Assignment #1

Notes:

Before you get started on the assignment, there are a few things to keep in minds.

As has been mentioned in class, we're going to be fairly demanding. When you're writing up your solutions, take some time to make sure your proofs are clear and complete. This also means that before each inductive proof we'd like you to give:

- a definition of P
- the universal you are proving for P
- the inductive principle used to prove that universal

It is also suggested that when asking for help, you show us as much work as you can (e.g. a table of values, inductive definitions of non-inductive sequences, etc.) If we don't see any of that work, we'll start helping you by working on those introductory steps. Your time, and ours, will be used far more efficiently if we can dive into the harder parts.

Problems:

1. (a) Give a symbolic statement that expresses

   Student S will pass CSC236 if he does his homework

   Please define any domains and properties that you need.

   (b) If student S passes CSC236, what can we say about his work ethic? What about if student S fails the course?

   (c) In a skating competition, skaters are given scores based on their performance and then ranked according to their score. The skater with the highest score wins, the skater with the second highest score comes in second, and so on.

   Let `S = the set of skaters competing`.
   Let `beat( s1, s2 ) = "skater s1 received a higher score than s2"`.
   Give a precise symbolic expression to express each of the following:

   - skater s won the competition
   - skater s came in third

2. Prove that $1 + r + r^2 + ... + r^n = \frac{1-r^{n+1}}{1-r}$ for $r \neq 1$ using induction.

3. Some of you may be familiar with the Tower of Hanoi puzzle. You are given three pegs. On one of the pegs is a tower of n rings, placed one on top of the other, such that as you move down the tower each successive ring has a larger diameter than the one above it. The other two pegs are empty. The object of the puzzle is to reconstruct the tower on one of the other pegs by moving one ring at a time, without ever moving a larger ring onto a smaller one.

   Prove that for any $n \in N$, the puzzle can be completed in $2^n - 1$ moves.

4. Consider the sequence

$$\begin{aligned} a_1 &= 1 \\ a_{n+1} &= 2a_n + 1 \end{aligned}$$

   (a) Write a recursive Java method that, given n, will compute a_n

   (b) Write an iterative Java method that, given n, will compute a_n

   (c) Prove that $\forall k \in N, k \geq 1, \quad a_k = 2^k - 1$

5. Let $T_n = \frac{n(n+1)}{2}$ and let $P_n = \frac{n(n+1)(n+2)}{6}$. Prove that $\forall n \geq 1, \quad \sum_{i=1}^{n} T_i = P_n$

6. Consider the following 'proof' by induction that there are no horses of a different colour:

   Let $P(n)$ be the statement that for every set of n horses, all horses in the set have the same colour coat. This is obviously true for $n = 1$. Now consider $n > 1$. Suppose that $P(n-1)$ is true. Given a group of n horses, consider all but the last horse in the group. This leaves us with $n - 1$ horses that all must have the same colour coat by the induction hypothesis. Now consider all but the first horse in the group. Again, the induction hypothesis tells us that that remaining $n - 1$ horses must all have the same colour coat. By combining these two facts, we find that all n horses have the same colour coat. Therefore, by induction, all horses have the same colour coat.

   This is quite obviously not true, so what is wrong with the argument?