A Reconciliation of Logical Representations of Space: from Multidimensional Mereotopology to Geometry

PhD Thesis Defence

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February 14, 2013

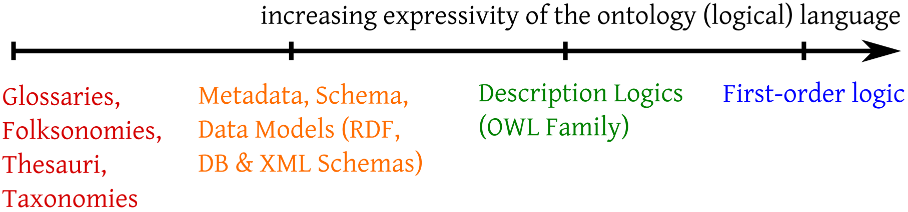
Spatial Ontologies

ontology (lowercase ‘o’)

an **artifact** designed with the purpose of **expressing** the intended meaning – **the semantics** – of a **vocabulary** (a set of concepts, relations, and functions) in terms of the nature and structure of the entities it refers to. Expressed as **axioms** in a logical language, usually a subset of FOL.

Purposes and uses:

- Communication (for humans and/or software systems)
- Standardization and re-use
- Explicit documentation of ontological assumptions for comparability
- Information and knowledge integration



Spatial Ontology

An explicit formalization of the semantics of a specific conceptualization of space in an **expressive logical language**.

Choice of ontology involves many decisions:

- What are the primitive spatial relations used?
- What spatial relations are defined?
- What are the assumptions made about space: *dimensionality, connectedness, atomicity, continuity*?
- Homeomorphic under what kind of operations: *topological, affine, metric*?

- Quantitative spatial ontologies:
  - Exact measurements (large, possibly infinite set of values) of distances, angles, and other sizes (areas, volumes)
  - Allows precise reasoning (calculations)
  - Motivated by classical geometric representation of physical space
- Qualitative spatial ontologies:
  - Small set of relations for connectivity, parthood, relative size (greater/equal/smaller), or order (before/after)
  - Restricted to high-level “qualitative” reasoning
  - Motivated by spatial relations used in human language

Why care about qualitative representations of space?

- Qualitative spatial reasoning is a promising approach for human-like high-level reasoning about space, which is sufficient for many tasks.
- To integrate less precise spatial knowledge from text sources, human descriptions, etc. with precise geometric spatial information.

Objectives

Research Problem and Objective 1

Mereological and topological relations lie at the heart of most qualitative representations of space.

- 1$^{\text{st}}$ Problem: **The limited expressiveness of previously available qualitative spatial ontologies is a main hindrance for their practical use, while multidimensional theories weaker than classical geometries are understudied.** (Chapters (3) and (4))
- Objective: Develop a qualitative theory of space that is
  - Multidimensional: allows models with entities of multiple dimensions;
  - Commonsensical: defines an intuitive set of spatial relations,
  - Dimension-independent: not dependent on specific combinations of absolute (numeric) dimensions,
  - Atomicity-neutral: admits discrete and continuous models,
  - Geometry-consistent: generalizes classical geometries.

Research Problem and Objective 2

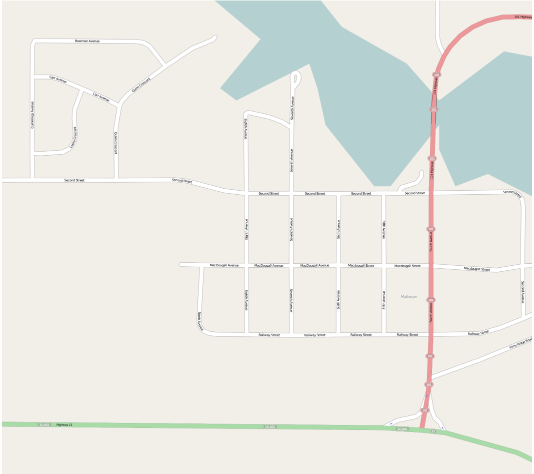
- 2^{nd} Problem: **How are the various available first-order spatial ontologies, including mereotopologies and geometries, related to the newly developed ontologies and to one another?**
- Objective: Semantically integrate them according to the

  Expressivity of their non-logical language: definability

  Which relations and functions are primitive?

  Restrictiveness of their axioms: non-conservative extensions.

Example 1: Simplified 2D Maps

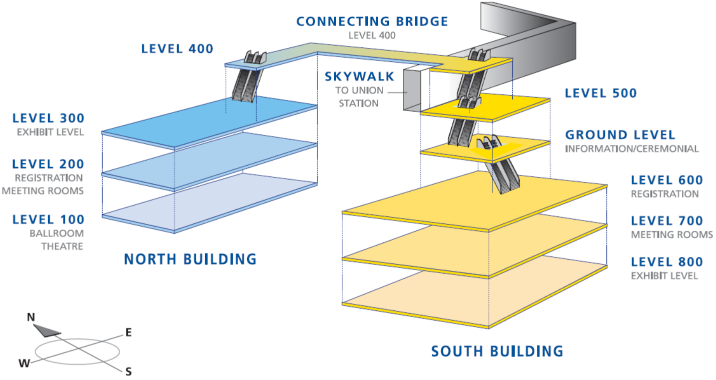


2D: cities, municipalities, lakes, parks;

1D: streets, rail lines;

0D: intersections, bridges, rail crossings.

Example 2: Building Maps



3D: entire building;
2D: each floor, stairs, escalators, rooms;
1D: walls, windows, doors;
0D: water fountains, telephones, internet outlets, etc.

⑤ What Are the Intended Structures?

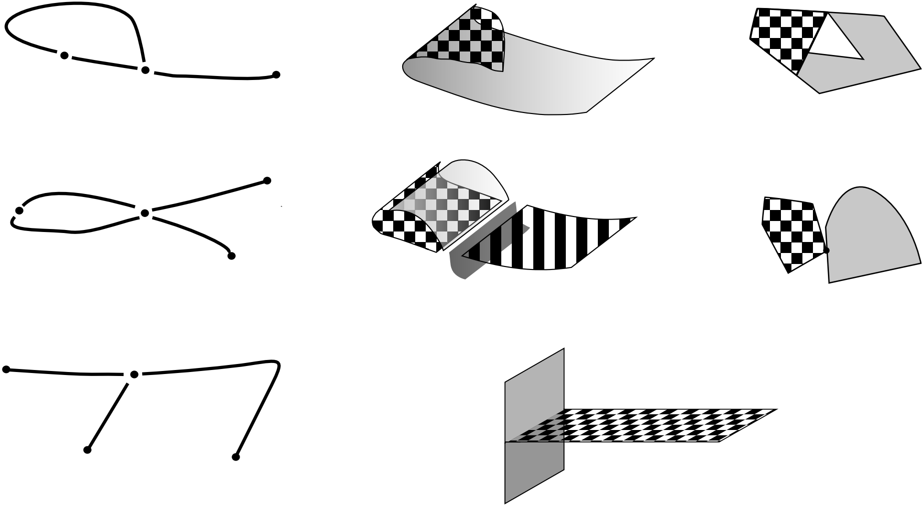
Need a model for multidimensional qualitative space as reference for evaluating our ontologies.

Idea **intended structures are topologically and dimensionally invariant transformations of simplicial complexes**

- Allows any kind of stretching, bending, rotating, curving, folding, etc.

- Specification of the class of intended structures similar to the definition of simplicial complexes from simplexes
  - Use m-manifolds with boundaries as primitive entities
  - Composite m-manifolds = finite sets of m-manifolds with boundaries of uniform dimension that do not meet in the interior
  - Class of intended multidimensional structures:
    complex m-manifolds = finite sets of composite m-manifolds
    (with closure under intersection and complementation)

Examples of Nonatomic Composite Manifolds



Methodology

② Methodology: Comparative ontology integration

(1) Construct **hierarchies of ontologies of equal expressivity** that are partially ordered by their axioms' *restrictiveness*

- Using **definability** (closure operations are defined: 7.1, 7.2, 7.5, 7.7) and **nonconservative extensions**

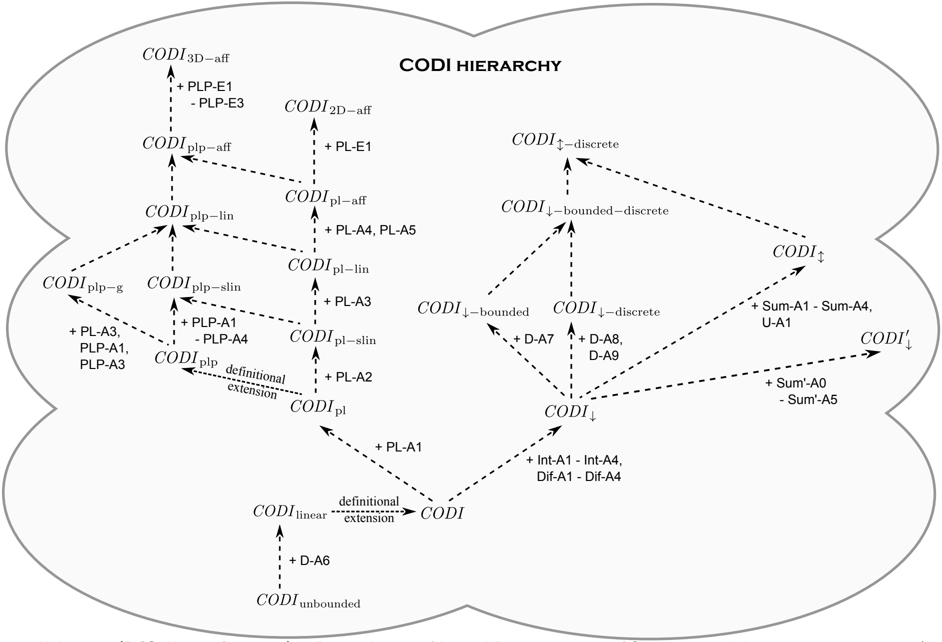
(2) **Partially order the hierarchies** by the *expressivity* of their primitives

- Using **nondefinability**

(3) Integration of external spatial ontologies using **theory relationships** to map them to extensions of *CODI*

- Full theory integration (definable equivalence between theories): 8.3
- Faithful interpretation (conservative extension, possibly language extension) established through model expansions: 10.2, 10.4, 10.6, 10.8
- Definable interpretation (possibly non-conservative extension) established when all models of the interpreting theory define models of the interpreted theory: 8.2, 10.1, 10.3, 10.5, 10.7, 10.9, 10.10
- Implicit interpretability via the intended structures: 9.5

Methodology: Hierarchy of Theories of Equal Expressivity



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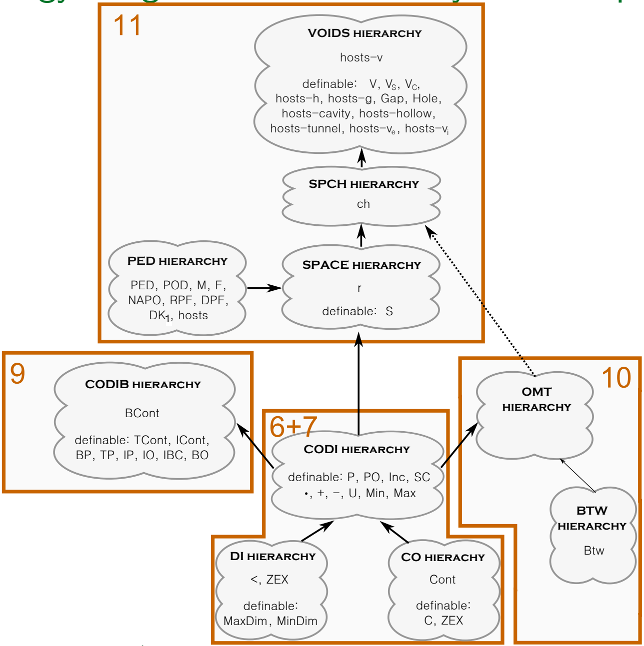
Methodology: Organize Hierarchies by Their Expressivity



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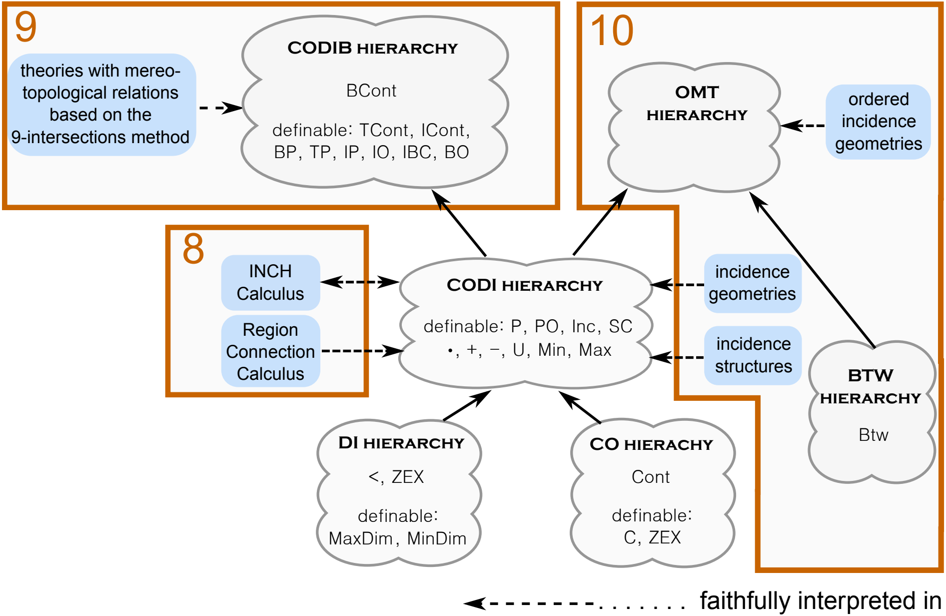
Methodology: Mappings to Ontologies in the Hierarchies



Methods Used to Verify the Developed Ontologies

- **JEPD relations:** classification of spatial relations; lends itself to spatial calculi (6.2, 9.1, 9.3, 9.4 (not disjoint), Ch. 11)
- **Satisfiability** w.r.t. the intended structures

T 7.4 Intended structures satisfy $CODI_{\downarrow}$

T 9.2 Intended structures satisfy $CODI_{\downarrow} \cup \{$BC-A1 – BC-A4$\}$

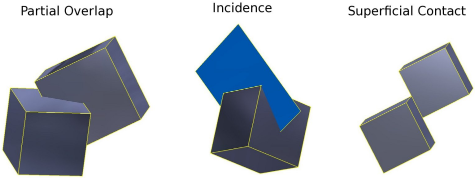
- **Model characterization:** understanding and verification of theories w.r.t. well-understood algebraic structures (4.2−4.10, 6.1, 7.6, 8.1)
  - The models of $CODI_{\updownarrow}$ are "stacks" of Boolean algebras
- **Cross-verification:** theory relationships to other ontologies
- **Competency questions:** proofs of expected properties
- **Non-trivial consistency:** constructed models to show that any relation can have a non-empty extension
  - Can prove consistency for even the most complex ontology (roughly 120 axioms, 60 distinct non-logical symbols, 40 existentials)

Some Details

Key: General Multidimensional Mereotopology

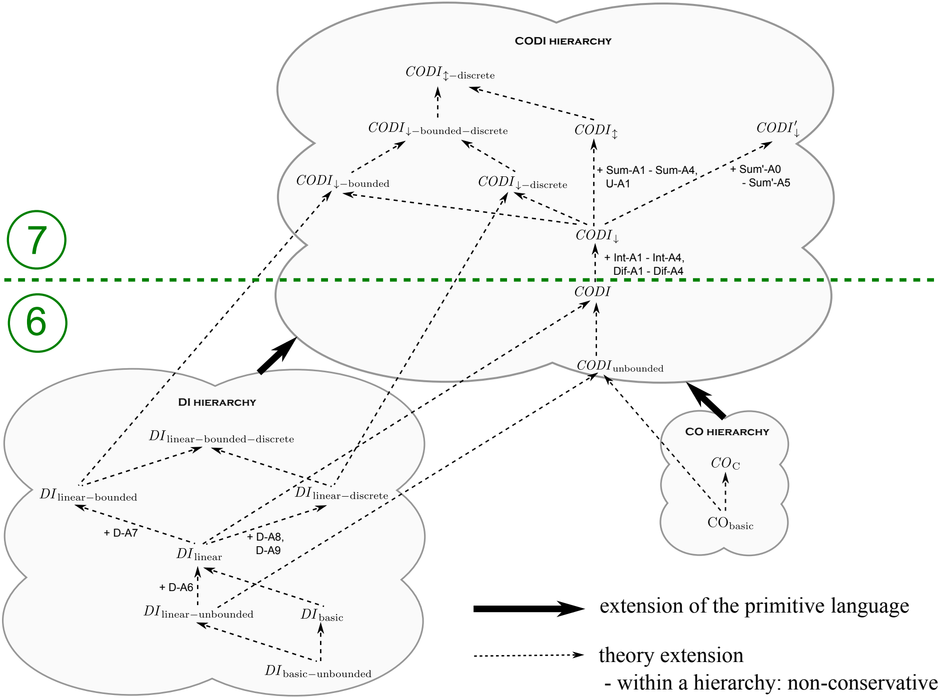
(6) Theory of multidimensional space with mereotopological relations

- Axiomatization of **linear relative dimension**: *DI* hierarchy
- Axiomatization of **spatial containment**: *CO* hierarchy
- Combination to $CODI_{\text{linear}}$
  - Three jointly exhaustive and pairwise disjoint (JEPD) types of contact: **Partial Overlap**, **Incidence**, **Superficial Contact** definable in *CODI*



(7) Extension of $CODI_{\text{linear}}$ with mereological closure operations intersection $\cdot$, difference $-$, sum $+$, and universal u

- Defined functions that are total


CODI hierarchy
$CODI_{\updownarrow-\text{discrete}}$
$CODI_{\downarrow-\text{bounded}-\text{discrete}}$
$CODI_{\updownarrow}$
$CODI'_{\downarrow}$
$CODI_{\downarrow-\text{bounded}}$
$CODI_{\downarrow-\text{discrete}}$
+ Sum-A1 - Sum-A4, U-A1
+ Sum'-A0 - Sum'-A5
$CODI_{\downarrow}$
+ Int-A1 - Int-A4, Dif-A1 - Dif-A4
7
6
$CODI$
$CODI_{\text{unbounded}}$
DI hierarchy
$DI_{\text{linear}-\text{bounded}-\text{discrete}}$
$DI_{\text{linear}-\text{bounded}}$
$DI_{\text{linear}-\text{discrete}}$
+ D-A7
+ D-A8, D-A9
DI_{linear}
+ D-A6
$DI_{\text{linear}-\text{unbounded}}$
DI_{basic}
$DI_{\text{basic}-\text{unbounded}}$
CO hierarchy
CO_{C}
CO_{basic}
extension of the primitive language
theory extension
- within a hierarchy: non-conservative

Extensions of *CODI*

(9) Boundaries and interiors [1,2]

- Motivation: *CODI* theories cannot distinguish between boundary and interior contact
- Introduces a new primitive relation of *boundary containment* → hierachy *CODIB*
- Defined relations tangential and interior containment/parthood
- More fine-grained relations that generalize the well-known 9-intersection relations to arbitrary finite (co-)dimensions

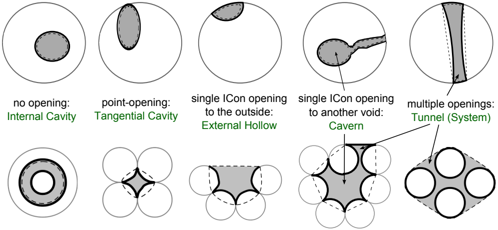
(10) Extension with betweenness: Geometries [1,2]

- Motivation: even when capturing space qualitatively we often want to preserve spatial orderings, for example, for street maps
- New quaternary primitive relation of *relativized betweenness* → hierarchy *BTW*
- Combining *BTW* and *CODI* results in ordered mereotopologies *OMT*; which are qualitative generalizations of ordered geometry

(11) Modelling Physical Space

Utilize the axiomatization of abstract space in a specific setting:
Ontology of Hydrogeology (rock formations and water bodies)

- New primitive relation of convex hull, which is not definable in every *ordered mereotopology* but only in very restricted ones
- 4 Classifications of physical voids
  - by the void's self-connectedness (simple vs. complex void)
  - by the host's self-connectedness (gap vs. hole)
  - by the void's external connectedness (cavity vs. hollow vs. tunnel)
  - by granularity distinction (voids in matter vs. voids in objects)



Summary

Summary of Results

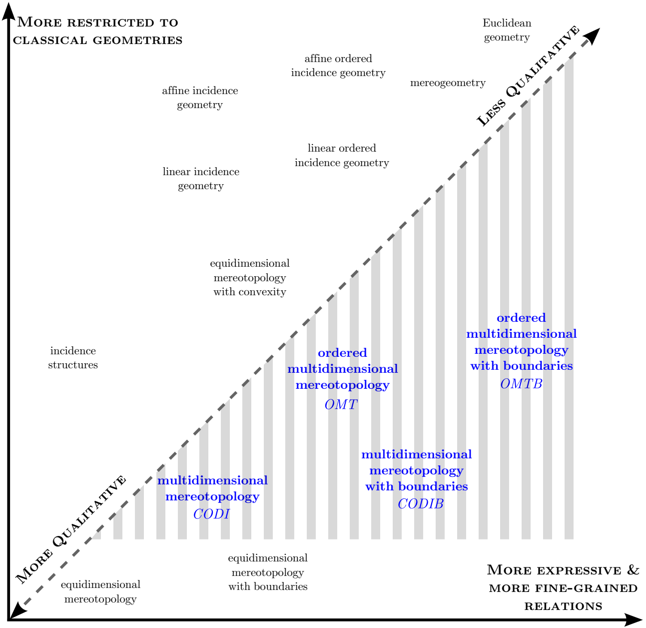
(1) Developed new qualitative ontologies of space that are more expressive than previously available mereotopologies and formally studied their expressivity and their logical relationships

- Proposed a characterization of multidimensional qualitative space
- First well-understood theory of multidimensional mereotopology
- Not fixed in number of dimensions, not tied to points or regions

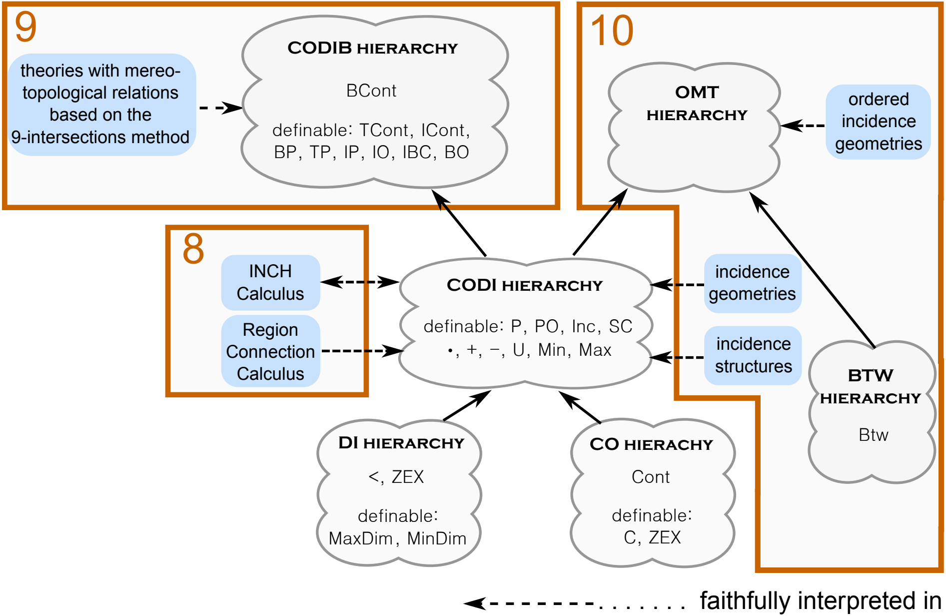
(2) Established formal relationships (theory interpretations and relationships between classes of models) to understand how various ontologies of space relate to one another

$\Rightarrow$ first step toward integration of spatial information

Bridging the Gap between Mereotopologies and Geometries



Integration Results



Future Work: Start Automate Verification and Integration

- Manual ontology verification and integration is arduous
  - Automated reasoning support is readily available
  - Automated reasoning often successful without much manual tweaking
  - ⇒ Suggests **ontology verification** and **ontology integration** can be largely automated in practise

Limitations and More Future Work

- **Static** view of **abstract space**
  - Can accommodate conflicting conceptualizations of space (surfaces as immaterial lower-dimensional abstractions or as thin layers of material) but cannot "solve" those conflicts
  - Only Chapter 11 shows how it can be used to model physical space that involves material objects and different levels of granularity
  - Ongoing work: physical containment and granularity
- More work needed to evaluate whether the ontologies are sufficient to capture maps or buildings qualitatively
- Investigate Convexity in Multidimensional Space