Computer Science 236F | November 22, 2006
UTM Campus | University of Toronto

Homework Assignment #4
Due: December 15

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*On the cover page of your assignment, you must write* ***and sign*** *the following statement: "I have read and understood the homework collaboration policy described in the Course Information handout." Without such a signed statement your homework will not be marked.*

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**Question 1.** (12 marks) Consider the first order language of arithmetic described in Section 6.2 and the structures $\mathcal{N}$ and $\mathcal{Z}$ described on page 152 in the notes. For each sentence below state whether it is true in $\mathcal{N}$, $\mathcal{Z}$, both or neither. Justify your answer by translating each sentence into a statement in precise English about numbers and then explain why that statement is true or false for the natural numbers or for the integers.

1. $\exists x \; \forall y(\neg \approx (x, y) \rightarrow L(x, y))$
2. $\forall x \; \forall y \; ((L(0, y) \wedge S(x, x, y)) \rightarrow L(x, 0))$
3. $\forall x \; \forall y \; \forall z \; ((L(x, 0) \wedge P(x, y, z) \wedge L(z, 0)) \rightarrow L(0, y))$
4. $\forall x \; \exists y \; \exists z \; \exists w \; (S(y, z, w) \wedge \neg L(w, x))$

**Question 2.** (8 marks) Consider the database schema for a library as defined in the lecture notes with some small additions. There are five predicates:

1. $Book(b, t, n)$: This is true if a book with id $b$, has title $t$ and is written by author $n$.
2. $Subscriber(s, n, a)$: This is true if a subscriber with SIN $s$ has name $n$ and lives in address $a$.
3. $Borrowed(s, b, d)$: This is satisfied when a subscriber with SIN $s$ has borrowed a book with id $b$ and it is due on date $d$.
4. $Likes\_donuts(s)$: This is true if a person with SIN $s$ likes to eat donuts.
5. $Contains(b, w)$: This is true if the word $w$ is contained in the title of the book with id $b$.

**a.** (4 marks) Give a first-order formula that expresses the query: Find the **names** of all subscribers who like donuts and who have borrowed at least one of the following books:
($i$) Donut recipes for dummies
($ii$) The role of donuts in modern politics.

**b.** (4 marks) Give a first-order formula that expresses the query: Find the **title** and **due date** of every book which is borrowed by Mr. Homer Simpson and which has the word donut in its title.

**Question 3.** (10 marks)
**a.** (5 marks) Write a formula in Prenex Normal Form that is logically equivalent to:

$$(\forall x \; P(x) \rightarrow \forall x \; Q(x)) \vee (\neg \forall x \; P(x) \rightarrow \neg \forall x \; Q(x))$$

**b.** (5 marks) For each formula below say whether it is valid, satisfiable or unsatisfiable. Justify your answer either with a formal proof using logical equivalences and/or by using appropriate structures.

1. $\forall x\ \forall y\ \forall z\ (\neg L(x,z) \rightarrow (\neg L(y,z) \vee \neg L(x,y)))$

2. $\forall x\ A(x) \rightarrow \neg \exists x A(x)$

**Question 4.** (30 marks)

**a.** (8 marks) An electrician is designing two-way switches so that the stairway lights of a building can be turned on or off from the first, second and third floor. She selects the alphabet $\Sigma = \{F, f, S, s, T, t\}$ with the meaning:

1. F means the first-floor switch is flipped up, f means it is flipped down.

2. S means the second-floor switch is flipped up, s means it is flipped down.

3. T means the third-floor switch is flipped up, t means it is flipped down.

A string over $\Sigma$ defines a sequence of switch flips. For example $SfTs$ means that the second-floor switch is flipped up, then the first-floor switch is flipped down, then the third-floor switch is flipped up and then the second-floor switch is flipped down. We can even have sequences where the same action repeats, like TT, which corresponds to a frustrated individual in the third floor who flips the third-floor switch up twice in a row, (not noticing in the second time that it is already flipped up).

Assume that initially all switches are down, i.e., the empty sequence $\epsilon$ leaves 0 switches up. Draw a DFSA for the language

$$L = \{x \in \Sigma^* : x \text{ leaves an even number of switches up}\}$$

For example $FSfT \in L$ whereas $FSf \notin L$. You do not need to prove that your automaton works but you should provide a brief justification of its correctness.

**b.** (12 marks) The electrician above decides that before moving on to her next assignment, it would be beneficial to practice more on regular expressions and finite automata. For each language below, provide a regular expression and draw a DFSA that accepts the language. You do not need to justify your answer.

1. $L = \{x \in \{0,1\}^* : x \text{ does not contain the substring } 110\}$.

2. $L = \{x \in \{0,1\}^* : \text{ every odd position of } x \text{ is } 1\}$.

3. $L = \{x \in \{0,1\}^* : x \text{ has length at least 3 and its third symbol is a } 0\}$.

**c.** (10 marks) The purpose of this question is to convince you (and the electrician) that in certain cases, a NFSA may be more convenient to represent a language than a DFSA. Draw a NFSA with at most 4 states for the language

$$L = \{x \in \{0,1\}^* : x \text{ has a 1 on the third position from the end}\}$$

For example $00100 \in L$ but $01000 \notin L$. Now draw a DFSA for the same language.

**Question 5.** (20 marks)

**a.** (10 marks) Let $\Sigma = \{0, 1\}$. Construct a DFSA that accepts a string $s \in \Sigma^*$ if and only if the decimal value of $s$ is divisible by 3. For example, your DFSA should accept 0 or 11 or 1001 but should not accept 10 or 101. Give a formal proof that your DFSA is correct along the lines of Example 7.12 of your textbook.

**b.** (10 marks) State whether the following statements are true or false. Briefly justify your answer. No credit will be given without justification.

1. The string 001101 belongs to the language denoted by $(0+1)(0^*10^*1)^*$.
2. Consider an infinite sequence of languages $L_0, L_1, L_2 ...$, such that each $L_i$ is regular. Then the infinite union of these languages $\cup_{i \in \mathbb{N}} L_i$ is also regular.
3. If $L$ is regular so is $L - \{x\}$, for any $x \in L$.
4. If $L$ is not regular then $\bar{L}$ is also not regular.

**Question 6.** (10 marks) For each of the following languages show whether they are regular or not. If they are you should provide either a regular expression or a DFSA or a NFSA and briefly justify its correctness. If the language is not regular you should provide a proof by contradiction, using the pumping lemma.

1. $L = \{xx : x \in \{0,1\}^*\}$.
2. $L = \{0^n 1^m : m + n \text{ is odd}\}$.

**Question 7.** (10 marks) **Extra credit:** Prove that the following unary language is not regular:

$$L = \{1^{n^2} : n \in \mathbb{N}\}$$