University of Toronto
Scarborough Campus
December 12, 2003

CSC B36 Final Examination

Aids allowed: One 8.5×11 handwritten, non-photocopied 'cheat sheet'

Duration: Three hours

- There should be 11 pages in this exam booklet, including this cover page.
- Answer all questions.
- Put all answers in this booklet, in the spaces provided.
- For rough work, use the backs of the pages; ***these will not be marked***.
- Good luck!

Family Name ______________________ Given Name ____________________

Student Number ____

Problem	Marks Rec'ved	Marks Worth
1.		20
2.		20
3.		10
4.		20
5.		20
6.		10
7.		20
8.		20
9.		10
TOTAL		150

QUESTION 1. (20 marks)

Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined as follows:

$$f(n) = \begin{cases} 1, & \text{if } n = 0 \\ 4 \cdot f(\lfloor \frac{n}{2} \rfloor) + (n-1), & \text{if } n \geq 1 \end{cases}$$

Present a specific constant c, as small as you can, and prove that for all integers $n \geq 1$, $f(n) \leq cn^2 - 3n$. You may use (without proof) the inequalities $(n-1)/2 \leq \lfloor n/2 \rfloor \leq n/2$.

ANSWER:

QUESTION 2. (20 marks)

Prove that the program below is correct with respect to the following pre/postcondition pair.

Precondition: $A[1..n]$ is an integer array of length at least 1, $f, \ell \in \mathbb{N}$ and $1 \leq f \leq \ell \leq n$.

Postcondition: $\textsc{Sorted}(A, f, \ell)$ returns 1 if $A[f..\ell]$ is sorted, i.e., if $A[i] \leq A[j]$ for all i, j such that $f \leq i \leq j \leq \ell$; and it returns 0 otherwise.

```
SORTED(A, f, ℓ)
if f = ℓ then
    return 1
else
    m := (f + ℓ) div 2
    if SORTED(A, f, m) = 0 or A[m] > A[m+1] or SORTED(A, m+1, ℓ) = 0 then
        return 0
    else
        return 1
    end if
end if
```

ANSWER:

[more space, if needed, is available on the next page]

[this page was deliberately left blank]

QUESTION 3. (10 marks)

Give a propositional formula that is logically equivalent to $(x \vee y) \wedge \neg z$ and uses ***only*** the connective $|$ (nand). Give a brief but informative explanation of how you obtained your formula.

ANSWER:

QUESTION 4. (20 marks)

Given below are four pairs (i)–(iv) of propositional formulas, P and Q.

(i)	$P:$	$x \vee y \vee z \vee w$	$Q:$	$x \vee y \vee z \vee \neg w$
(ii)	$P:$	$x \rightarrow \big(y \rightarrow (z \rightarrow w)\big)$	$Q:$	$(x \wedge y \wedge z) \rightarrow w$
(iii)	$P:$	$(x \wedge \neg x) \rightarrow (y \vee z \vee \neg w)$	$Q:$	$(x \vee y) \leftrightarrow (z \wedge w)$
(iv)	$P:$	$\big((x \wedge y) \leftrightarrow \neg x\big) \wedge (z \vee w)$	$Q:$	$x \wedge y \wedge z \wedge w$

For each pair select the one of the following five options (a)–(e) that is true for that pair. Do not justify your answer.

(a) P logically implies Q, but Q does not logically imply P.

(b) P does not logically imply Q, but Q logically implies P.

(c) P and Q are logically equivalent.

(d) Neither one of P, Q logically implies the other.

(e) None of the above.

ANSWER:

Pair	Answer (one of (a)–(e))
(i)	
(ii)	
(iii)	
(iv)	

QUESTION 5. (20 marks)

Consider the database schema for a library defined in the notes. It contains three predicates:

- $Book(b, t, n)$: Book (with id) b has title t and is written by author n.
- $Subscriber(s, n, a)$: Subscriber (with SIN) s is named n and lives in address a.
- $Borrowed(b, s, d)$: Copy with book id b was borrowed by subscriber s and is due on d.

a. (10 marks) Give a formula that expresses the following query: "Find the name and address of each subscriber who has borrowed a copy of the Odyssey."

Formula:

b. (10 marks) Assume that there is exactly one subscriber called Simpson, and that the predicate *Borrowed* describes all copies of books that have ever been borrowed — not only those that are presently checked out. Which of the following formulas expresses the query: "Find the name and address of each subscriber who has borrowed every copy borrowed by Simpson." (For brevity we write "*Sub*" for "*Subscriber*" and "*Bor*" for "*Borrowed*".)

(i) $\exists s \left(Sub(s,n,a) \land \forall b \, \forall s' \, \forall a' \, \forall d \, \exists d' \left(\left(Sub(s', \text{Simpson}, a') \land Bor(s', b, d) \right) \rightarrow Bor(s, b, d') \right) \right)$

(ii) $\exists s \left(Sub(s,n,a) \land \forall b \, \forall s' \, \forall a' \, \forall d \, \forall d' \left(\left(Sub(s', \text{Simpson}, a') \land Bor(s', b, d) \right) \rightarrow Bor(s, b, d') \right) \right)$

(iii) $\exists s \left(Sub(s,n,a) \land \exists b \, \exists s' \, \exists a' \, \exists d \, \exists d' \left(\left(Sub(s', \text{Simpson}, a') \land Bor(s', b, d) \right) \rightarrow Bor(s, b, d') \right) \right)$

(iv) $\exists s \left(Sub(s,n,a) \land \exists b \, \exists s' \, \exists a' \, \exists d \, \exists d' \left(Sub(s', \text{Simpson}, a') \land Bor(s', b, d) \land Bor(s, b, d') \right) \right)$

Circle the correct answer from the following choices:

(a) Only (i) correctly expresses the query.

(b) Only (ii) correctly expresses the query.

(c) Only (iii) correctly expresses the query.

(d) Only (iv) correctly expresses the query.

(e) All of (i)–(iv) correctly express the query.

(f) None of (i)–(iv) correctly expresses the query.

QUESTION 6. (10 marks)

Consider a first-order language with a ternary predicate symbol A, binary predicate symbol B, and unary predicate symbols C and D. Transform the following first-order formula of this language into an equivalent Prenex Normal Form formula. Demonstrate the steps through which you obtain your final formula.

$$\forall x \left(\exists y\, A(x, y, z) \wedge B(x, y)\right) \rightarrow \left(\neg \exists z\, C(z) \vee \forall x\, D(x)\right)$$

ANSWER:

QUESTION 7. (20 marks)

Let L be the language defined as follows:

$$L = \{x \in \{0,1\}^* : |x| \geq 4 \text{ and the last (rightmost) four symbols of } x \text{ contain at least one } 0\}$$

a. (10 marks) Give the diagram of a NFSA M that accepts L. You need not prove that your automaton is correct — but it should be!

b. (10 marks) Present a regular expression R that denotes L. You need not prove that your regular expression is correct — but it should be!

QUESTION 8. (20 marks)

State whether each of the following statements is true or false. Do not justify your answers.

Do not guess: 4 for each correct answer, 0 for no answer, −2 for wrong answer.

(a) The string 01010 is in the language denoted by $(1^*0^*)^*$.

(b) The regular expressions $0(10)^*1$ and $(01)^*$ are equivalent.

(c) For every language L, if L is not regular then $\overline{L}$ (the complement of L) is also not regular.

(d) For all regular languages L and L', the language of all strings that are in neither of L, L' is also regular.

(e) For all languages L and L', if L is regular and $L \cup L'$ is regular then L' is also regular.

QUESTION 9. (10 marks)

Following is a grammar that generates the set of binary strings with as many 0s as 1s. The grammar has variables S, A and B, with start symbol S, and the following productions:

$$\begin{array}{lll} S \to 0B & A \to 0 & B \to 1 \\ S \to 1A & A \to 0S & B \to 1S \\ & A \to 1AA & B \to 0BB \end{array}$$

a. (5 marks) Show a derivation of 11101000 in the grammar.

ANSWER:

b. (5 marks) Is there a right-linear context-free grammar that generates the same language? If so, give such a grammar. If not, explain why.

ANSWER:

THE END