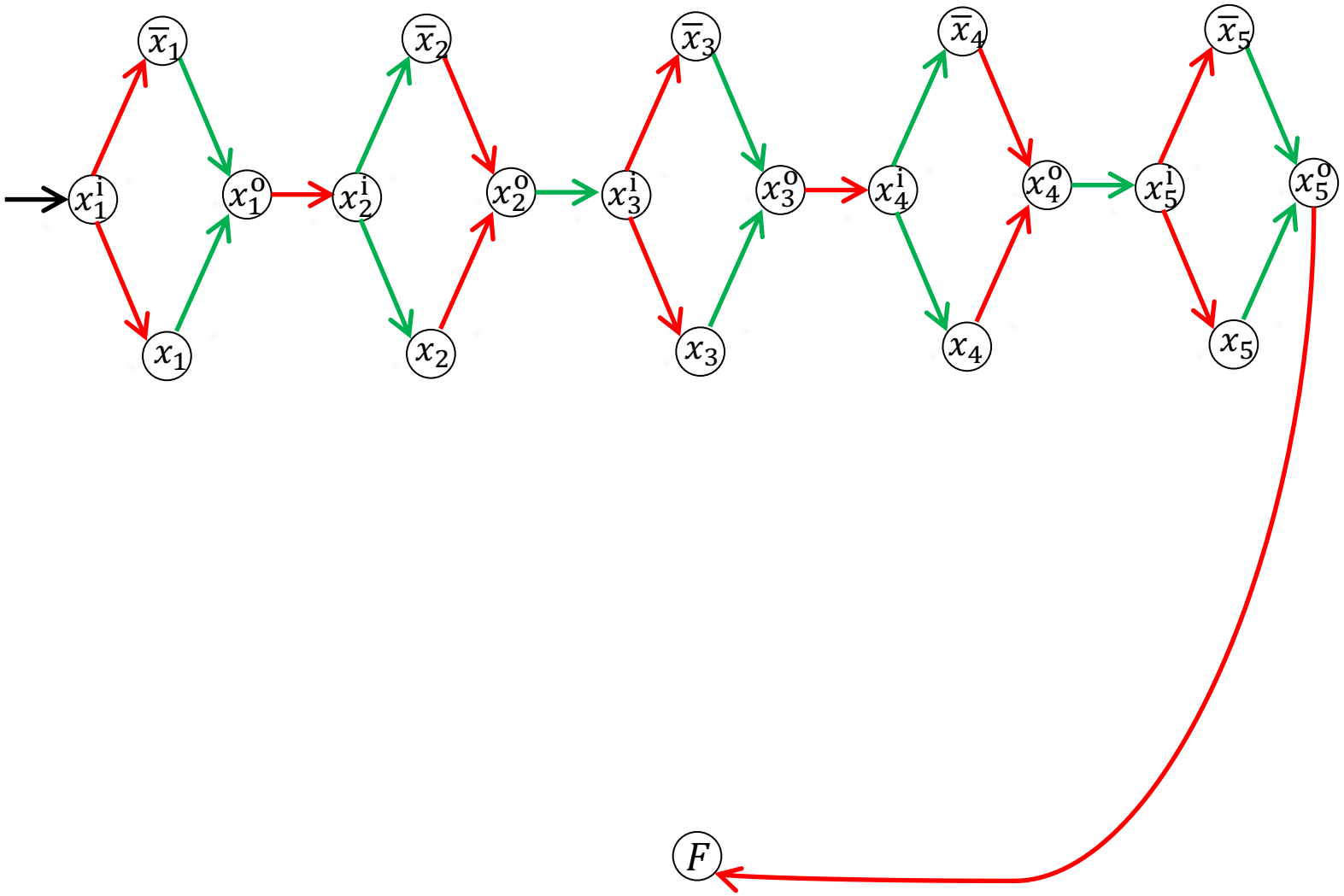

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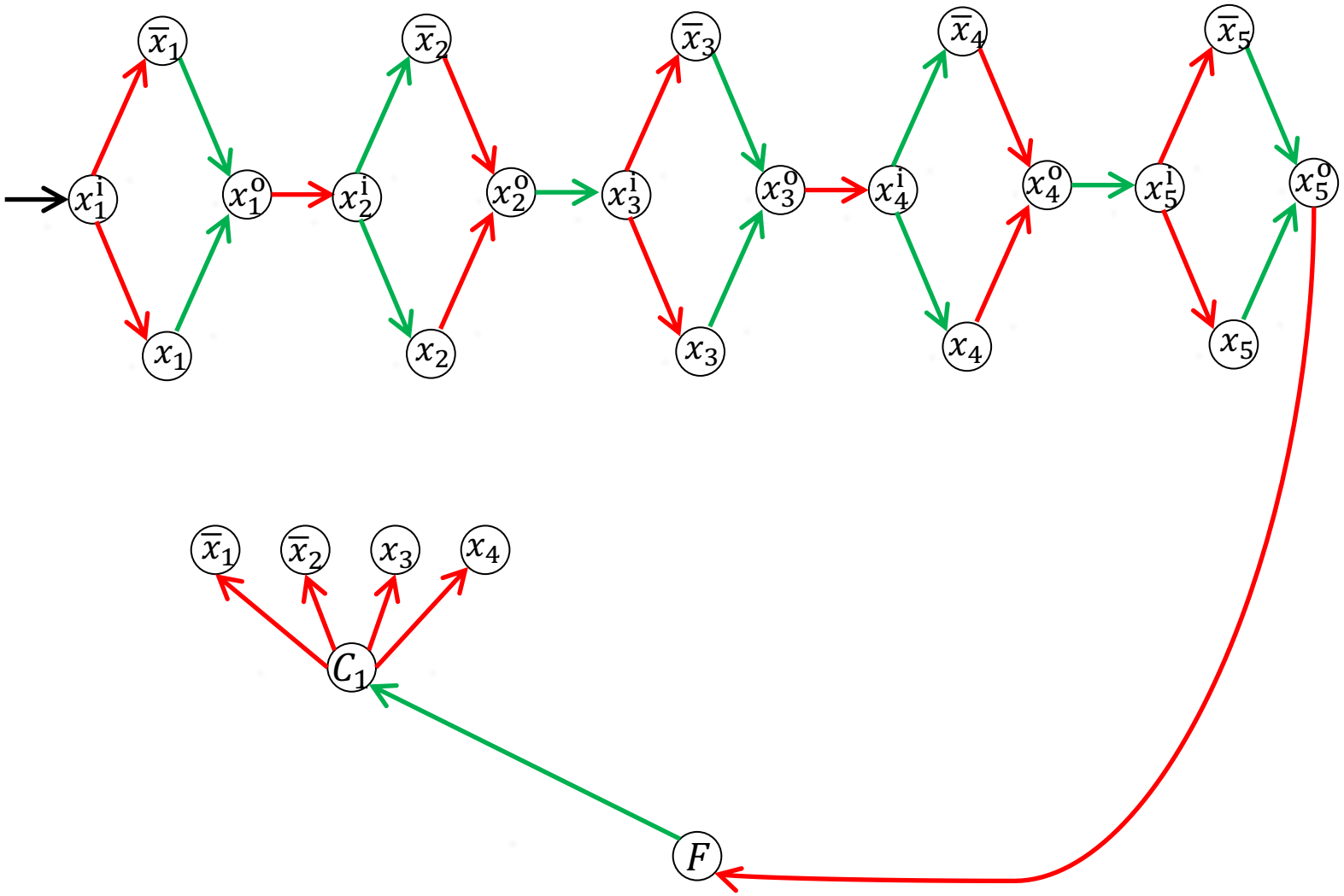
C_1



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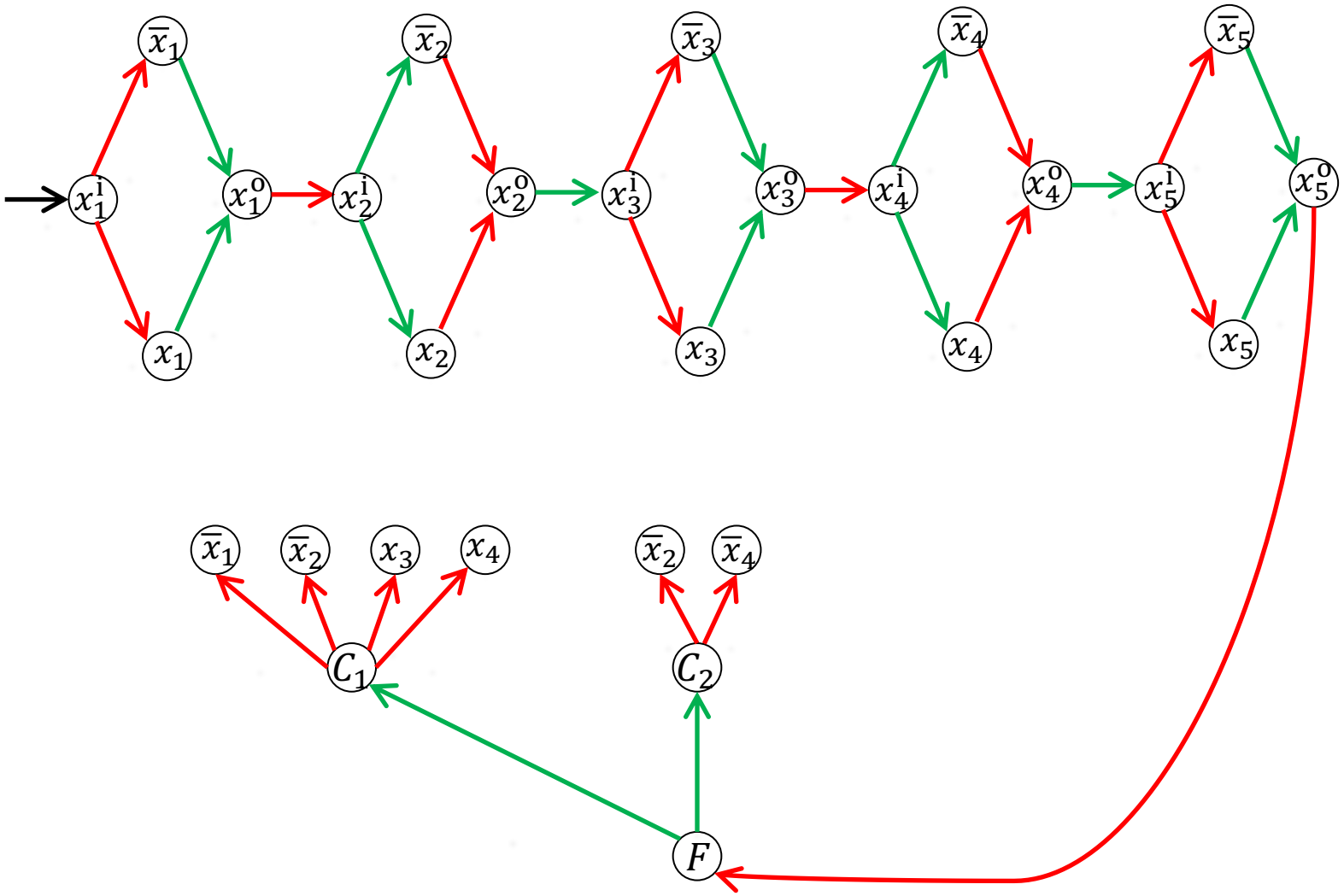
C_1 C_2



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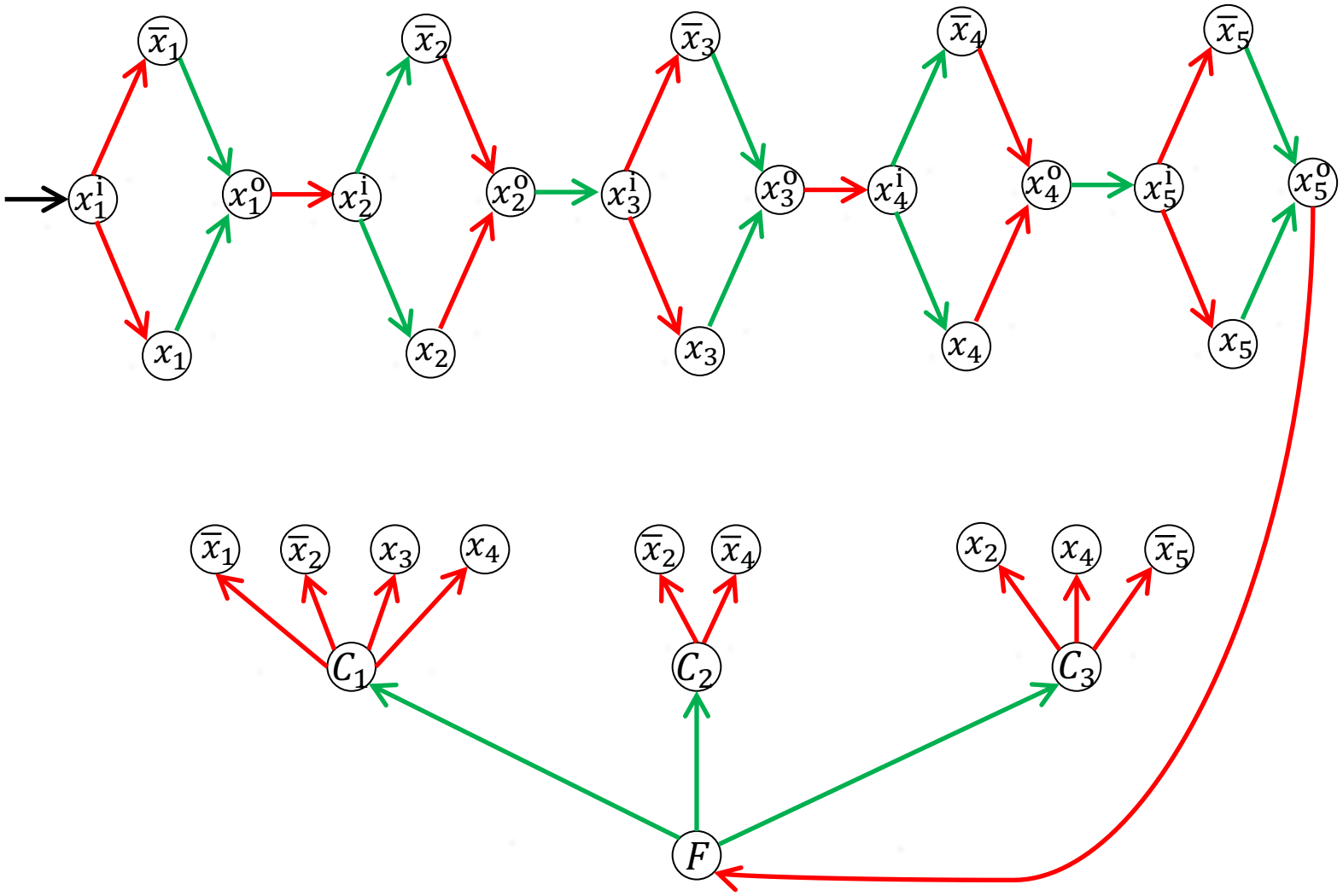
C_1 C_2 C_3



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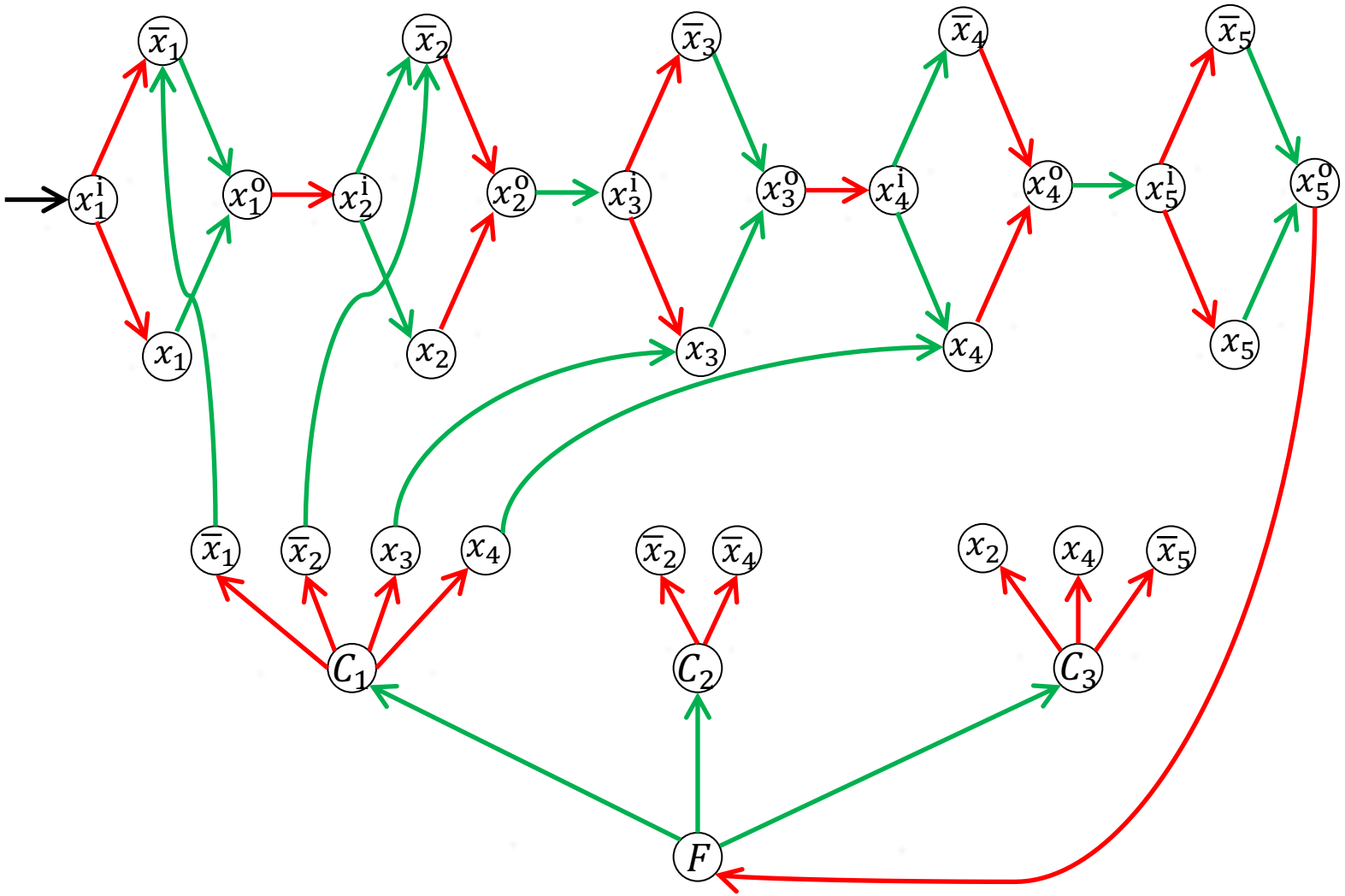

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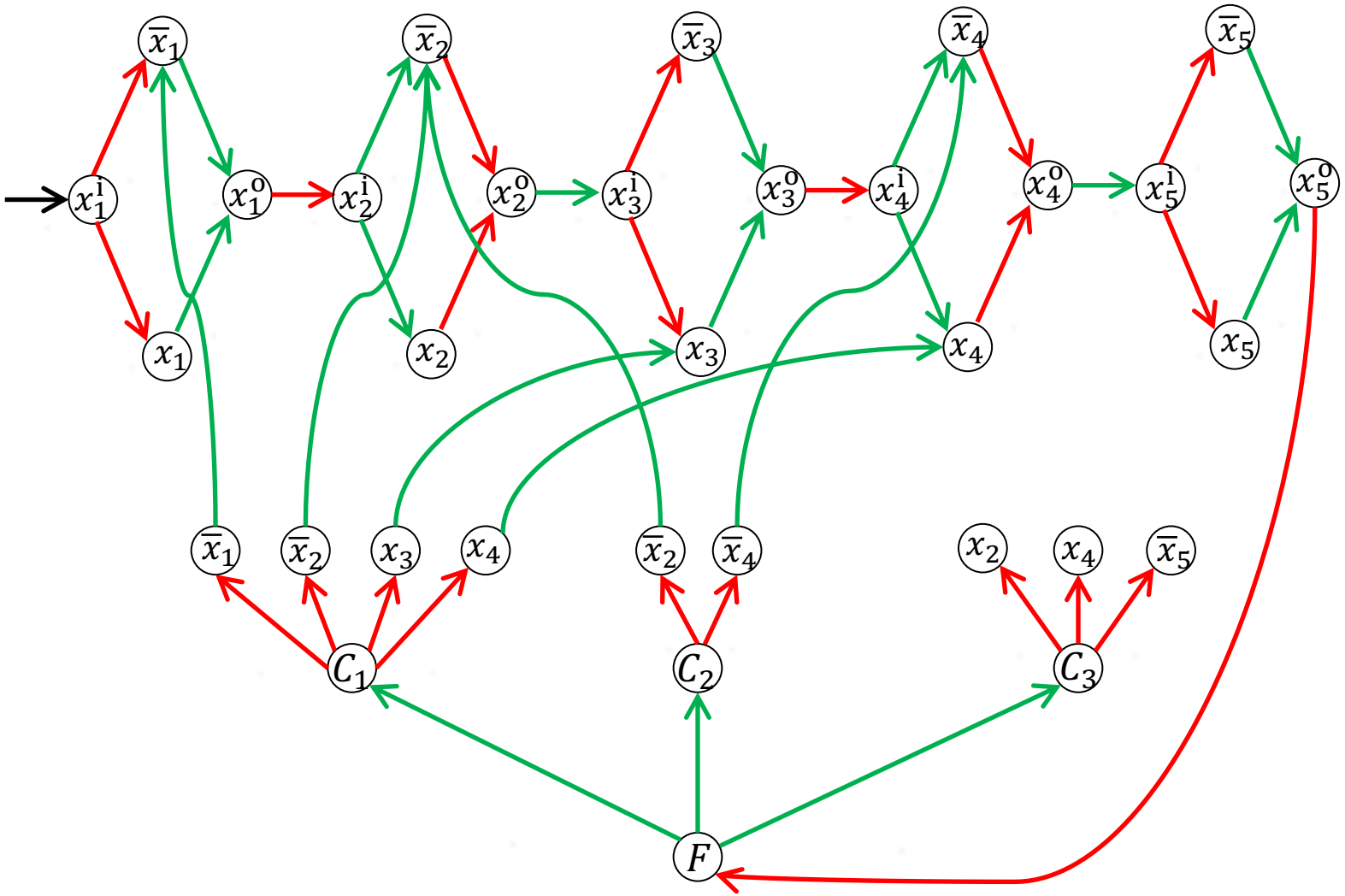
C_1 C_2



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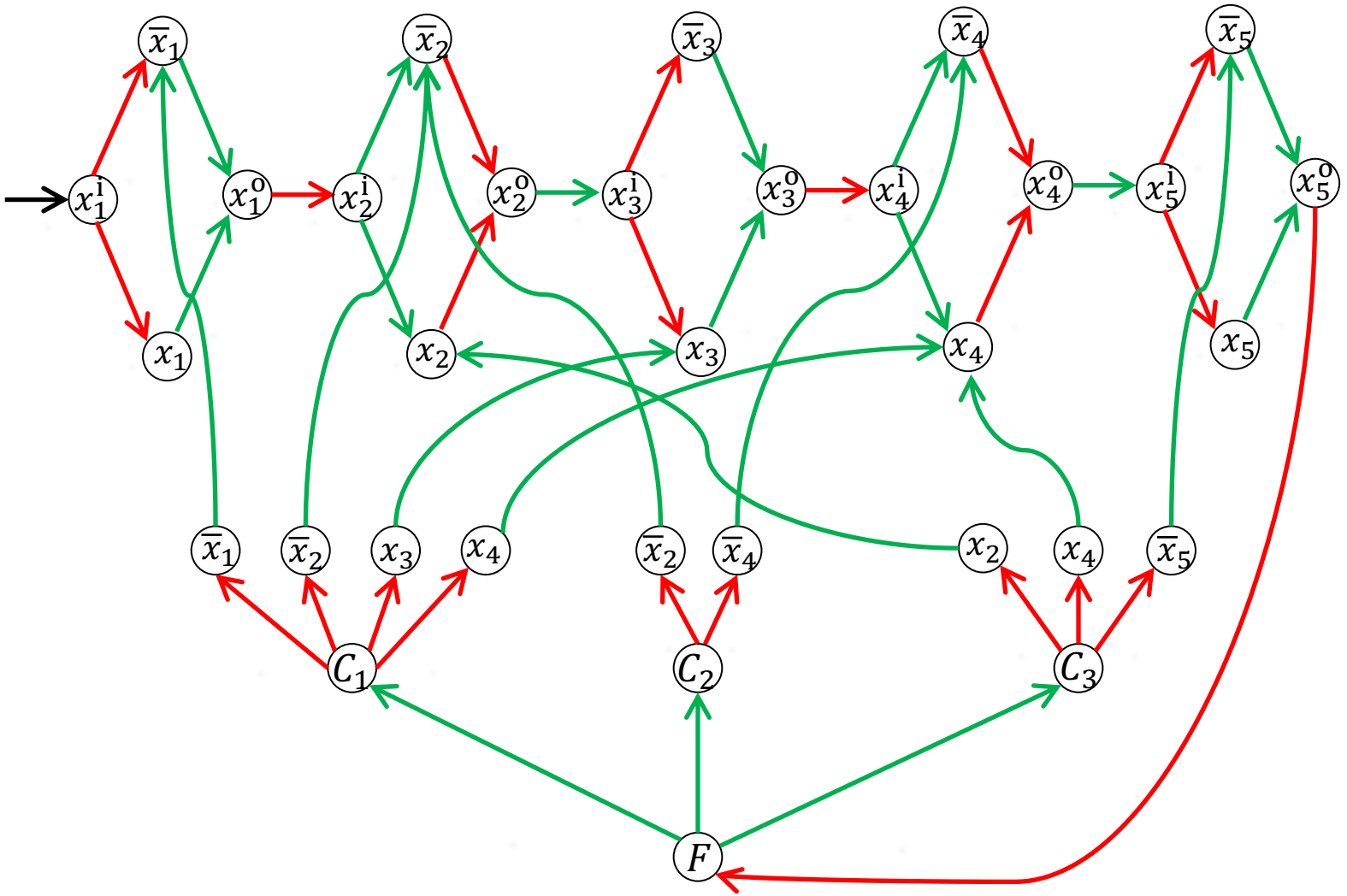
C_1 C_2 C_3



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