Kinds of Full Physical Containment

Torsten Hahmann and Boyan Brodaric *

Department of Computer Science, University of Toronto, Toronto, ON, Canada
Geological Survey of Canada, Natural Resources Canada, Ottawa, ON, Canada

Abstract. Full physical containment is the relation in which one physical entity is completely inside another. It is central to the description of natural resources held in reservoirs above or below the surface. Previous ontological representations of containment are located in abstract space, incomplete, or insufficiently incorporate voids, so in this paper we develop a complete taxonomy for the full containment relation that is situated in physical space and integrates voids. The taxonomy is formalized in a mereotopological theory and specializes the DOLCE foundational ontology, thus advancing hydro ontology development.

Keywords: knowledge representation, geospatial ontology, spatial relations, hydro ontology, hydrogeology, physical containment, taxonomy, container schema, physical void, hole, material, immaterial, DOLCE

1 Introduction

Containment is critical to the geosciences. It plays a foundational role in the description of both surface and subsurface resources, such as water, petroleum, and natural gas, where a nuanced notion is required to capture the subtle ways that something can be inside or surrounded by something else. For example, estimates of subsurface fluids or gases need to know how much underground space exists, how much of it is fillable and open to flow, and how much is actually filled or flowing. A container's spaces—its voids—assume particular significance in these scenarios, as it is their size and arrangement (together known as porosity [17]) that is used to determine storage and flow. This is particularly evident in models and simulations of subsurface resources, which inherently rely on a sophisticated containment schema that delineates three things, a container, its voids, and a containee, as first-class entities. However, these distinctions are largely absent in emerging domain representations, such as data standards for water and geology [1, 21], which either ignore containment or omit voids from containment relations. Their omission is problematic given the expected role that such standards will play in delivering data from distributed databases, and sensor networks, to modeling and simulation tools. Theoretical work on qualitative topological relations [4, 5, 9, 10, 14, 16] can help bridge this representation gap, but the relations are modeled in an abstract mathematical-geometric conception of space rather than in physical space, accounting for topological constraints but not for physical constraints. The relations also hold only between untyped regions, which are akin to mathematical objects, rather than

* We gratefully acknowledge support from the Groundwater Program of Natural Resources Canada.

between physically typed entities, such as voids or material bodies, thus the physical significance of the relation is not captured. In short, what is required is an ontological interpretation of topological relations in which abstract containment is considered physically.

In this paper, we begin such an interpretation motivated by various containment relations associated with both surface and subsurface water. An ultimate endpoint for this work is its incorporation into a hydro ontology. The paper thus aims to accomplish a specific piece of ontology engineering, and in doing so it makes the following original contributions: (1) it provides a taxonomy of the full containment relation, in which the relation and its components are interpreted physically (see Figure 4); (2) it specializes the DOLCE foundational ontology [19] to provide types for the components of the relation; and (3) it grounds these physical distinctions in a first-order theory of abstract space, which provides mereotopological and mereogeometrical relations.

The paper is organized as follows: Section 2 illustrates motivating examples; Section 3 discusses related work; Section 4 introduces some background material; Sections 5–7 describe the full containment relation, first generically and then partitioned into dependent and detachable containment; Section 8 provides some additional discussion and the paper concludes in Section 9 with a recap and future directions.

2 Containment scenario

Various types of physical containment that motivate this work are illustrated in Figure 1. Included are the following relations:

$contains(\mathrm{LB}, \mathrm{SWB})$ $contains(\mathrm{LB}, \mathrm{Rock})$ $contains(\mathrm{LB}, \mathrm{Hole})$
$contains(\mathrm{Hole}, \mathrm{SWB})$ $contains(\mathrm{Hole}, \mathrm{Rock})$ $contains(\mathrm{SWB}, \mathrm{Rock})$
$contains(\mathrm{AQ}, \mathrm{GWB})$ $contains(\mathrm{AQ}, \mathrm{RM})$ $contains(\mathrm{AQ}, \mathrm{CT})$
$contains(\mathrm{AQ}, \mathrm{Gaps})$ $contains(\mathrm{Gaps}, \mathrm{GWB})$ $contains(\mathrm{Gaps}, \mathrm{CT})$
$contains(\mathrm{GWB}, \mathrm{CT})$

Described is a lake: a surface water body (SWB) located in a lakebed (LB) and thus also in its associated hole. Also described is a rock at the bottom of the lake, a groundwater body (GWB) in the gaps of an aquifer's (AQ) rock matter (RM), and a contaminant (CT) within the groundwater body. These examples show that full physical containment is primarily considered here, mainly for scoping reasons, but also because it is most frequently encountered in the water domain. Full physical containment involves complete immersion or enclosure of one entity within another, such as a rock completely immersed in the lake, or a body of water completely enclosed by the lakebed, but excludes partial containment such as a boat floating partially submerged on the lake.

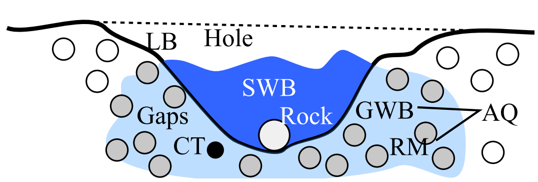

Fig. 1. Containment examples in surface and subsurface water. See text for details.

Despite this narrowing of scope, it is apparent that a physical interpretation of full containment can lead to useful specializations, e.g. being inside a hole is different from, but associated with, being surrounded by a material container. These and many other distinctions are elaborated in Sections 6 and 7.

3 Related work

Containment relations have been considered in image schemata, qualitative spatial relations, formal top-level ontologies, and geoscience representations. In the work to date, however, containment relations are either detached from an associated physical ontology, the participation of voids is limited, or the taxonomy is incomplete.

Containment image schema is a cognitive-driven template for containment. Diversely studied in terms of its formalization and uses [18, 24], it typically exhibits a rudimentary ontology consisting of the roles played by participants (container/containee), their potential behaviour (moving in/out), and states (inside/outside). However, this ontology lacks a key participant—the void afforded by a container and occupied by a containee—and also generally lacks a physical typing of participants, which could lead to erroneous interpretations. Also typically lacking are physical constraints, such as the existential dependence of a void on its host container.

Qualitative spatial relations have been studied extensively, most notably in the 9-intersection and RCC paradigms, which describe the possible topological relations that occur between abstract regions as well as their compositions and inferences [4, 5, 9, 10]. Voids can implicitly participate in the defined abstract containment relations, inasmuch as a void can be represented as an abstract region, but there is no mechanism for typing the region as a void [12]. These relations are, then, subject to concerns similar to those expressed about image schema: a lack of physical typing and associated constraints.

Formal top-level ontologies such as DOLCE and BFO [13, 19], include neither physical containment relations nor voids, but provide a superstructure from which they can be specialized. Indeed, DOLCE physical entities and their dependants have been extended to encompass containers and voids, respectively [15], laying the groundwork for a physical containment relation. The upper ontology SUMO [20] does include two basic containment relations, 'contains' and 'inside': while 'contains' is a partial containment relation that is further specialized into a proper and a complete version, the 'inside' relation lacks a clear definition and other kinds of containment are missing. Cyc [11], a large collection of commonsense knowledge, possesses a rich suite of physical containment relations that are, however, also incomplete: for example, a void cannot contain something, except implicitly via the very general 'inside' relation, which has no constraints on its domain and range. Conversely, voids can explicitly contain material entities in one of four containment relations in [8], but the remaining relations occur between abstract regions, again without physical typing and constraints. Similarly, holes (but not gaps) are fillable in [2], though other types of containment relations are missing.

Relevant geoscience representations for water or geology, such as ontologies or data transfer standards, e.g. [21, 23], do not include containment relations nor voids. An exception is the Groundwater Markup language (GWML) [1], in which water reservoirs offer a limited notion of voids, and where a containment relation holds between a reservoir and the matter filling it, but GWML is not expressed formally and lacks a general approach to containment.

4 Background

This work follows in the footsteps of [15] and relies on a distinction between *physical space* and abstract spatial regions of purely topological/geometrical nature. For the former we specialize the DOLCE foundational ontology, and for the latter we reuse the first-order spatial ontology from [14, 16], which is a multidimensional version of the first-order axiomatization of the Region Connection Calculus (RCC) [4]. This section reviews the relevant DOLCE categories of physical entities, the theory of abstract space, and the formalization of physical voids from [14, 15]. We follow [14] in notation and axiom numbering. Free variables in our logical sentences are assumed to be implicitly universally quantified.

Physical endurants —real entities of primarily physical nature that populate *physical space* form the domain of interest for our study of physical containment. These physical entities are captured by the DOLCE category PED, as shown in Figure 2. They can be physical objects (e.g. rocks), POB, amounts of matter (e.g. clay), M, or physical features, F. Features are either relevant part features (e.g. a bump or an edge), RPF, which are constituted by a portion of the matter of their associated physical object, or dependent place features (e.g. a hole or a shadow), DPF, which are *immaterial*, i.e., not constituted of any matter. All other physical endurants are *material*, denoted by the predicate $mat(x)$. The predicate $DK_1(x, y)$, called *direct primary constitution*, denotes that x is the immediate matter constituting an object or relevant part y.

(Mat-D) $mat(x) \leftrightarrow POB(x) \lor M(x) \lor RPF(x)$ (material endurant)
($\mathbf{DK_1}$-D) $DK_1(x, y) \rightarrow M(x) \land [POB(y) \lor RPF(y)] \land P(\mathrm{r}(x), \mathrm{r}(y))$
(primary direct constitution of an object or relevant-part feature by matter)

Physical endurants are entities extended in physical space; following the ideas of [7, 8] we assign them a location in abstract space using the $\mathrm{r}(x)$ function. The resulting abstract regions, i.e., the entities satisfying $S(x) \leftrightarrow x = \mathrm{r}(x)$, are disjoint from the set

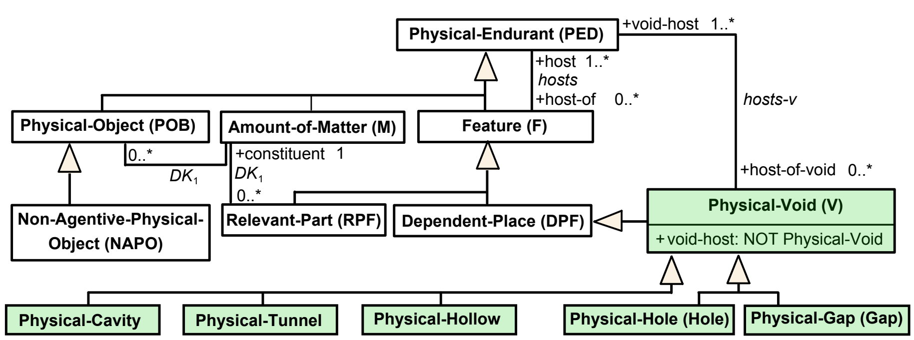

Fig. 2. The DOLCE category of physical endurants and its refinements, as UML diagram.

of physical endurants that populate physical space. This distinction between physical endurants and abstract regions allows us to consider abstract space as a mathematical-geometrical construct, which provides greater flexibility in all spatial operations. For example, many physical features of physical endurants, such as physical boundaries, are defined in terms of their regions.

Abstract regions of space are interpreted as topologically closed regions of abstract space obtainable by gluing together finite sets of *manifolds with boundaries* [14]. Their dimension can vary, e.g., both a 2D area and its 1D boundary are valid regions. We compare regions dimensionally using the predicates $<_{\mathrm{dim}}$, $=_{\mathrm{dim}}$, and $\prec_{\mathrm{dim}}$ that denote lower, equal, and next lower dimension, respectively. We assume that all physical endurants are located in a region of maximal dimension, that is, of codimension 0. For example, all physical endurants in a 3D space must also be 3D.

(MaxDim-D) $MaxDim(x) \leftrightarrow S(x) \wedge \neg\exists y[x <_{\mathrm{dim}} y]$ (regions of maximal dim.)
(S-A8) $PED(x) \rightarrow MaxDim(\mathrm{r}(x))$ (PEDs are located in regions of maximal dim.)

Regions can be in various spatial relationships. These relationships are all founded on a single primitive spatial relation $Cont(x, y)$ that expresses 'x is a subregion (of equal or lower dimension) of y' and that is reflexive, antisymmetric, and transitive[1]. For mathematical simplicity we include an empty region of lowest dimension, denoted as $ZEX(x)$. Among other relations, we define contact C, parthood P, proper parthood PP, overlap PO, and superficial contact SC; those apply only to regions[2], but we can, for example, write $C(\mathrm{r}(x), \mathrm{r}(y))$ to state that the physical endurants x and y are in contact. Regions that overlap exactly the same set of regions are considered equivalent (PO-E1).

(C-D) $C(x, y) \leftrightarrow \exists z\,[Cont(z, x) \wedge Cont(z, y)]$ (contact: a shared entity exists)
(EP-D) $P(x, y) \leftrightarrow Cont(x, y) \wedge x =_{\mathrm{dim}} y$ (equi-dimensional parthood)
(EPP-D) $PP(x, y) \leftrightarrow P(x, y) \wedge \neg P(y, x)$ (equi-dimensional proper parthood)
(PO-D) $PO(x, y) \leftrightarrow \exists z\,[P(z, x) \wedge P(z, y)]$ (overlapping in a part)
(SC-D) $SC(x, y) \leftrightarrow C(x, y) \wedge \neg\exists z[Cont(z, x) \wedge P(z, y)] \wedge \neg\exists z[P(z, x) \wedge Cont(z, y)]$ (superficial contact)
(PO-E1) $S(x) \wedge S(y) \rightarrow \big[\forall z[PO(x, z) \leftrightarrow PO(y, z)] \rightarrow x = y\big]$
(PO-extensionality: two regions with the same PO-extension are equivalent)

The set of regions of maximal dimension is assumed mereologically closed, that is, for every pair of regions x and y with $MaxDim(x)$ and $MaxDim(y)$ and thus for every pair of regions corresponding to physical endurants, the intersection $x \cdot y$, the difference $x - y$, and the sum $x + y$ are defined (they may yield the zero region); for their precise definitions we refer to [14]. Moreover, we assume that a universal region S_{u} of maximal dimension exists with $MaxDim(S_{\mathrm{u}})$ and $\forall x[S(x) \wedge \neg ZEX(x) \rightarrow Cont(x, S_{\mathrm{u}})]$, so that all regions of maximal dimension have a complement $x' = S_{\mathrm{u}} - x$. These mereological closure operations apply only to regions; we do not force the set of physical entities to be closed in the same way. The operations allow us to define strong contact, $C_{\mathrm{S}}(x, y)$, as sharing a region of the next lower dimension. E.g., 3D bodies are in strong contact

[1] The relation $Cont(x, y)$ is equivalent to the relation $x \subseteq y$ used in [15].
[2] This deviates from the more relaxed use of the predicates in [15].

if they touch in a 2D surface, but not if they only touch in a curve segment or in points. This, in turn, lets us define (self-)connectedness and internal (self-)connectedness (also known as *strong self-connectedness*), meaning that the interior of $x+y$ is a single piece. The universal region S_u is assumed to be internally connected, that is, $ICon(S_\mathrm{u})$.

(C$_\mathrm{S}$-D) $C_\mathrm{S}(x,y) \leftrightarrow SC(x,y) \wedge x =_\mathrm{dim} y \wedge r(x) \cdot r(y) \prec_\mathrm{dim} x$ (strong contact)
(Con-D) $Con(x) \leftrightarrow \forall y[PP(y,x) \rightarrow C(y, r(x) - r(y))]$ (connected)
(ICon-D) $ICon(x) \leftrightarrow \forall y[PP(y,x) \rightarrow C_\mathrm{S}(y, r(x) - r(y))]$ (internally connected)

We further use a primitive function $\mathrm{ch}(x)$ to denote the convex hull region of x. It is needed to specify necessary spatial conditions for the existence of voids. See [14] for a more complete axiomatization of ch based on ideas from [4, 8].

Voids , V, are special kinds of immaterial dependent place features, as they depend on some hosting, non-void, physical endurant. Voids include holes (following [2]) and gaps, which are differentiated according to whether their hosts are internally connected or not, respectively [15]. Any void must be located in the convex hull $\mathrm{ch}(x)$ of a host x, but the region $\mathrm{ch}(x) - \mathrm{r}(x)$ need not be completely covered by voids. In other words, part of the convex hull of an endurant may be neither the location of a material part nor of a void of the endurant. For example, the space between the base and the bulb of a wine glass is typically not called a void. As identifying voids is thus somewhat arbitrary, $hosts\text{-}v$ is a primitive, i.e., undefined relation. It can be refined in various ways [15], such as hosting a hole or a gap (depending on the host's internal connectedness); hosting a cavity, hollow, or tunnel (depending on the void's *opening*, see V-A12); or hosting an external or internal void (depending on the contact of the void's opening to the exterior or to other voids in the same host).

(V-A1′) $hosts\text{-}v(y,x) \rightarrow PED(y) \wedge \neg V(y) \wedge V(x) \wedge P(\mathrm{r}(x), \mathrm{ch}(y)) \wedge C_\mathrm{S}(\mathrm{r}(x), \mathrm{r}(y)) \wedge PO(\mathrm{r}(x), \mathrm{r}(y))$ (hosting a void: relation between a void x and its physical host y)
(V-D) $V(x) \leftrightarrow \exists y[hosts\text{-}v(y,x)]$ (all voids are hosted)
(V-A11) $hosts\text{-}v(x,v) \rightarrow \mathrm{op}(x,v) = \mathrm{r}(v) \cdot (\mathrm{r}(x) + \mathrm{r}(v))'$
(the opening of a void v is the part of its boundary that is not shared with its host)
(V-A12) $hosts\text{-}cav(x,y) \leftrightarrow hosts\text{-}v(x,y) \wedge \mathrm{op}(x,y) \not\prec_\mathrm{dim} \mathrm{r}(x)$ (hosting a cavity: hosting a void with an opening that is not of the dimension of its host's boundary)

We can ascribe voids to the level of granularity at which they occur [15]: a physical object directly hosts larger, macroscopic voids, while more minuscule voids are hosted by the object's constituent matter. The region encompassing all voids within an object's matter defines its *pore space*—it is the difference between the object's region and the region corresponding to its constituent matter. This two-level model of physical space can be extended to multiple levels to represent, for example, that a rock body constituted by grains of rock matter (first level) is also constituted by crystals (second level) and then molecules and atoms (third level). To accommodate multiple levels of granularity, we introduce a set of hosting relations expressing that a physical endurant hosts a void at the n-th level of granularity, written as $hosts\text{-}v_n(y,x)$ with n being a natural number. $hosts\text{-}v(y,x)$ is then equivalent to $hosts\text{-}v_0(y,x)$. The relations $hosts\text{-}v_n$ will be properly defined and axiomitized in subsequent work. Here, the relation $hosts\text{-}v_\mathrm{any}(y,x)$ is used to express that y hosts the void x at *some* level of granularity—assuming a finite number of granularity levels. Then the entire void space of a physical endurant x, $\mathit{voidspace}_{\mathrm{all}}(x)$, is defined as the sum of the regions of all voids therein, independent of their level of granularity. This region must correspond to a void itself, which is intuitively hosted by the object's constituent matter and encompasses all the empty spaces within an object that can be filled by matter of the lowest granularity.

(V$_{\mathbf{any}}$-D) $\mathit{hosts}\text{-}v_{\mathrm{any}}(y,x) \leftrightarrow \exists(n \geq 0)\big[\mathit{hosts}\text{-}v_n(y,x)\big]$
(hosting a void at any granularity level)

(V-A25) $PO(y, \mathit{voidspace}_{\mathrm{all}}(x)) \leftrightarrow \exists v[\mathit{hosts}\text{-}v_{\mathrm{any}}(x,v) \wedge PO(y, \mathrm{r}(v))]$
(the union of the voids of all levels of granularity define $\mathit{voidspace}_{\mathrm{all}}(x)$)

(V-A26) $\mathit{mat}(x) \wedge \neg \mathit{ZEX}(\mathit{voidspace}_{\mathrm{all}}(x)) \rightarrow \exists y, h[\mathrm{r}(y) = \mathit{voidspace}_{\mathrm{all}}(x) \wedge \mathit{hosts}\text{-}v(h,y)]$ (some void is located in a material endurant's nonempty void space)

5 Generic physical containment

Three restrictions are imposed here on full physical containment relations. First, all participants must be physical endurants, i.e. physical objects, amounts of matter, or related features such as voids. Second, the full physical containment relation addresses spatial inclusion in physical space, eliminating other forms of non-physical or non-spatial containment, such as a file containing data, a book containing information, or my heart containing feelings. Third, only static containment relations are considered: change, motion, and other time-related ideas are beyond the scope of this paper. The predicate $\mathit{fully}\text{-}\mathit{phys}\text{-}\mathit{contains}(y,x)$ is used to denote this generic kind of full physical containment, which has the necessary condition that x's region is a subregion of y's convex hull. Any two physical endurants—material or immaterial—can participate in generic full physical containment. However, full containment in immaterial containers is still more restrictive: the containee x must not only be located within container y's convex hull, but within y's region. Figures 3(a) and (b) demonstrate the need for this restriction.

(FPCont-D) $\mathit{fully}\text{-}\mathit{phys}\text{-}\mathit{contains}(y,x) \leftrightarrow \mathit{PED}(x) \wedge \mathit{PED}(y) \wedge P(\mathrm{r}(x), \mathrm{ch}(y)) \wedge \big[\neg \mathit{mat}(y) \rightarrow P(\mathrm{r}(x), \mathrm{r}(y))\big]$ (for x to be generically fully physically contained in y, both x and y must be physical endurants and x's region must be within y's convex hull and, if y is immaterial, within y's region)

Where feasible, we identify relationships to the containment relations presented in [8, 5] and give explicit mappings (labelled **Mx**) to the relations from [5]. The relation $\mathit{fully}\text{-}\mathit{phys}\text{-}\mathit{contains}(y,x)$ resembles the union of Donnelly's generic and material-region containment predicates $\mathit{CNT}\text{-}\mathit{IN}_{\mathrm{g}}(x,y)$ and $\mathit{CNT}\text{-}\mathit{IN}_{\mathrm{mr}}(x,y)$ [8], as well as the $\mathit{INSIDE}(x,y)$ predicate defined in [5], but differs by its addition of physical typing.

(M1) $\mathit{fully}\text{-}\mathit{phys}\text{-}\mathit{contains}(y,x) \rightarrow \mathit{INSIDE}(\mathrm{r}(x), \mathrm{r}(y)) \wedge [\mathit{CNT}\text{-}\mathit{IN}_{\mathrm{g}}(x,y) \vee \mathit{CNT}\text{-}\mathit{IN}_{\mathrm{mr}}(x,y)]$ (y generically physically contains x implies $\mathit{INSIDE}(x,y)$)

For brevity, we subsequently drop the qualification "physical" with the understanding that *all* containment relations in this paper occur exclusively between two physical endurants. Two types of generic containment relations are distinguished first, using the physical dependence between participants as a discriminator: dependent containment and detachable containment, as illustrated by the initial division of full physical containment in the taxonomy in Figure 4. Intuitively, this division delineates whether the

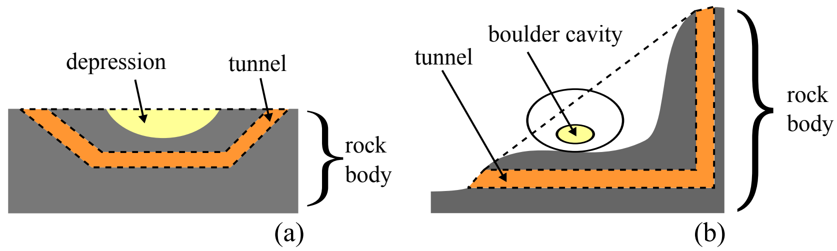

Fig. 3. Examples of a physical endurant spatially contained in the convex hull of an immaterial physical endurant, but not in its region. On the left, the depression is within the tunnel's convex hull, but not physically contained in the tunnel. On the right, the boulder's cavity is within the tunnel's and rock body's convex hulls, but is not physically contained in the tunnel or rock body.

topological attachment between participants is necessary or accidental, respectively. Each of these two branches is subsequently delineated according to the (im)materiality of the container and containee, leading to two more horizontal levels of division in the taxonomy: in the first the container is either a material endurant or a void, and in the next the containee varies similarly. The last level in the taxonomy represents refinements of the previous distinctions; these are a partial selection from a greater number of possibilities, and they denote common uses that could eventually be further expanded. For example, containment involving material detachable containers is subdivided by the manner of enclosure, i.e. whether the container fully, partially, or incidentally encloses the containee. Containment with immaterial detachable containers is subdivided by the spatial positioning of the material containee, i.e. whether it splits or fills the void container; and containment in which the dependent container and containee are both im(material) is subdivided by parthood, i.e. whether the containee is a spatial part of the container. Other possible subdivisions are discussed where applicable within the relevant sections below. Note the taxonomy in Figure 4 is the product of a set of theorems, labelled **JPEDx**, which prove that the subrelations at each division are pairwise disjoint and jointly exhaustive with respect to the relation they refine.

While all kinds of physical endurants are valid participants in full physical containment relations, we restrict our study to voids as the only kind of immaterial physical endurant. In other words, voids are the only subtype of DOLCE's DPF category that are included. Other subtypes of DPF, such as shadows, might also participate in full physical containment relations, but they are beyond the scope of this paper.

6 Dependent containment

Dependent containment is denoted as $dep\text{-}contains(y, x)$, meaning that '$y$ physically contains x through some inherent physical dependency between x and y'. This (undirected) physical dependency is denoted by the symmetric primitive predicate $dep(x, y)$ meaning 'there is a dependency between the endurants x and y'. Dependent physical containment is further specialized in this section according to the types of physical endurants participating in the relation. Both containers and containees can be either material endurants or voids, resulting in four possible specializations. Figure 5 illustrates a selection of these dependent containment relations. It is noteworthy that while some specializations of $dep(x, y)$ might be strongly related to a form of ontolog-

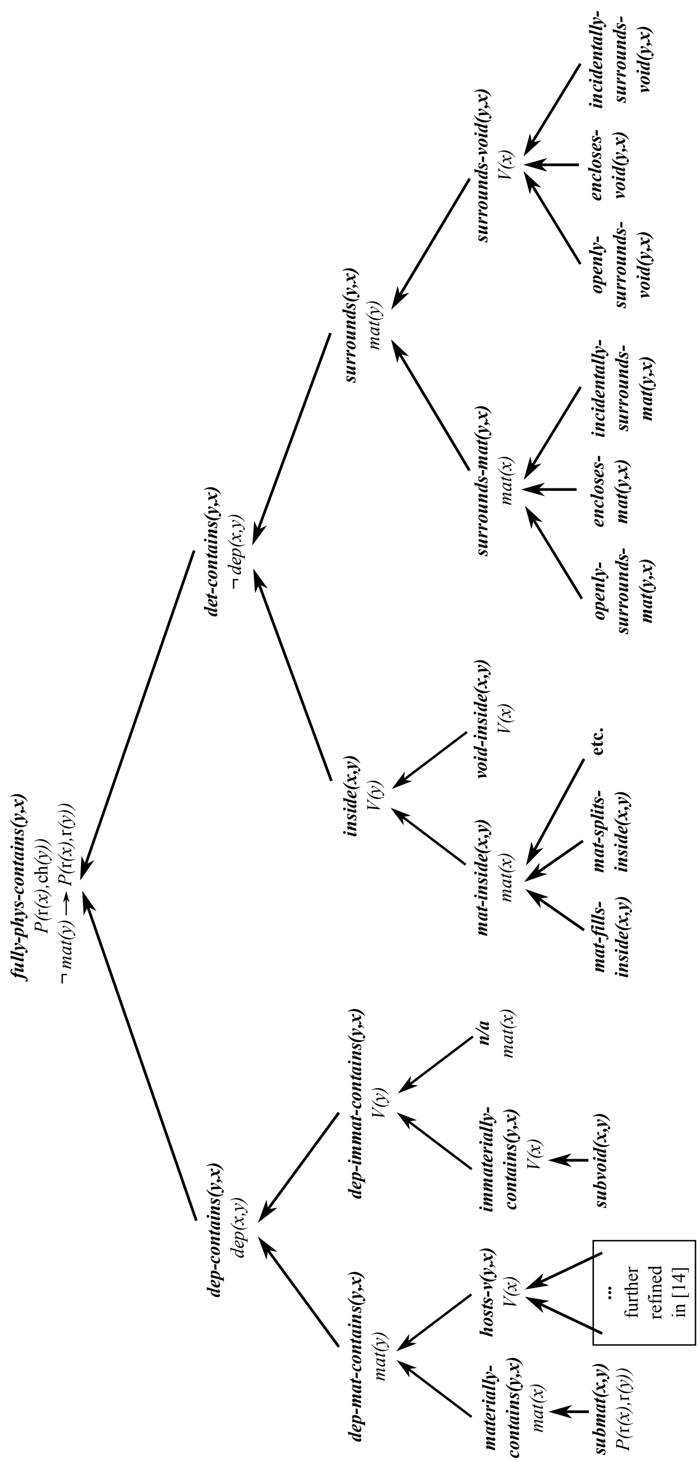

Fig. 4. The taxonomy of full physical containment relations.

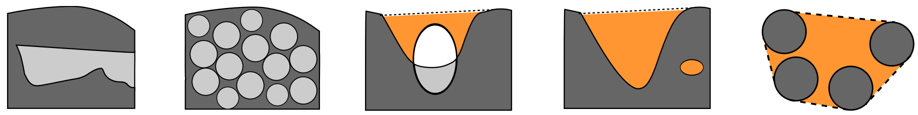

Fig. 5. Examples of two dependent physical endurants in a containment relation. The rock body (dark) (a) contains a submaterial part (light), (b) contains its constituent matter (light) as submaterial, (c) the rock body (dark and medium grey) materially contains a boulder (medium grey and white) without the latter being a submaterial, (d) hosts two holes: a depression and a cavity (both orange/medium grey), and (e) hosts a gap, a tunnel system (orange/medium grey).

ical dependence, e.g. if x exists then y must exist [22], we interpret $dep(x, y)$ physically and topologically, rather than existentially, as seems more appropriate. The relationship to ontological dependence is left to future work. The four specializations of dependent containment are next described individually in Subsections 6.1–6.4.

(Dep-A1) $dep(x, y) \rightarrow PED(x) \wedge PED(y)$
(dependence is a relation between physical endurants)
(Dep-A2) $dep(x, y) \rightarrow dep(y, x)$ (dependence is symmetric, i.e. undirected)
(DepCont-D) $dep\text{-}contains(y, x) \leftrightarrow fully\text{-}phys\text{-}contains(y, x) \wedge dep(y, x)$
(dependent containment is generic containment where x and y are dependent)

6.1 Material containment

An obvious kind of dependent containment is *material containment*: a material endurant x that is physically contained in a second material endurant y, and whose region overlaps y's region, is materially contained in y. The most frequently encountered case occurs when x is located completely within y's region, which is known as x being a *submaterial* of y, as shown in Figures 5(a),(b), and (c). In this case, the dependency between x and y manifests itself in that all matter that constitutes x also constitutes y, at least in part, e.g. a particular rock formation within an aquifer, or the water in a bay and in the corresponding lake. The case of y materially containing x, but x not being a submaterial of y, is far less common but can occur, e.g., in a coral reef. The coral material—the containee—consists of dead as well as living corals. The dead corals are essentially a kind of rock matter, thus also part of the rock body that hosts and contains the reef, while the living corals are not part of the rock body.

(Dep-A3) $mat(x) \wedge mat(y) \rightarrow [dep(x, y) \leftrightarrow PO(\mathrm{r}(x), \mathrm{r}(y))]$
(material endurants are dependent iff they spatially overlap)
(MCont-D) $materially\text{-}contains(y, x) \leftrightarrow dep\text{-}contains(y, x) \wedge mat(x) \wedge mat(y)$
(material containment is dependent containment between material endurants)
(SubMat-D) $submaterial(x, y) \leftrightarrow materially\text{-}contains(y, x) \wedge P(\mathrm{r}(x), \mathrm{r}(y))$
(x is a submaterial of y iff y materially contains x and x is located in a subregion of y)
(SubMat-T1) $DK_1(x, y) \rightarrow submaterial(x, y)$
(the constituent matter x of y is a submaterial of y)
(SubMat-T2) $P(\mathrm{r}(x), \mathrm{r}(y)) \wedge mat(x) \wedge mat(y) \rightarrow submaterial(x, y)$
(a physical endurant whose region is located in part of another material endurant's region is a submaterial thereof)

The relation $submaterial(x, y)$ is the material version of Donnelly's *region containment* [8], written as $CNT\text{-}IN_{\mathrm{r}}(x, y)$.

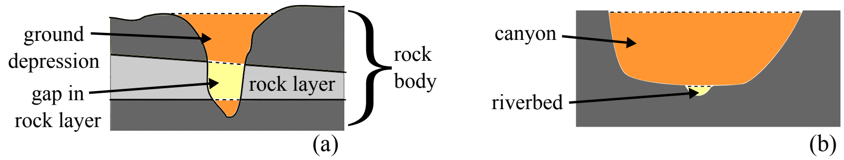

Fig. 6. Examples of two dependent voids in a containment relation. In (a) the gap hosted by the layer of rock is immaterially contained in the depression hosted by the rock body. In (b), the riverbed and canyon are both hosted by the rock body (voids do not need to be maximal), thus the riverbed is immaterially contained in the canyon.

6.2 Hosting a void

The second kind of dependent containment arises from *hosting a void*. Any void x hosted by a physical endurant y also depends on the host, because the void would not be present without the host. For this to be transitive over material containment, V-A27 specifies how voids are preserved when hosted by a material part [3].

(V-A27) $mat(y) \wedge mat(z) \wedge hosts\text{-}v(y, x) \wedge P(\mathrm{r}(y), \mathrm{r}(z)) \wedge \neg PO(\mathrm{r}(z), \mathrm{r}(x)) \rightarrow hosts\text{-}v(z, x)$ (any void x hosted by a material part y of z that is not even partially filled by z is also hosted by z)

(Dep-A4) $V(x) \wedge mat(y) \rightarrow [dep(y, x) \leftrightarrow hosts\text{-}v_{\mathrm{any}}(y, x)]$
(a void and a material endurant are dependent iff they are in a hosts relation)

(DepCont-T1) $hosts\text{-}v_{\mathrm{any}}(y, x) \leftrightarrow dep\text{-}contains(y, x) \wedge V(x) \wedge mat(y)$
(y hosts void x iff y is a material endurant that dependently contains void x)

6.3 Immaterial containment

The third kind of dependent containment is *immaterial containment*. In order to define it, we first capture dependency between voids: two overlapping voids are dependent if their material hosts occupy overlapping regions. Then, a void that is dependent on another void, and that occupies a subregion of the other void, is also immaterially contained in the other void.

(Dep-A5) $V(x) \wedge V(y) \rightarrow \big[dep(x, y) \leftrightarrow PO(\mathrm{r}(x), \mathrm{r}(y)) \wedge \exists h_x, h_y[hosts\text{-}v(h_x, x) \wedge hosts\text{-}v(h_y, y) \wedge mat(h_x) \wedge mat(h_y) \wedge \big(P(\mathrm{r}(h_x), \mathrm{r}(h_y)) \vee P(\mathrm{r}(h_y), \mathrm{r}(h_x))\big)]\big]$
(voids are dependent iff they overlap and have spatially nested material hosts)

(ImCont-D) $immaterially\text{-}contains(y, x) \leftrightarrow dep\text{-}contains(y, x) \wedge V(x) \wedge V(y)$
(immaterial containment is dependent containment between voids)

(ImCont-T1) $immaterially\text{-}contains(y, x) \rightarrow P(\mathrm{r}(x), \mathrm{r}(y))$
(immaterial containment requires that x is located in a subregion of y)

In immaterial containment, the dependency relation is more implicit because of the inclusion relation between their hosts' regions. Intuitively, if the larger host (or one of its material parts) does not host the smaller void, the larger void would not exist in its present form. A general form of immaterial containment is illustrated in Figure 6(a). A special case is illustrated in Figure 6(b), where two voids have the same host; then we call the contained void a *subvoid* of the container void.

[3] V-A27 is required in addition to the weaker condition previously imposed by V-A7 in [14, 15].

(SubVoid-D) $subvoid(x, y) \leftrightarrow immaterially\text{-}contains(y, x) \land \exists h[hosts\text{-}v(h, y) \land hosts\text{-}v(h, x)]$
(x is a subvoid of y iff y immaterially contains x and they have a common host)

6.4 Can a material endurant be dependently contained in a void?

So far, we have considered three types of dependent containment between material or void endurants. The remaining combination involves a void as container and a dependent material endurant as containee. It is difficult to imagine this case unless the container void is the entire space of interest, e.g., physical space. However, that does not fit our framework requiring voids to be hosted by some physical endurant, because space as such is not physically hosted. Hence voids and physical space are distinct notions, with the latter being a container for all empirical entities, including voids. Physical space should then not be confused with related notions of voids, e.g. the term "outer space" is more precisely described as the space "that separates the planets, stars, and galaxies" [6, p. 132], which is in fact a gap (a void) hosted by celestial bodies. In essence, void containers cannot dependently contain material endurants. We can prove this in our formalization with the help of Dep-A4 and DepCont-D.

(Dep-T1) $dep\text{-}contains(y, x) \land mat(x) \to \neg V(y)$
(a material entity cannot be dependently contained in a void)

The inclusion of physical space as a participant in containment relations remains a potential future task.

6.5 Classification of dependent containment

Because voids cannot dependently contain material endurants, the remaining three relations are exhaustive subrelations of $dep\text{-}contains(y, x)$. Their typing immediately entails they are disjoint relations.

(JEPD1) $[mat(x) \lor V(x)] \land [mat(y) \lor V(y)] \land dep\text{-}contains(y, x) \to [materially\text{-}contains(y, x) \lor immaterially\text{-}contains(y, x) \lor hosts\text{-}v(y, x)]$
(JEPD2) $\neg materially\text{-}contains(y, x) \lor \neg immaterially\text{-}contains(y, x)$
(JEPD3) $\neg materially\text{-}contains(y, x) \lor \neg hosts\text{-}v(y, x)$
(JEPD4) $\neg immaterially\text{-}contains(y, x) \lor \neg hosts\text{-}v(y, x)$

To complete the dependent containment taxonomy, we consider notions of containment in which the container type is fixed, but the containee can vary as material endurant or void: dependent material containment, $dep\text{-}mat\text{-}contains$, has a material container, and dependent immaterial containment, $dep\text{-}immat\text{-}contains$, has an immaterial container. Dependent material containment is thus specialized by the material containment and the hosting relations, and dependent immaterial containment has immaterial containment as its only feasible specialization, so that their extensions are equivalent.

(DepMCont-D) $dep\text{-}mat\text{-}contains(y, x) \leftrightarrow dep\text{-}contains(y, x) \land mat(y)$
(dependent material containment)
(DepImCont-D) $dep\text{-}immat\text{-}contains(y, x) \leftrightarrow dep\text{-}contains(y, x) \land V(y)$
(dependent immaterial containment)
(JEPD5) $dep\text{-}mat\text{-}contains(y, x) \leftrightarrow materially\text{-}contains(y, x) \lor hosts\text{-}v(y, x)$
(JEPD6) $dep\text{-}immat\text{-}contains(y, x) \leftrightarrow immaterially\text{-}contains(y, x)$
(JEPD7) $\neg dep\text{-}mat\text{-}contains(y, x) \lor \neg dep\text{-}immat\text{-}contains(y, x)$

7 Detachable containment

Detachable containment holds between physical endurants that are not physically dependent. It occurs when the containee is physically contained strictly due to a non-necessary spatial arrangement. The term 'detachable' emphasizes that the two physical endurants are independent and, in principle, separable[4].

(DetCont-D) $\mathit{det\text{-}contains}(y, x) \leftrightarrow \mathit{fully\text{-}phys\text{-}contains}(y, x) \wedge \neg \mathit{dep}(y, x)$
(detachable containment is generic containment between independent endurants)

By the definitions DepCont-D and DetCont-D, detachable and dependent containment are subrelations of generic physical containment; now we can prove that they are jointly exhaustive, pairwise disjoint (JEPD) subrelations.

(JEPD8) $\mathit{fully\text{-}phys\text{-}contains}(y, x) \leftrightarrow \mathit{dep\text{-}contains}(y, x) \vee \mathit{det\text{-}contains}(y, x)$
(JEPD9) $\neg \mathit{dep\text{-}contains}(y, x) \vee \neg \mathit{det\text{-}contains}(y, x)$

To achieve our objective of classifying and formalizing the various kinds of *detachable containment*, we study its specializations that arise from the four combinations of material endurants and voids as container and containee.

7.1 An endurant inside a void

Arguably, the most foundational form of detachable containment involves something being inside a void. This relation is denoted by the predicate $\mathit{inside}(x, y)$ understood as 'the physical endurant x is spatially located within the void y'. Being located inside a void requires that the containee's region $\mathrm{r}(x)$ is completely within the region of the void container, that is, $P(\mathrm{r}(x), \mathrm{r}(y))$, and not just within the container's convex hull. Beware that this relation switches the positions of the two parameters: x is inside the void y, that is, $\mathit{inside}(x, y)$, means y detachably contains x, that is, $\mathit{det\text{-}contains}(y, x)$.

(INSIDE-D) $\mathit{inside}(x, y) \leftrightarrow \mathit{det\text{-}contains}(y, x) \wedge V(y)$
($\mathit{inside}(x, y)$ is detachable containment in a void container)
(INSIDE-T1) $\mathit{inside}(x, y) \rightarrow P(\mathrm{r}(x), \mathrm{r}(y))$
($\mathit{inside}(x, y)$ requires x to be located within the region of y)

Next, the $\mathit{inside}(x, y)$ relation is further specialized by the containee's type.

A material endurant inside a void If a material endurant is inside a void, we use the relation $\mathit{mat\text{-}inside}(x, y)$. This relation is equivalent to the only relation discussed in [8] that explicitly involves immaterial entities, namely *material-region containment*, $\mathit{CNT\text{-}IN}_{\mathrm{mr}}(x, y)$.

(MINSIDE-D) $\mathit{mat\text{-}inside}(x, y) \leftrightarrow \mathit{inside}(x, y) \wedge \mathit{mat}(x)$
($\mathit{mat\text{-}inside}(x, y)$ denotes that the material endurant x is inside the void y)

More fine-grained specializations of $\mathit{mat\text{-}inside}(x, y)$ can be derived according to (1) the kind of container void, and (2) the location of the containee within the container void. The first choice includes distinctions based on (1a) the void's internal connectedness (whether it is a simple or complex void), (1b) the host's internal connectedness

[4] Two physical endurants in a detachable containment relation may be inseparable for reasons other than a physical dependence, for example, because they are interlocked, glued or otherwise fused together, or because one endurant fully encloses the other.

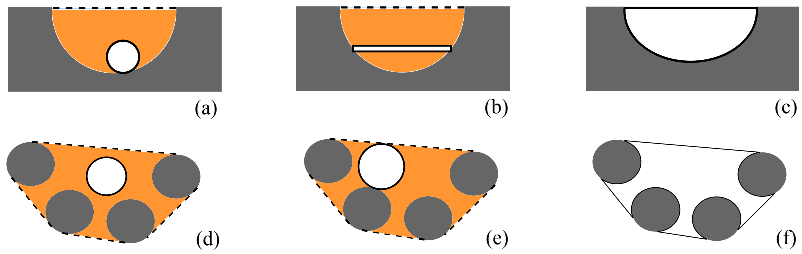

Fig. 7. A material endurant (white) being inside a void (orange/medium grey) and being surrounded by another material endurant (dark). The void is a hole in the top row, and a gap in the bottom row. The white containee may split (center column) or fill (right column) the void.

(whether the void is a hole or a gap), (1c) the void's opening (whether the void is a cavity, a hollow, or a tunnel), and (1d) the void's connection to other voids within the same host (whether the void is an internal or external void). As these choices simply mirror the classification of voids in our earlier work, we invite the reader to consult [15] for the relevant axioms necessary to expand the taxonomy accordingly.

The second choice includes distinctions based on the containee's connection to (2a.I) the container's host, (2a.II) the outside, and (2a.III) other voids in the container's host. These three distinctions involve the container void's host and are thus not definable using the void alone. But they can be coupled with further distinctions that fall within the second choice, namely whether the containee (2b.I) splits the void into disconnected parts, (2b.II) completely fills the void, or (2b.III) is "stuck" in the void. Except for "being stuck" in a void, which is an intricately shape-based relation, these relations are definable using our underlying mereotopological theory. Figure 7 gives examples of splitting and filling a void. $\mathit{mat\text{-}fills\text{-}inside}$ is equivalent to the intersection of SUMO's [20] relations 'completelyFills' and 'properlyFills'.

(MSINSIDE-D) $\mathit{mat\text{-}splits\text{-}inside}(x, y) \leftrightarrow \mathit{mat\text{-}inside}(x, y) \wedge PP(\mathrm{r}(x), \mathrm{r}(y)) \wedge ICon(\mathrm{r}(y)) \wedge \neg ICon(\mathrm{r}(y) - \mathrm{r}(x))$ (a material containee x splits a void y iff it is located in a proper subregion of y, the void y is internally connected, and the part of y that is not occupied by the containee is not internally connected)

(MFINSIDE-D) $\mathit{mat\text{-}fills\text{-}inside}(x, y) \leftrightarrow \mathit{mat\text{-}inside}(x, y) \wedge \mathrm{r}(x) = \mathrm{r}(y)$
(a containee fills a void if it is material and its region saturates the entire void)

(JEPD10) $\neg \mathit{mat\text{-}splits\text{-}inside}(x, y) \vee \neg \mathit{mat\text{-}fills\text{-}inside}(x, y)$

A void inside another void Voids can be inside other voids without their hosts being in a containment relation. We capture this relation between two independent voids using the predicate $\mathit{void\text{-}inside}(x, y)$.

(VINSIDE-D) $\mathit{void\text{-}inside}(x, y) \leftrightarrow \mathit{inside}(x, y) \wedge V(x)$
($\mathit{void\text{-}inside}(x, y)$ is the relation of a void x inside another void y)

Figure 8(a) and (b) illustrate this relation. As the example (a) demonstrates, it is not required that the containee's host (A) spatially overlaps the container's convex hull ($\mathrm{r}(V_B) + \mathrm{r}(B)$). However, this is often the case, as shown in example (b). A special case of $\mathit{void\text{-}inside}(x, y)$ occurs when the convex hull of the containee's host is entirely contained in the container void, as demonstrated by Figure 8(c). Then *every* void within that particular host z is inside the void y.

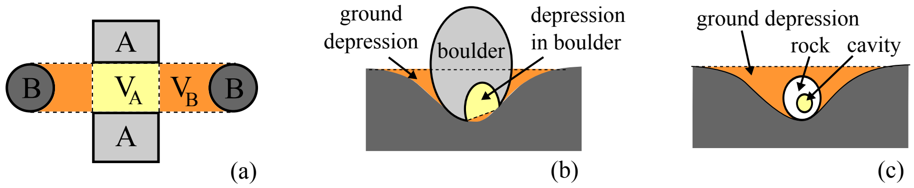

Fig. 8. Examples of a void detachably containing another void.

(VINSIDE-T1) $P(\mathrm{ch}(z), \mathrm{r}(y)) \wedge mat(z) \wedge V(y) \rightarrow \forall v[hosts\text{-}v_{\mathrm{any}}(z, v) \rightarrow void\text{-}inside(v, y)]$ (if the convex hull of a material endurant z is completely contained in the void y, then every void v hosted by z is inside y as well)

Classification of *inside* The two relations $mat\text{-}inside$ and $void\text{-}inside$ are JEPD subrelations of $inside$.

(JEPD11) $inside(x, y) \leftrightarrow mat\text{-}inside(x, y) \vee void\text{-}inside(x, y)$
(JEPD12) $\neg mat\text{-}inside(x, y) \vee \neg void\text{-}inside(x, y)$

7.2 A material endurant surrounding another endurant

When a material endurant y detachably contains another endurant x, we say 'y surrounds x' denoted by the predicate $surrounds(y, x)$.

(SUR-D) $surrounds(y, x) \leftrightarrow det\text{-}contains(y, x) \wedge mat(y)$
($surrounds(y, x)$ is detachable containment with a material container)

This relation is equivalent to *surround containment*, $CNT\text{-}IN_{\mathrm{s}}(x, y)$, from [8]. It can be further refined according to the (im)materiality of the containee, as discussed in the remainder of this section.

A material endurant surrounding another material endurant A material endurant detachably contained in another material endurant is *materially surrounded* by the latter. We denote this relation using the predicate $surrounds\text{-}mat(y, x)$, which is read as 'the material endurant y partially or fully surrounds the material endurant x'.

(MSUR-D) $surrounds\text{-}mat(y, x) \leftrightarrow surrounds(y, x) \wedge mat(x)$
(material endurant y surrounds a material endurant x)

$surrounds\text{-}mat(y, x)$ does not rule out the case in which x's region is within y's convex hull, but outside any void (or set of voids) hosted by y. For example, the boulder in Figure 9(a) is surrounded by the rock body, yet is not contained in the rock body. This can occur when the convex hull of a material container has spaces that are neither material nor voids. There is no principled way to identify voids amongst candidate spaces at this time [3], hence their identification is somewhat arbitrary. When another material endurant is located in such a non-void space, the relation is called *incidentally materially surrounds*, written as $incidentally\text{-}surrounds\text{-}mat(y, x)$.

(IMSUR-D) $incidentally\text{-}surrounds\text{-}mat(y, x) \leftrightarrow surrounds\text{-}mat(y, x) \wedge \neg P(\mathrm{r}(x), voidspace_{\mathrm{all}}(y))$ (y incidentally materially surrounds x iff y materially surrounds x, but x's region is not within y's entire void space)

A special case of the material surrounds relation is *fully materially surrounds*, written as $encloses\text{-}mat(y, x)$. In this case, the containee must be located within some

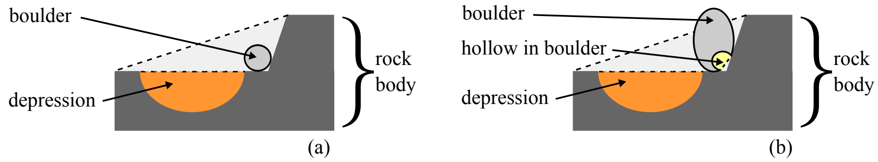

Fig. 9. Example of a (a) material endurant (a boulder) or a (b) void (a hollow in a boulder) located within the convex hull of a material endurant (the rock body) without being located within any of its voids (the depression). Hence, the boulder in (a) and the hollow in (b) are only incidentally surrounded by the rock body.

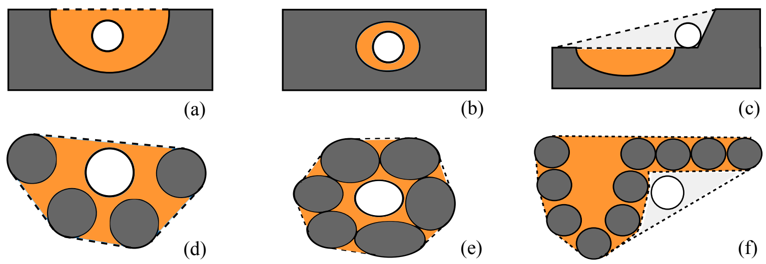

Fig. 10. A material endurant (dark) surrounding another material endurant (white) with the surrounding container's void (orange/medium grey) being either a hole (top row) or a gap (bottom row). From left to right: openly surrounds, encloses, and incidentally surrounds.

cavity of the container. Examples of a container fully surrounding a containee are water in a closed bottle, or water in the subterranean cavity of a rock body.

(MENCL-D) $\mathit{encloses\text{-}mat}(y,x) \leftrightarrow \mathit{surrounds\text{-}mat}(y,x) \wedge \exists v[\mathit{hosts\text{-}cav}_{\text{any}}(y,v) \wedge P(\mathrm{r}(x),\mathrm{r}(v))]$ (a material container y encloses a material containee x iff it hosts a cavity wherein x is located)

The relation $\mathit{encloses\text{-}mat}(y,x)$ is a physical version of being *topologically inside*, $\mathit{TOP\text{-}INSIDE}(x,y)$, as defined in [5], for detachable material endurants.

(M2) $\mathit{encloses\text{-}mat}(y,x) \leftrightarrow \mathit{TOP\text{-}INSIDE}(\mathrm{r}(x),\mathrm{r}(y)) \wedge \mathit{mat}(y) \wedge \mathit{mat}(x) \wedge \neg \mathit{dep}(x,y)$ ($\mathit{encloses\text{-}mat}$ is the *topologically inside* relation for detachable material endurants)

It is obvious that $\mathit{incidentally\text{-}surrounds\text{-}mat}$ and $\mathit{encloses\text{-}mat}$ are disjoint relations. When the containee is neither incidentally surrounded nor enclosed by its material container, the container *openly materially surrounds* the containee, denoted as $\mathit{openly\text{-}surrounds\text{-}mat}(y,x)$. Notice that $\mathit{openly\text{-}surrounds\text{-}mat}(y,x)$ is agnostic about whether the containee can exit the container—we consider physical accessibility to be an associated but different relation. The three relations $\mathit{openly\text{-}surrounds\text{-}mat}$, $\mathit{incidentally\text{-}surrounds\text{-}mat}$, and $\mathit{encloses\text{-}mat}$ form a set of JEPD subrelations of $\mathit{surrounds\text{-}mat}$. Examples for each are given in Figure 10.

(OMSUR-D) $\mathit{openly\text{-}surrounds\text{-}mat}(y,x) \leftrightarrow \mathit{surrounds\text{-}mat}(y,x) \wedge \neg \mathit{encloses\text{-}mat}(y,x) \wedge \neg \mathit{incidentally\text{-}surrounds\text{-}mat}(y,x)$ (openly materially surrounds)

(JEPD13) $\neg \mathit{incidentally\text{-}surrounds\text{-}mat}(y,x) \vee \neg \mathit{encloses\text{-}mat}(y,x)$

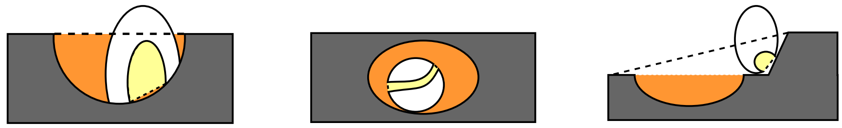

Fig. 11. Examples of a material endurant (dark) (a) openly surrounding, (b) enclosing, and (c) incidentally surrounding an independent void (light grey) hosted by the white object. The case for a hole (orange/medium grey) is depicted, but works equally for a gap.

(JEPD14) $surrounds\text{-}mat(y,x) \leftrightarrow openly\text{-}surrounds\text{-}mat(y,x) \vee encloses\text{-}mat(y,x) \vee incidentally\text{-}surrounds\text{-}mat(y,x)$

Together, $openly\text{-}surrounds\text{-}mat$ and $incidentally\text{-}surrounds\text{-}mat$ are the physical version of being *geometrically inside*, $GEO\text{-}INSIDE(x,y)$, from [5], for detachable material endurants.

(M3) $openly\text{-}surrounds\text{-}mat(y,x) \vee incidentally\text{-}surrounds\text{-}mat(y,x) \leftrightarrow GEO\text{-}INSIDE(\mathrm{r}(x),\mathrm{r}(y)) \wedge mat(y) \wedge mat(x) \wedge \neg dep(x,y)$
($encloses\text{-}mat$ and $incidentally\text{-}surrounds\text{-}mat$ together are the geometrically inside relation for detachable material endurants)

A material endurant surrounding a void If a material endurant y surrounds a void x that is independent of y, we write $surrounds\text{-}void(y,x)$ and say y *void-surrounds* x.

(VSUR-D) $surrounds\text{-}void(y,x) \leftrightarrow surrounds(y,x) \wedge V(x)$
(a material endurant y surrounds a void x)

Again, this kind of surrounds relation may be incidental, in which case we say y *incidentally void-surrounds* x and write $incidentally\text{-}surrounds\text{-}void(y,x)$. The enclosed and open analogues can also be defined: y detachably *void-encloses*, that is, fully surrounds the void x, if x is located in a cavity of y. When the void containee is neither incidentally surrounded nor enclosed by its material container, then it is *openly void-surrounded* by the container. Examples are given in Figure 11. These three subrelations of $surrounds\text{-}void$ are the physical versions of *topologically* and *geometrically inside* involving a material container and a void, and they are JEPD subrelations.

(IVSUR-D) $incidentally\text{-}surrounds\text{-}void(y,x) \leftrightarrow surrounds\text{-}void(y,x) \wedge \neg P(\mathrm{r}(x), \mathrm{voidspace}_{\mathrm{all}}(y))$ (y incidentally void-surrounds the void x iff y void-surrounds x but y's void space does not spatially contain x)

(VENCL-D) $encloses\text{-}void(y,x) \leftrightarrow surrounds\text{-}void(y,x) \wedge \exists v[hosts\text{-}cav_{\mathrm{any}}(y,v) \wedge P(\mathrm{r}(x),\mathrm{r}(v))]$
(a material container y encloses a void x iff it hosts a cavity wherein x is located)

(OVSUR-D) $openly\text{-}surrounds\text{-}void(y,x) \leftrightarrow surrounds\text{-}void(y,x) \wedge \neg encloses\text{-}void(y,x) \wedge \neg incidentally\text{-}surrounds\text{-}void(y,x)$ (openly void-surrounds)

(JEPD15) $\neg incidentally\text{-}surrounds\text{-}void(y,x) \vee \neg encloses\text{-}void(y,x)$

(JEPD16) $surrounds\text{-}void(y,x) \leftrightarrow openly\text{-}surrounds\text{-}void(y,x) \vee incidentally\text{-}surrounds\text{-}void(y,x) \vee encloses\text{-}void(y,x)$

Classification of *surrounds* It is easy to see that the relations $surrounds\text{-}mat(y,x)$ and $surrounds\text{-}void(y,x)$ are JEPD subrelations of $surrounds(y,x)$. Both relations are further specialized into three JEPD subrelations as already discussed.

(JEPD17) $surrounds(y, x) \leftrightarrow surrounds\text{-}mat(y, x) \vee surrounds\text{-}void(y, x)$
(JEPD18) $\neg surrounds\text{-}mat(y, x) \vee \neg surrounds\text{-}void(y, x)$

The following three subrelations of $surrounds$ are introduced for convenience only.
(ISUR-D) $incidentally\text{-}surrounds(y, x) \leftrightarrow$
$incidentally\text{-}surrounds\text{-}void(y, x) \vee incidentally\text{-}surrounds\text{-}mat(y, x)$
(ENCL-D) $encloses(y, x) \leftrightarrow encloses\text{-}void(y, x) \vee encloses\text{-}mat(y, x)$
(OSUR-D) $openly\text{-}surrounds(y, x) \leftrightarrow$
$openly\text{-}surrounds\text{-}void(y, x) \vee openly\text{-}surrounds\text{-}mat(y, x)$

7.3 Classification of detachable containment

If we only consider material endurants and voids as possible participants, then $inside$ and $surrounds$ are the only kinds of detachable containment, that is, they are JEPD subrelations of $det\text{-}contains$.
(JEPD19) $[mat(x) \vee V(x)] \wedge [mat(y) \vee V(y)]$
$\rightarrow [det\text{-}contains(y, x) \leftrightarrow inside(x, y) \vee surrounds(y, x)]$
(JEPD20) $\neg inside(x, y) \vee \neg surrounds(y, x)$

8 Discussion

The various types of containment described above lead to several implications. First, material constitution is a special case of full physical containment.
(DetCont-T1) $DK_1(x, y) \rightarrow materially\text{-}contains(y, x)$

Second, the surrounds and inside relations are somewhat reciprocal: the two main kinds of the surrounds relation, namely $openly\text{-}surrounds(y, x)$ and $encloses(y, x)$, can always be traced back to the relation of x being inside y's entire void space, $\mathrm{voidspace}_{\mathrm{all}}(y)$, which must exist according to V-A26. This accounts for the case where the surrounded entity x is distributed across voids at multiple levels of granularity within the container y. For example, an amount of water surrounded by a rock body can be located partly in the rock body's macroscopic voids and partly in the rock matter's microscopic voids. In the other direction, $inside(x, y)$ entails that any host of y openly surrounds or encloses x.
(DetCont-T2) $openly\text{-}surrounds(y, x) \vee encloses(y, x) \rightarrow \exists z[inside(x, z) \wedge$
$P(\mathrm{r}(z), \mathrm{voidspace}_{\mathrm{all}}(y))]$ (y openly surrounding or enclosing x requires x to be inside some void located in y's void space)
(DetCont-T3) $inside(x, y) \rightarrow$
$\forall h\big[hosts\text{-}v_{\mathrm{any}}(h, y) \rightarrow openly\text{-}surrounds(h, x) \vee encloses(h, x)\big]$
(x being inside void y requires any host of y to openly surround or to enclose x)

Third, the most interesting relations, $mat\text{-}inside$ and $surrounds\text{-}mat$, might be refined further if we take not only the relative location of the two participating endurants into account, but also the location of the indirectly involved host or void. DetCont-T2 and DetCont-T3 demonstrate that such a third, indirect, participant must exist. Consider the example of a material containee inside a void hosted by some material endurant, e.g. a rock in a hole hosted by a lakebed. In this case, it is possible to distinguish whether the containee is (non)tangentially inside the void or (non)tangentially surrounded by the host. Similar to the definition of (non)tangential parthood in [4], this can be expressed in a multidimensional setting using a definable relation of tangential containment, $TCont$, that specializes the spatial inclusion relation $Cont$ [14]. We can then express more sophisticated relations, such as whether the rock protrudes from the lake. A protruding rock is not only tangentially contained in the water body but is also in contact, by means of partial overlap or tangential contact, with the lake's exterior—the space where neither the lakebed nor the water body nor the water body's voids are located.

Lastly, all relations in our motivating example (Figure 1) can now be much more specifically expressed using the various kinds of full physical containment:

$\textit{openly-surrounds-mat}(\mathrm{LB}, \mathrm{SWB})$	$\textit{openly-surrounds-mat}(\mathrm{LB}, \mathrm{Rock})$
$\textit{hosts-v}(\mathrm{LB}, \mathrm{Hole})$	$\textit{mat-inside}(\mathrm{SWB}, \mathrm{Hole})$
$\textit{mat-inside}(\mathrm{Rock}, \mathrm{Hole})$	$\textit{openly-surrounds-mat}(\mathrm{SWB}, \mathrm{Rock})$
$\textit{materially-contains}(\mathrm{AQ}, \mathrm{GWB})$	$\textit{materially-contains}(\mathrm{AQ}, \mathrm{RM})$
$\textit{encloses-mat}(\mathrm{AQ}, \mathrm{CT})$	$\textit{hosts-v}_{\mathrm{any}}(\mathrm{AQ}, \mathrm{Gaps})$
$\textit{mat-inside}(\mathrm{Gaps}, \mathrm{GWB})$	$\textit{mat-inside}(\mathrm{Gaps}, \mathrm{CT})$
$\textit{encloses-mat}(\mathrm{GWB}, \mathrm{CT})$	

By accounting for physical constraints in the definitions of the various containment relations we further ensured that the rock is not physically contained in the gaps, i.e. that $\neg \textit{fully-phys-contains}(\mathrm{Gaps}, \mathrm{Rock})$, because while the rock's region is fully inside the gap's convex hull, it is not inside the gap's region.

We further have two examples of the incidental surrounds relation:

$\textit{incidentally-surrounds-mat}(\mathrm{AQ}, \mathrm{SWB})$ $\textit{incidentally-surrounds-mat}(\mathrm{AQ}, \mathrm{Rock})$

Some of the detachable containment relations, notably a void being inside another void, or a void being surrounded by a material endurant, are included here merely for completeness. They are not required to express the various containment relations in our motivating example, and will likely not play a prominent role in practical settings.

9 Conclusion

Full physical containment—the notion that one physical entity is completely inside or surrounded by another—plays a central role in describing many natural resources, especially water. To date, ontological representations of the full physical containment relation are limited to abstract space, incomplete, or they insufficiently incorporate voids. In this paper we argue that a thorough interpretation of this relation must accommodate both voids and material entities as containers and containees, and must account for the physical differences between voids and material entities. From this we develop a taxonomy, summarized in Figure 4, in which such containment relations are first differentiated according to the dependency between container and containee, and then according to their (im)materiality. This results in a delineation of full physical containment that is grounded in physical space and more comprehensive than prior efforts. The taxonomical distinctions are expressed using a formal multidimensional mereotopological theory, inspired by RCC, and are integrated into the DOLCE foundational ontology, as another step towards a rigorous hydro ontology. Potential future research directions include extensions to partial containment, and to containment across various levels of physical granularity, e.g. to better examine the relation between a containee held in the gaps of some matter (such as a contaminant), and the physical object constituted by the matter. Work is also underway to integrate the notion of physical space—distinct from voids—as a container for all physical entities.

Acknowledgements We are grateful for the anonymous reviewers' thoughtful and meticulous comments, which helped improve the final version of the paper.

References

1. Boisvert, E., Brodaric, B.: GroundWater Markup Language (GWML) – enabling groundwater data interoperability in spatial data infrastructures. J. Hydroinformatics **14**(1) (2012) 93–107
2. Casati, R., Varzi, A.C.: Holes and other Superficialities. MIT Press (1994)
3. Casati, R., Varzi, A.C.: Parts and Places. MIT Press (1999)
4. Cohn, A.G., Bennett, B., Gooday, J.M., Gotts, N.M.: Qualitative spatial representation and reasoning with the Region Connection Calculus. GeoInformatica **1** (1997) 275–316
5. Cohn, A.G., Randell, D.A., Cui, Z.: Taxonomies of logically defined qualitative spatial relations. Int. J. Hum.-Comput. St. **43**(5–6) (1995) 831–846
6. Dainton, B.: Space and Time. McGill-Queens Press (2001)
7. Donnelly, M.: Layered mereotopology. In: Int. Joint Conf. on Artif. Intell. (IJCAI-03). (2003) 1269–1274
8. Donnelly, M.: Containment relations in anatomical ontologies. In: Symp. of the Amer. Medical Inform. Assoc. (AMIA-2005). (2005) 206–210
9. Egenhofer, M.J., Clementini, E., Di Felice, P.: Topological relations between regions with holes. Int. J. Geogr. Inf. Sci. **8**(2) (1994) 129–144
10. Egenhofer, M.J., Vasardani, M.: Spatial reasoning with a hole. In: Conf. on Spatial Inf. Theory (COSIT-07). LNCS 4736, (2007) 303–320
11. Foxvog, D.: Cyc. In R.Poli, M.Healy, A.Kameas, eds.: Theory and Applications of Ontology: Computer Applications. Springer (2010) 259–278
12. Grenon, P.: Tucking RCC in Cyc's ontological bed. In: Int. Joint Conf. on Artif. Intell. (IJCAI-03). (2003) 894–899
13. Grenon, P., Smith, B.: SNAP and SPAN: towards dynamic spatial ontology. J. Spat. Cogn. Comput. **4**(1) (2004) 69–104
14. Hahmann, T.: A Reconciliation of Logical Representations of Space: from Multidimensional Mereotopology to Geometry. PhD thesis, Univ. of Toronto, Dept. of Comp. Science (2013)
15. Hahmann, T., Brodaric, B.: The void in hydro ontology. In: Conf. on Formal Ontology in Inf. Systems (FOIS-12), IOS Press (2012) 45–58
16. Hahmann, T., Grüninger, M.: A naïve theory of dimension for qualitative spatial relations. In: Symp. on Logical Formalizations of Commonsense Reasoning, AAAI Press (2011)
17. Hook, J.R.: An introduction to porosity. Petrophysics **44**(3) (2003)
18. Kuhn, W.: An image-schematic account of spatial categories. In: Conf. on Spatial Inf. Theory (COSIT-07). LNCS 4736 (2007) 152–168
19. Masolo, C., Borgo, S., Gangemi, A., Guarino, N., Oltramari, A.: WonderWeb Deliverable D18 – Ontology Library (final report). Technical report, ISTC-CNR, Trento (2003)
20. Niles, I., Pease, A.: Towards a standard upper ontology. In: Conf. on Formal Ontology in Inf. Systems (FOIS-01), IOS Press (2001) 2–9
21. Sen, M., Duffy, T.: GeoSciML: development of a generic geoscience markup language. Comput. Geosci. **31**(9) (2005) 1095–1103
22. Simons, P.: Parts - A Study in Ontology. Clarendon Press (1987)
23. Tripathi, A. Babaie, H.: Developing a modular hydrogeology ontology by extending the SWEET upper-level ontologies. Comput. Geosci. **34**(9) (2008) 1022–1033
24. Walton, L., Worboys, M.: An algebraic approach to image schemas for geographic space. In: Conf. on Spatial Inf. Theory (COSIT-09). LNCS 5756, (2009) 357–370