University of Toronto
Scarborough Campus
October 27, 2006

CSC B36 Midterm Examination

Aids allowed: One 8.5×11 'cheat sheet' (may be written on both sides)

Duration: One hour and fifty minutes

- There should be 6 pages in this exam booklet, including this cover page.
- Answer all questions.
- Put all answers in this booklet, in the spaces provided.
- For rough work, use the backs of the pages; ***these will not be marked***.
- Good luck!

Family Name ______________________________ Given Name ______________________

Student Number ____

Problem	Marks Rec'ved	Marks Worth
1.		15
2.		15
3.		20
4.		20
5.		30
TOTAL		100

QUESTION 1. (15 marks)

Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined recursively as follows:

$$f(n) = \begin{cases} 0, & \text{if } n = 0 \\ 2, & \text{if } n = 1 \\ 4, & \text{if } n = 2 \\ f(n-3) + 2n, & \text{if } n \geq 3 \end{cases}$$

Prove that for all integers $n \geq 6$, $f(n) \leq n^2/2$.

PROOF:

QUESTION 2. (15 marks)

Recall the binary Boolean connective $|$ (nand): A truth assignment satisfies the propositional formula $(P \mid Q)$ if and only if it falsifies at least one of P and Q. Thus, $(P \mid Q)$ is logically equivalent to $\neg(P \wedge Q)$ (hence the name "not-and" or "nand").

Give a formula that is logically equivalent to $(x \rightarrow \neg y) \vee \neg z$ and uses only the connective $|$. In the space below your formula, explain why it is correct.

FORMULA:

EXPLANATION:

QUESTION 3. (20 marks)

Let P be any ***unsatisfiable*** formula, and Q, R be any ***tautologies***. For each of the statements (a)–(e) below, indicate whether it is true or false. Do not justify your answers.

Do not guess: 4 for each correct answer, 0 for no answer, -2 for wrong answer.

(a) P logically implies Q

(b) Q logically implies P

(c) $Q \vee \neg R$ is logically equivalent to R

(d) $Q \rightarrow P$ is logically equivalent to P

(e) $(P \leftrightarrow Q) \rightarrow R$ is a tautology

QUESTION 4. (20 marks)

Recall the binary Boolean connective $\oplus$ (exclusive-or): A truth assignment satisfies the propositional formula $(P_1 \oplus P_2)$ if and only if it satisfies exactly one of P_1 and P_2. Following is a recursive definition of the set $\mathcal{F}$ of propositional formulas that use only the binary Boolean connectives $\vee$ and $\oplus$. $\mathcal{F}$ is the smallest set of formulas such that:

BASIS: Every propositional variable is in $\mathcal{F}$.

INDUCTION STEP: If P_1 and P_2 are in $\mathcal{F}$ then so are the formulas $(P_1 \vee P_2)$ and $(P_1 \oplus P_2)$.

a. (15 marks) Let τ_0 be the truth assignment that assigns the value 0 (false) to each propositional variable. Prove that τ_0 falsifies every formula in $\mathcal{F}$.

PROOF:

b. (5 marks) Use part (a) to prove that the set of connectives $\{\vee, \oplus\}$ is not complete.

PROOF:

QUESTION 5. (30 marks)

The program DIV(m, n) below computes the quotient of the division of m by n. More precisely, the program satisfies the following specification:

Precondition: $m, n \in \mathbb{N}$ and $n \neq 0$.

Postcondition: DIV(m, n) returns an integer q such that $m = q \cdot n + r$, where r is some integer in the range $0 \leq r < n$.

```
DIV(m, n)
x := m
i := 0
while x ≥ n do
    x := x − n
    i := i + 1
end while
return i
```

Prove that this program is correct with respect to its specification.

PROOF OF CORRECTNESS:

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THE END