Assignment #5

Due Date: Wednesday, April 11 at 10:00am

1. Let $\mathcal{LG}$ be a first-order language having an infinite set of variables that includes x, y, and z, and predicates S, P, and M of arity 2, 3, and 1, respectively. Consider an interpretation where the the domain D is the set of all people, and the predicates are defined as follows:

   - let $S(x, y)$ be "x and y are siblings"
   - let $P(x, y, z)$ be "x and y are the parents of z"
   - let $M(x)$ be "x is male"

   (a) Give the precise, concise English meaning of

   $$\forall u(\exists x \exists y(P(x, y, u) \land P(x, y, v)) \rightarrow S(u, v))$$

   (b) Write a formula that expresses the claim "x has no uncle". Please also explain your formula in clear English.

2. Give a formula in Prenex Normal Form that is logically equivalent to

   $$(P(x, y) \land \neg\exists z R(x, y, z)) \rightarrow \exists y(P(y, x) \lor \forall u R(u, x, y))$$

3. Suppose we have the alphabet $\Sigma = \{0, 1\}$. Consider the language

   $$\mathcal{L} = \{x \in \Sigma^* : x \text{ contains an odd number of 0s}\}$$

   (a) Give a regular expression that corresponds to $\mathcal{L}$.

   (b) Prove that the language denoted by your regular expression in part (a) is indeed the same language as $\mathcal{L}$.

4. Again, let our alphabet be $\Sigma = \{0, 1\}$ as in the previous question. For each of the following languages, give a FSA that accepts the language.

   (a) $\mathcal{L}_1 = \{x \in \Sigma^* : x \text{ contains an even number of 1s}\}$

   (b) $\mathcal{L}_2 = \{x \in \Sigma^* : x \text{ contains an odd number of 0s and an odd number of 1s}\}$

   (c) $\mathcal{L}_3 = \{x \in \Sigma^* : x \text{ contains both 101 and 010 as a substring}\}$

   For full marks, your FSA must be reasonably efficient.

5. The exponentiation operation on languages is defined inductively by

   $$\mathcal{L}^k = \begin{cases} \{\epsilon\} & \text{if } k = 0 \\ \mathcal{L}^{k-1}\mathcal{L} & \text{if } k > 0 \end{cases}$$

   Show that if $\mathcal{L}$ is a regular language, then $\mathcal{L}^n$ is a regular language for $n \geq 0$.