Applications of Gaussian Process Regression

We will concentrate on a few successful applications in computer vision

1. Multiple kernel learning: object recognition
2. Active Learning: object recognition
3. GPs as an optimization tool: weakly supervised segmentation
4. Human pose estimation from single images
5. Flow Classification and ROI detection
6. Shape from shading
7. Online Shopping
1) Object Recognition

- **Task:** Given an image $x$, predict the class of the object present in the image $y \in Y$

  $y \rightarrow \{\text{car, bus, bicycle}\}$

- Although this is a classification task, one can treat the categories as real values and formulate the problem as regression.
1) Object Recognition

- **Task:** Given an image $\mathbf{x}$, predict the class of the object present in the image $y \in \mathcal{Y}$.

$$y \rightarrow \{\text{car, bus, bicycle}\}$$

- Although this is a classification task, one can treat the categories as real values and formulate the problem as regression.
How do we do Object Recognition?

- Given this two images, we will like to say if they are of the same class.

- Choose a representation for the images
  - Global descriptor of the full image
  - Local features: SIFT, SURF, etc.

- We need to choose a way to compute similarities
  - Histograms of local features (i.e., bags of words), pyramids, etc.
  - Kernels on global descriptors, e.g., RBF
  - ...
Multiple Kernel Learning (MKL)

- Why do we need to choose a single representation and a single similarity function?
- Which one is the best among all possible ones?
- Multiple kernel learning comes at our rescue, by learning which cues and similarities are more important for the prediction task.
- Simplest form:

\[ K = \sum_i \alpha_i K_i \]
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Supposed we want to emulate a 1-vs-all strategy as $|\mathcal{Y}| > 2$

- We define $y \in \{-1, 1\}^{\mathcal{Y}}$
- We can employ maximum likelihood and learn all the parameters for all classifiers at once

$$\min_{\theta, \alpha > 0} - \sum_i \log p(y^{(i)}|X, \theta, \alpha) + \gamma_1 \|\alpha\|_1 + \gamma_2 \|\alpha\|_2$$

with $y^{(i)} \in \{-1, 1\}$ each of the individual problems.

- Efficient as we can share the covariance across all classes
Caltech 101 dataset
Results: Caltech 101


Comparison with SVM kernel combination: kernels based on Geometric Blur (with and without distortion), dense PMK and spatial PMK on SIFT, etc.

Figure: Average precision.

Figure: Time of computation.
Results: Caltech 101


Figure: Comparison with the state of the art as in late 2008.
Other forms of MKL

Convex combination of kernels is too simple (not big boost reported), we need more complex (non-linear) combinations

- Localized comb. (the weighting varies locally) (Christioudias et al. 09)

\[ K^{(v)} = K^{(v)}_{np} \odot K^{(v)}_p \]

use structure to define \( K^{(v)}_{np} \), e.g., low-rank

- Bayesian co-training (Yu et al. 07)

\[ K_c = \left[ \sum_j (K_j + \sigma^2_j I)^{-1} \right]^{-1} \]
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Check out Mario Christoudias PhD thesis for more details
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2) Active Learning: user in the loop

- Labeling is typically an expensive process (now less with Mechanical Turk).
- Label as little as possible to reach a certain performance level.
- In **active learning**, we ask the human annotators to label not randomly, but where the classifier is more uncertain about a label.
- Trade-off between exploration and exploitation
Active Learning Algorithm

\textbf{repeat}

Select $x_u$ to labeled using active learning criteria
As the user to label $x_u$
Re-train classifier with previous labels + new label point

\textbf{until} budget reached
Active Learning Criteria

Let $X_U$ be the pool of unlabeled data, we could use

- SVM-like criteria of distance to the margin
  $$\min_{x_u \in Y_U} |y_u^*|$$

- Most uncertain point (i.e., variance)
  $$\max_{x_u \in Y_U} \Sigma_u^*$$

- Trade off between exploitation and exploration
  $$\min_{x_u \in Y_U} \frac{|y_u^*|}{\sqrt{\Sigma_u^* + \sigma^2}}$$
Some examples on Caltech 101

Does it always help?

Supposed you have a function that you want to optimize, but it is non-differentiable and also computationally expensive to evaluate, you can

- Discretize your space and evaluate discretized values in a grid (combinatorial)
- Randomly sample your parameters
- Utilize "active learning" style analysis and GPs to query where to look
Suppose we want to compute $\max f(x)$, we can simply

**repeat**

Choose $x_t = \arg \max_{x \in D} \mu_{t-1}(x) + \sqrt{\beta_t} \sigma_{t-1}(x)$

Evaluate $y_t = f(x_t) + \epsilon_t$

Evaluate $\mu_t$ and $\sigma_t$

**until** budget reached
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GPs as an optimization tool in vision

[A. Vezhnevets, V. Ferrari and J. Buhmann, CVPR 2012]

- Image segmentation in the weakly supervised setting, where the only labels are which classes are present in the scene.

\[ y \in \{ \text{sky}, \text{building}, \text{tree} \} \]

- Train based on expected agreement, if I partition the dataset on two sets and I train on the first, it should predict the same as if I train on the second.

- This function is sum of indicator functions and thus non-differentiable.
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Examples of Good Segmentations and Results

[A. Vezhnevets, V. Ferrari and J. Buhmann, CVPR 2012]

<table>
<thead>
<tr>
<th>Image</th>
<th>Ground truth</th>
<th>GMIM result</th>
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<td><img src="gmim1" alt="GMIM result" /></td>
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<tr>
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<tr>
<td><img src="image3" alt="Image" /></td>
<td><img src="ground3" alt="Ground truth" /></td>
<td><img src="gmim3" alt="GMIM result" /></td>
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<th>[Vezhnevets 11]</th>
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4) Discriminative Approaches to Human Pose Estimation

- **Task:** given an image $x$, estimate the 3D location and orientation of the body parts $y$.

- We can treat this problem as a multi-output regression problem, where the input are image features, e.g., BOW, HOG, etc.

- The main challenges are
  - Poor imaging: motion blurred, occlusions, etc.
  - Need of large number of examples to represent all possible poses: represent variations in appearance and in pose.
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Challenges for GPs

- GP have complexity $O(n^3)$, with $n$ the number of examples, and cannot deal with large datasets in their standard form.
- This problem cannot be solved directly as a regression task, since the mapping is multimodal: an image observation can represent more than one pose.
Dealing with multimodal mappings

- We can represent the regression problem as a mixture of experts, where each expert is a local GP.
- The experts should be selected online to avoid the possible boundary problems of clustering.

Advantages:
- Probabilistic estimates.
- Reliable estimation of hyperparameters.
- Strategy for pruning unnecessary examples and detecting outliers.
- Fast solution with up to millions of examples if combined with fast NN retrieval, e.g., LSH.
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**Online Algorithm**

**ONLINE: Inference** of test point $x_*$

- $T$: number of experts, $S$: size of each expert

1. Find NN in $x$ of $x_*$
2. Find Modes in $y$ of the NN retrieved
3. For $i = 1 \ldots T$ do
   - Create a local GP for each mode $i$
   - Retrieve hyper-parameters
   - Compute mean $\mu$ and variance $\sigma$
4. End for

$$p(f_*|y) \approx \sum_{i=1}^{T} \pi_i N(\mu_i, \sigma_i^2)$$
Online vs Clustering

- Full GP
- Local Offline GP
- Local online GP

Urtasun & Lawrence ()

GP tutorial

June 16, 2012 24 / 38
Single GP vs Mixture of Online GPs

Global GP

Local Online GP
Results: Humaneva

[R. Urtasun and T. Darrell, CVPR 2008]

Table: Comparison with state of the art (error in cm).

- **Caviat**: Oracle has to select the optimal mixture component
5) Flow Estimation with Gaussian Process

- Model a trajectory as a continuous dense flow field from a sparse set of vector sequences using Gaussian Process Regression.
- Each velocity component modeled with an independent GP.
- The flow can be expressed as

\[ \phi(x) = y^{(u)}(x)i + y^{(v)}(x)j + y^{(t)}(x)k \in \mathbb{R}^3 \]

where \( x = (u, v, t) \)

- Difficulties:
  - How to model a GPRF from different trajectories, which may have different lengths.
  - How to handle multiple GPRF models trained from different numbers of trajectories with heterogeneous scales and frame rates.

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- Solution: normalize the length of the tracks before modeling with a GP, as well as the number of samples
- Classification based on the likelihood for each class
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Classification based on the likelihood for each class
Flow Classification and Anomaly Detection

[K. Kim, D. Lee and I. Essa, ICCV 2011]
Detecting Regions of Interest

- Detect the regions of interests in moving camera views of dynamic scenes with multiple moving objects
- Important for cheap streaming of events on the internet, e.g., NBA playoffs on TNT
- Extract a global motion tendency that reflects the scene context by tracking movements of objects in the scene
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6) Shape from Shading

- 3D shapes were obtained using a motion capture system and taking small $5 \times 5$ patches from the reconstructed shapes.

- Corresponding intensities were obtained using a Computer Graphics software and the calibrated lighting environment.

- To reduce the dimensionality, performed PCA on the 3D patch shapes and on the patch intensities separately.

- Local intensities and shapes are then represented as

$$I = I_0 + \sum_{i=1}^{N_I} x_i I_i,$$

$$D = D_0 + \sum_{i=1}^{N_D} y_i D_i.$$
Gaussian Process Mapping

- We want to learn a mapping from intensity to 3D patch shape
- This is equivalent to learn a mapping from $x$ to $y$

Use GPs to learn this mapping
- Illumination modeled as weighted sum of spherical harmonics
- Illumination parameters estimated using a light probe
Handling Multimodality

- Unfortunately, this mapping is multimodal (i.e., similar intensities may correspond to different 3D shapes)

- These ambiguities are strongly related to the first two shape component, which define out-of-plane rotation

- Use local GPs (Urtasun et al., 08) to handle multiple modes

- Utilize an MRF to patch the local patches.
Real Data Experiments

[A. Varol, A. Shaji, M. Salzmann and P. Fua, PAMI 2011]
7) 3D Shape Recovery for Online Shopping

- Interactive system for quickly modelling 3D body shapes from a single image
- Obtain their 3D body shapes so as to try on virtual garments online
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Creating the 3D Shape from Single Images

Manually annotate a set of five 2D measurements

- Well-defined by the anthropometric positions, easy to discern and unambiguous to users.
- Good correlations with the corresponding tape measurements and convey enough information for estimating the 3D body shape
- User’s effort for annotation should be minimised
The role of the GPs

- A body shape estimator is learned to predict the 3D body shape from user’s input, including both image measurements and actual measurements.
- Training set is (CAESAR) dataset (Robinette et al. 99), with 2000 bodies.

- Register each 3D instance in the dataset with a 3D morphable human body
- A 3D body is decomposed into a linear combination of body morphs
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<th></th>
<th>Chest</th>
<th>Waist</th>
<th>Hips</th>
<th>Inner leg length</th>
</tr>
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<tbody>
<tr>
<td>Error (cm)</td>
<td>1.52 ± 1.36</td>
<td>1.88 ± 1.06</td>
<td>3.10 ± 1.86</td>
<td>0.79 ± 0.90</td>
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[Y. Chen and D. Robertson and R. Cipolla, BMVC 2011]