Computer Science B36
Scarborough Campus

October 11, 2006
University of Toronto

Homework Assignment #3

Due: November 1, 2006, by 12 noon
(in the course drop box)

Appended to this document is a cover page for your assignment. Fill it out, staple your answers to it, and deposit the resulting document into the course drop box. Please do ***not*** *enclose your assignment in an envelope.*

Question 1. (10 marks) For each of the following four pairs of propositional formulas P and Q, state whether (i) P logically implies Q, and (ii) Q logically implies P. Justify your answer. If for some pair you claim that both (i) and (ii) are true, i.e., that P and Q are logically equivalent, then you should prove this fact using only logical equivalences in Section 5.6 (i.e., without using truth tables).

a. $P = (x \wedge y) \to z$ and $Q = (x \to z) \wedge (y \to z)$.

b. $P = (x \vee y) \to z$ and $Q = (x \to z) \vee (y \to z)$.

c. $P = (x \vee y) \to z$ and $Q = (x \to z) \wedge (y \to z)$.

d. $P = (x \leftrightarrow y) \to z$ and $Q = (x \to z) \leftrightarrow (y \to z)$.

Question 2. (10 marks) Consider truth assignments involving only the propositional variables x_0, x_1, x_2, x_3 and y_0, y_1, y_2, y_3, y_4. Every such truth assignment gives a value of 1 (representing true) or 0 (representing false) to each variable. We can therefore think of a truth assignment τ as determining a four-bit integer x_τ depending on the values given to x_0, x_1, x_2 and x_3, and a five-bit integer y_τ depending on the values given to y_0, y_1, y_2, y_3 and y_4. More precisely, we can define the integers $x_\tau = \tau(x_0) + 2 \cdot \tau(x_1) + 4 \cdot \tau(x_2) + 8 \cdot \tau(x_3)$ and $y_\tau = \tau(y_0) + 2 \cdot \tau(y_1) + 4 \cdot \tau(y_2) + 8 \cdot \tau(y_3) + 16 \cdot \tau(y_4)$.

Write a formula that is satisfied by exactly those truth assignments τ for which $y_\tau = x_\tau + 2$. Your formula may use any of the Boolean connectives discussed in the notes. Explain how you obtained your formula, and justify its correctness.

Note: This can be done by writing down a truth table for nine propositional variables — i.e., a truth table with $2^9 = 512$ rows. This is too much boring work. It can be done more easily (and interestingly) by expressing as a propositional formula the condition under which the five-bit number $y_4y_3y_2y_1y_0$ is exactly two more than the four-bit number $x_3x_2x_1x_0$. Solutions based on large truth tables will receive no credit.

Question 3. (10 marks) There are $2^{(2^n)}$ Boolean functions of n variables. (To see why, review Proposition 1.6 in the Notes.) For each of the sixteen Boolean functions of two variables x_1 and x_2, write a propositional formula that represents that function. Your formulas may use any of the Boolean connectives discussed in the notes.

continued

Question 4. (10 marks) Prove that $\{\rightarrow, \oplus\}$ is a complete set of Boolean connectives.

Question 5. (10 marks)

a. Let X be a finite set of even size, and let T and T' be two subsets of X, both of which have even size. Also, let $\overline{T}$ and $\overline{T}'$ be the complements of T and T' with respect to X. Prove that either all of $T \cap T'$, $\overline{T} \cap T'$, $T \cap \overline{T}'$, $\overline{T} \cap \overline{T}'$ have even cardinality, or all of them have odd cardinality.

b. Prove that $\{\leftrightarrow, \oplus\}$ is not a complete set of connectives. (Hint: Prove that the number of truth assignments that satisfy any formula that uses only the connectives $\leftrightarrow$ and $\oplus$ is even. In doing so, you may find part (a) useful.)

Cover page for
CSCB36 Homework #3

Submitted by

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Tutorial: ______________________

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By virtue of submitting this homework I/we acknowledge that I am/we are aware of the policy on homework collaboration for this course.