Grouping and Structure from Motion

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Example of grouping techniques

- K-means style clustering, e.g., SLIC superpixels
- Normalized cuts
- Graph-based superpixels
- Mean-shift
- Watershed transform
Simple K-means

Find three clusters in this data

Figure: From M. Tappan
Simple K-means

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Results

[R. Achanta and A. Shaji and K. Smith and A. Lucchi and P. Fua and S. Susstrunk, PAMI12]
Joint Segmentation and Depth Estimation

- Let $S = \{s_1, \cdots, s_m\}$ be the set of superpixel assignments
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We can define the total energy of a pixel as:

$$E(p) = E_{l,r\text{col}}(p, cs, \theta_{sp}) + \lambda_{pos}E_{pos}(p, \mu_{sp}) + \lambda_{disp}E_{l,r\text{disp}}(p, \theta_{sp}),$$

and

$$E_{pos}(p, \mu_{sp}) = ||p - \mu_{sp}||^2 \quad / \quad g E_{col}(p, cs = (I_t(p) - cs)^2)$$

and

$$E_{disp}(p, \theta_{sp}) = \begin{cases} d(p, \theta_{sp}) - \hat{d}(p) & \text{if } p \in F \\ \lambda & \text{otherwise} \end{cases}$$
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where $E_{col}(p, c_{sp}, \theta_{sp})$ is the color consistency term, $E_{pos}(p, \mu_{sp})$ is the positional consistency term, and $E_{disp}(p, \theta_{sp})$ is the depth disparity term.
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We can use:

$$E_{\text{pos}}(p, \mu_{sp}) = \|p - \mu_{sp}\|^2 / g$$

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and

$$E_{\text{disp}}(p, \theta_{sp}) = \begin{cases} (d(p, \theta_{sp}) - \hat{d}(p))^2 & \text{if } p \in \mathcal{F} \\ \lambda & \text{otherwise} \end{cases}$$
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Segmentation as a mincut problem

- Examines the **affinities** (similarities) between nearby pixels and tries to separate groups that are connected with weak affinities.

\[
\begin{pmatrix}
0 & 1 & 3 & \infty & \infty \\
1 & 0 & 4 & \infty & 2 \\
3 & 4 & 0 & 6 & 7 \\
\infty & \infty & 6 & 0 & 1 \\
\infty & 2 & 7 & 1 & 0
\end{pmatrix}
\]

*Weight Matrix: W*

- The cut separate the nodes into two groups
The cut between two groups $A$ and $B$ is defined as the sum of all the weights being cut:

$$cut(A, B) = \sum_{i \in A, j \in B} w_{i,j}$$

Problem: Results in small cuts that isolates single pixels.

We need to normalize somehow.
Normalized Cuts

- Better measure is the normalized cuts

\[ N_{\text{cut}}(A, B) = \frac{\text{cut}(A, B)}{\text{assoc}(A, V)} + \frac{\text{cut}(A, B)}{\text{assoc}(B, V)} \]

with \( \text{assoc}(A, A) = \sum_{i \in A, j \in A} w_{ij} \) is the association term within a cluster and \( \text{Assoc}(A, V) = \text{assoc}(A, A) + \text{cut}(A, B) \) is the sum of all the weights associated with nodes in A.

- We want minimize the disassociation between the groups and maximize the association within the groups
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Normalized Cuts

- Computing the optimal normalized cut is NP-Complete.
- Instead, relax by computing a real value assignment

Let $d = W$ be the row sums of the symmetric matrix $W$, and $D = \text{diag}(d)$ be the corresponding diagonal matrix. Shi and Malik, compute the cut by solving

$$\min_y y^T (D - W) y y^T D y$$

relaxing $y$ to be real-value

$D - W$ is the Laplacian
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Let \( d = W1 \) be the row sums of the symmetric matrix \( W \), and \( D = \text{diag}(d) \) be the corresponding diagonal matrix.
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Minimizing this **Rayleigh quotient** is equivalent to solving the generalized eigenvalue system

\[(D - W)y = \lambda Dy\]

This is a normal eigenvalue problem

\[(I - N)z = \lambda z\]

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This is an example of a spectral method for segmentation, solution is the second smallest eigenvector/eigenvalue.
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\[w_{i,j} = \exp \left( -\frac{\|F_i - F_j\|_2^2}{\sigma_f^2} - \frac{\|p_i - p_j\|_2^2}{\sigma_s^2} \right)\]

for pixels within a radius \(\|p_i - p_j\|_2 < r\), and \(F\) is a feature vector with color, intensities, histograms, gradients, etc.
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Algorithm

1. Given an image or image sequence, set up a weighted graph $G = (V, E)$ and set the weight on the edge connecting two nodes to be a measure of the similarity between the two nodes.
2. Solve $(D - W)x = \lambda Dx$ for eigenvectors with the smallest eigenvalues.
3. Use the eigenvector with the second smallest eigenvalue to bipartition the graph.
4. Decide if the current partition should be subdivided and recursively repartition the segmented parts if necessary.
Figure: Shi and Malik N-Cuts
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Graph-based Superpixels

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- Define edges between neighboring pixels, with whatever definition of neighboring
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- Let $G(V, E)$ be the graph, we want to segment $V$ into components $(C_1, \ldots, C_r)$
- Felzenswald and Hutterlocker defined a simple greedy algorithm which can be shown to have some interesting global properties
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1. Sort $E$ into $\pi = (o_1, \cdots, o_m)$ by non-decreasing weights

2. Start with segmentation $S^0$, where each vertex is its own component (i.e., as many superpixels as pixels)
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4. Construct $S^q$ given $S^{q-1}$ as follows. Let $o_q = (v_i, v_j)$. If $v_i$ and $v_j$ are disjoint components of $S^{q-1}$ and $w(o_q)$ is small compared to the internal difference of both components of $S^{q-1}$, then merge the two components. Otherwise do nothing $S^q = S^{q-1}$
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The internal difference is defined as the largest weight in the minimum spanning tree of the component.

$$Int(C) = \max_{e \in MST(C,E)} w(e)$$
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[P. Felzenszwalb and D. Huttenlocher, IJCV04]
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Basics of Kernel Density Estimation

- We have a bunch of points drawn from some distribution
- What’s the distribution that generated these points?

[Source: M. Tappen]
We can fit a parametric distribution, e.g., mixture of Gaussians

KDE idea: Use the data to define the distribution
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KDE idea: Use the data to define the distribution.

If I were to draw more samples from the same probability distribution, then those points would probably be close to the points that I have already drawn.
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KDE idea: Use the data to define the distribution.
- If I were to draw more samples from the same probability distribution, then those points would probably be close to the points that I have already drawn.
- Build distribution by putting a little mass of probability around each data-point.

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[Source: M. Tappen]
Figure 2-2: Kernel density estimates of the density function shown in Figure 2-1(a). Figure (a) shows the estimate found with a relatively small number of samples. It is uneven and does not approximate the true density well. (b) With more samples, the estimate of the density improves significantly.

[Source: M. Tappen]
We approximate the density by

\[ \hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K_H(x - x_i) \]

with \( x_i \) the points, and \( K_H(x - x_i) \) the kernel

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Alternative way to think about this, put 1 wherever you have a sample and convolve with a Gaussian
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What is mean-shift

- The density will have peaks (also called modes)
- If we started at point and did gradient-ascent, we would end up at one of the modes
- Cluster based on which mode each point belongs to

[Source: M. Tappen]
No need for gradient ascent

- A set of iterative steps can be taken that will monotonically converge to a mode
- No worries about step sizes
No need for gradient ascent

- A set of iterative steps can be taken that will monotonically converge to a mode
- No worries about step sizes
- This is an adaptive gradient ascent, for each iteration

\[ y_{j+1} = \frac{\sum_{i=1}^{n} x_i g(\|y_j - x_i\|_2^2)}{\sum_{i=1}^{n} g(\|y_j - x_i\|_2^2)} \]

with \( g = \frac{d}{du} k(u) \), and \( k(x) = C \sum_{i=1}^{n} k(\|y_j - x_i\|_2^2) \)
No need for gradient ascent

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  - This procedure gives you one mode, how to get all?
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[Source: M. Tappen]
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[Source: M. Tappen]
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Let’s look at Structure from Motion
Structure from motion

- We saw in class how 2D and 3D point sets could be aligned
- We also saw how this alignment could be used to estimate camera pose and internal parameters
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Structure from motion

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This process typically involves simultaneous estimation of 3D geometry (structure) and camera pose (motion).
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Structure from motion

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Triangulation

- It is the problem of determining a point’s 3D position from a set of corresponding image locations and known camera positions.
- This problem is the converse of the pose estimation problem.

Simplest solution: find 3D point $\mathbf{p}$ that lies closest to all the 3D rays corresponding to the 2D feature locations $\{x_j\}$ observed by cameras $\mathbf{P}_j = \mathbf{K}_j [\mathbf{R}_j | \mathbf{t}_j]$, with $\mathbf{t}_j = -\mathbf{R}_j \mathbf{c}_j$. 
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The rays originate at $c_j$ in a direction $\hat{v}_j = \mathcal{N}(R_j^{-1}K_j^{-1}x_j)$.

The nearest point to $p$ is the point $q_j = c_j + d_j\hat{v}_j$ such that

$$\min_{d_j} ||c_j + d_j\hat{v}_j - p||^2_2$$
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$$\mathbf{q}_j = \mathbf{c}_j + (\hat{\mathbf{v}}_j \hat{\mathbf{v}}_j^T)(\mathbf{p} - \mathbf{c}_j) = \mathbf{c}_j + (\mathbf{p} - \mathbf{c}_j)_\parallel$$
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q_j = c_j + (\hat{v}_j \hat{v}_j^T)(p - c_j) = c_j + (p - c_j)_\parallel
$$

The square distance is then

$$
r_j^2 = \|(I - \hat{v}_j \hat{v}_j^T)(p - c_j)\|^2 = \|(p - c_j)_\perp\|^2
$$
Triangulation

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  \]

- The optimal value of $p$ can be computed by least-squares
  \[
  p = \frac{\sum_j (I - \hat{v}_j \hat{v}_j^T)c_j}{\sum_j (I - \hat{v}_j \hat{v}_j^T)}
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Triangulation

The nearest point to \( p \) is the point \( q_j \) such that

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This has a minimum at \( d_j = \hat{v}_j \cdot (p - c_j) \), thus

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q_j = c_j + (\hat{v}_j \hat{v}_j^T)(p - c_j) = c_j + (p - c_j) \parallel
\]

The square distance is then

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\[
p = \frac{\sum_j (I - \hat{v}_j \hat{v}_j^T)c_j}{\sum_j (I - \hat{v}_j \hat{v}_j^T)}
\]
Alternative formulation

- Can produce significantly better estimates if some cameras are closer to the 3D point than others is to minimize the residual in the measurement equations

\[
\begin{align*}
  x_j &= \frac{p_{j0} X + p_{j1} Y + p_{j2} Z + p_{j3} W}{p_{j20} X + p_{j21} Y + p_{j22} Z + p_{j23} W} \\
  y_j &= \frac{p_{j10} X + p_{j11} Y + p_{j12} Z + p_{j13} W}{p_{j20} X + p_{j21} Y + p_{j22} Z + p_{j23} W}
\end{align*}
\]

where \((x_j, y_j)\) are the measured 2D feature locations, and \(\{p_{j0}, \ldots, p_{j23}\}\) are the known entries in the camera matrix \(P\), and \(p = (X, Y, Z, W)\) in homogeneous coordinates

- Why is this better?
- How do we solve this now?
Two frame structure from motion

Figure: Images from N. Snavely

- Simultaneous recovery of 3D structure and pose from image correspondences
- Why is this not the stereo problem?
Two frame structure from motion

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- Why is this not the stereo problem?
- What can we do?

\[ x' \begin{bmatrix} F \end{bmatrix} x = 0 \]
Two frame structure from motion

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- We can estimate the fundamental matrix given enough correspondences!

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\[ x'^T F x = 0 \]
8 Point Algorithm

- We can write a system of equations

\[
\begin{bmatrix}
    u_1 u'_1 & v_1 u'_1 & u'_1 & u_1 v'_1 & v_1 v'_1 & v'_1 & u_1 & v_1 & 1 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    u_n u'_n & v_n u'_n & u'_n & u_n v'_n & v_n v'_n & v'_n & u_n & v_n & 1
\end{bmatrix}
\begin{bmatrix}
f_{11} \\
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    f_{33}
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- Shouldn’t \( F \) have rank 2?

\[
\min_{F'} \|F - F'\|_2^2
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- What happens in the presence of noise?

- Normalize the points to have mean 0 and unit variance (Hartley 99)
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- This works better in practice
[Source: N. Snavely]
Results

[Source: N. Snavely]
Given the fundamental matrix, we can calibrate the cameras

Given this calibration we can triangulate

This is a chicken and egg problem so we can re-iterate this process.
Pure Translational Motion (known rotation)

- If we know the rotation, we can pre-rotate all the points in the second image to match the viewing direction of the first.
- The resulting set of 3D points move towards (or away from) the focus of expansion (FOE)
- See exercise about this ...

![Diagram](adapted from Gibson 1978)

- What if it's a purely rotational motion?
Pure Translational Motion (known rotation)

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What if it's a purely rotational motion?
- What happens in the general case if we know K?
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What if its a purely rotational motion?
- What happens in the general case if we know \( K \)?
The geometry of three views is described by a $3 \times 3 \times 3$ tensor called the **trifocal tensor**

The geometry of four views is described by a $3 \times 3 \times 3 \times 3$ tensor called the **quadrifocal tensor**

After this it starts to get complicated ...

We will not see this in class
Structure from motion

Given many images, how can we

- figure out where they were all taken from?
- build a 3D model of the scene?

[Source: N. Snavely]
Structure from Motion

- **Input:** images with points in correspondence \( p_{i,j} = (u_{i,j}, v_{i,j}) \)
- **Output:**
  - **structure:** 3D location \( x_i \) for each point \( p_i \)
  - **motion:** camera parameters \( R_j, t_j \) possibly \( K_j \)
- **Objective function:** minimize reprojection error

[Source: N. Snavely]
Reconstructions from Video

[Source: N. Snavely]
How do we get correspondences?

- Feature detection and matching
- We can construct a graph of matches
- Use RANSAC to estimate fundamental matrices between each pair

[Source: N. Snavely]
Structure from Motion Problem

\[ \prod_1 X_1 \sim p_{11} \]

\[ \text{minimize } \ g(R, T, X) \]

\[ \text{non-linear least squares} \]

[Source: N. Snavely]
Problem Size

- What are the variables?
- How many variables per camera?
- How many variables per point?
- E.g., Trevi Fountain collection, 466 input photos, $>100,000$ 3D points

[Source: N. Snavely]
Bundle Adjustment

- Minimize sum of squared reprojection errors:

\[
g(X, R, T) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \| P(x_i, R_j, t_j) - \begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix} \|_2^2
\]

with \( w_{ij} \) indicator whether they are visible or not, \( P(x_i, R_j, t_j) \) the predicted image location, and \( (u_{i,j}, v_{i,j}) \) the observed image location

- Minimizing this is called bundle adjustment
Bundle Adjustment

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- We saw some other versions in class.
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Optimized using non-linear least squares, e.g. Levenberg-Marquardt
Bundle Adjustment

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- Optimized using non-linear least squares, e.g., Levenberg-Marquardt.
- Initialization is very important.
Minimize sum of squared reprojection errors:

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Initialization is very important.
Failure cases

- Necker reversal

[Source: N. Snavely]
Failure cases

- Repetitive Patterns

[Source: N. Snavely]
Incremental Structure From Motion

[Source: N. Snavely]
What’s the problem with this approach?
More Results

[Source: N. Snavely]
Applications: 3D Reconstruction from Photo Collections

Figure 1: Our system takes unstructured collections of photographs such as those from online image searches (a) and reconstructs 3D points and viewpoints (b) to enable novel ways of browsing the photos (c).

[N. Snavely et al. Siggraph 2006]