Computer Science 236F
UTM Campus

November 1, 2006
University of Toronto

Homework Assignment #3
Due: November 17

On the cover page of your assignment, you must write ***and sign*** *the following statement: "I have read and understood the homework collaboration policy described in the Course Information handout." Without such a signed statement your homework will not be marked.*

Question 1. (20 marks) Let x, b be natural numbers such that $x \geq 1$ and $b \geq 2$. The **integer logarithm of** x **base** b is the unique natural number l such that $b^l \leq x < b^{l+1}$ (which is also equal to $\lfloor \log_b x \rfloor$). Prove that the program below is correct with respect to the given precondition/postcondition pair. You should come up with an appropriate loop invariant and use the technique that we have used for iterative programs.

Precondition: $x, b \in \mathbb{N}$, $x \geq 1$ and $b \geq 2$.

Postcondition: The program returns the integer logarithm of x base b.

```
int intlog(int x, int b)
r = 0
t = 1
while (b * t ≤ x) do
    r = r + 1
    t = t * b
end while
return r
```

Question 2. (10 marks) Prove that if the precondition below is true, the program zigzag(n) terminates.
Precondition: n is an integer.

```
void zigzag(int n)
s = 1
if (n < 0) then
    s = -1
end if
while (n ≠ 0) do
    n = n − s
    s = s ∗ (−1)
    n = n ∗ (−1)
end while
```

Question 3. (10 marks) Write a paragraph (at least 10 lines) on the scientific contributions of one of the following scientists: Kurt Gödel, Alan Turing, Tony Hoare, Volker Strassen, Edsger Dijkstra. You should provide at least two references. One of your sources can be a website but at least one should be either a book or an article that you found in the library or elsewhere.

Question 4. (10 marks) In this question you should prove your assertions using only the logical equivalences of Section 5.6 (i.e., without using truth tables).

a. (5 marks) Prove that:
$x \rightarrow (y \leftrightarrow z)$ LEQV $\neg(x \wedge ((y \rightarrow z) \vee (z \rightarrow y)) \wedge \neg(\neg y \wedge \neg z) \wedge \neg(y \wedge z))$

b. (5 marks) Find a CNF formula that is logically equivalent to the formulas of Part (a). Prove your assertion.

Question 5. (20 marks)

a. (10 marks) Prove that $\{\oplus, \rightarrow\}$ is a complete set of connectives.

b. (10 marks) Prove that $\{\oplus, \vee\}$ is not a complete set of connectives.

Question 6. (10 marks) Consider truth assignments containing only the propositional variables x_0, x_1, x_2, x_3, x_4 and y_0, y_1, y_2, y_3, y_4. Every such truth assignment gives a value of 1 (representing true) or 0 (representing false) to each variable. Therefore we can think of a truth assignment τ as determining a 5-bit integer x_τ, where the most significant bit is x_4 and the least significant bit is x_0. In particular, $x_\tau = \tau(x_0) + 2\tau(x_1) + 4\tau(x_2) + 8\tau(x_3) + 16\tau(x_4)$. Similarly the assignment τ determines a 5-bit integer $y_\tau = \tau(y_0) + 2\tau(y_1) + 4\tau(y_2) + 8\tau(y_3) + 16\tau(y_4)$.

Write a propositional formula in the variables $x_0, ..., x_4, y_0, ..., y_4$ that is satisfied by exactly those truth assignments τ for which $y_\tau = x_\tau \textbf{ mod } 4$. You may use any of the Boolean connectives discussed in the notes. Justify your answer.

Hint: Consider a 5-bit integer x and the result of $x \textbf{ mod } 4$ as another 5-bit integer y. How are the bits of y related to the bits of x?