Detecting Physical Defects: A Practical 2D-Study of Cracks and Holes

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Qualitative Spatial Reasoning: Idea [Cohn 1997]

Common-sensical reasoning about the physical world

- In the absence of complete knowledge
- In the absence of precise quantitative descriptions
- If quantitative reasoning is intractable

- Abstracts away irrelevant properties for a certain reasoning task
  - Collapses indistinguishable values into equivalence classes of values, so-called qualitative values
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Benchmarks depend on the qualitative properties.

Qualities in QSR

Various spatial qualities definable for spatial entities:

- *Topology*: connection/contact
- *Mereology*: parthood
- *Morphology*: shape (curvature, convexity, congruence, etc.)
- *Dimension*: point, line, surface/region, etc.
- *Direction & Orientation*: geographic (N, W, SW), right/left, inside/outside, parallel
- *Size and Distances*: small/big, close/far
- *Fuzzy & approximate qualities*: vagueness

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Unlike SAT, CSP, QBF benchmarking:
varying expressiveness amongst QSR formalisms.

Example domain: Metal sheet cutting and punching

Simple Two-dimensional domain: but requires diverse spatial qualities

- Punching & drilling processes
- Cutting processes
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Detecting production errors & sorting out faulty products

Errors can be caused by:

- Loose drill
- Wrong positioning
- Moving during processing
- etc.

Queries: detecting processing faults

Queries: Did we ...

- punch only a single (connected)?
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Need mereotopology, but what other qualities?

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Benchmarking for QSR

Two dimensions when talking about benchmarking

1. Expressiveness benchmarking
   - Not applicable to traditional benchmarks in SAT, CSP, QBF, Planning
   - QSR is rich in languages: each formalisms has a different expressiveness
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Define a super-language for expressing QSR problems?

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⇒ Need other ways to compare expressiveness!

- *Well-defined for Formal Ontologies*

Problems with expressiveness

Diverse expressiveness hinders direct comparison of QSR formalisms

- Benchmarks of diverse expressiveness $\Rightarrow$ everyone has their own?
- Compare only formalisms of same expressiveness?
  - *Need to understand the expressiveness.*
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Example: mereotopology = mereology + topology

- Lots of theories available ⇒ perfect for benchmarking
- But difficult to think of (practical) pure mereotopological applications
- Some work on extensions of mereotopology:
  - Morphological [Borgo et al. 1996; Tarski 1956; Bennett et al., 2000]
  - Direction, Orientation [Moratz et al. 2000; Sharma 1996]
  - Distances, Size [Hernández et al. 1995]

How can formal ontologies help?

Ontology

Ontology $\equiv$ Computer-interpretable specification of a domain to

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Can establish semantic mappings between ontologies of QSR formalisms.
$\Rightarrow$ *Give a formal (model-theoretic) characterization of expressiveness.*

Example: A first-order mereotopological ontology [Asher, Vieu 1995]

A1. $C(x, x)$ (*C* reflexive)

A2. $C(x, y) \rightarrow C(y, x)$ (*C* symmetric)

A3. $\forall z\,(C(z, x) \equiv C(z, y)) \rightarrow x = y$ (*C* extensional)

A4. $\exists x \forall u\,[C(u, x)]$ (Universe a^*)

⋮

D1. $P(x, y) \equiv \forall z\,[C(z, x) \rightarrow C(z, y)]$ (Parthood)

D3. $O(x, y) \equiv \exists z\,[P(z, x) \wedge P(z, y)]$ (Overlap)

D4. $EC(x, y) \equiv C(x, y) \wedge \neg O(x, y)$ (External Connection)

D5. $TP(x, y) \equiv P(x, y) \wedge \exists z\,[EC(z, x) \wedge EC(z, y)]$ (Tangential Part)

⋮

Model-theoretic Analysis of Ontologies

Model $\mathcal{M}$ of a first-order ontology T:

$\mathcal{M} = \langle \mathcal{D}, \mathcal{I} \rangle$ is a model iff it satisfies all axioms of the ontology T.

Model-theory for **comparing expressiveness** of ontologies

Largely independent from algorithmic complexity and performance

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Model-theory for **comparing expressiveness** of ontologies

Largely independent from algorithmic complexity and performance

- **Relative Interpretations: Compare ontologies relatively**
  - Definitional extensions (subset of models)
  - Definitional equivalence (equal sets of models)
  - Definability: find sentences that discriminate two models
- *Representation Theory*: use other well-understood mathematical structures to capture the models up to elementary equivalence
- *Classification Theorems*: Identify invariants and classes of models

Expressiveness analysis within ontologies: Definability

Let T_Σ be an ontology in a (first-order) language Σ.

Assume $\mathfrak{M}'_\Sigma$ and $\mathfrak{M}_\Sigma$ to be models of T_Σ. I.e. $\mathfrak{M}'_\Sigma \models T_\Sigma$ and $\mathfrak{M}_\Sigma \models T_\Sigma$.

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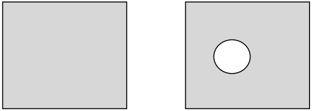
Definability

Can we discriminate $\mathfrak{M}'_\Sigma$ from $\mathfrak{M}_\Sigma$ by a sentence $\Phi \in \Sigma$
s.t. $\mathfrak{M}'_\Sigma \models \Phi$ but $\mathfrak{M}_\Sigma \not\models \Phi$?

If NO such sentence Φ exists, $\mathfrak{M}'_\Sigma$ and $\mathfrak{M}_\Sigma$ are model-theoretically equivalent (elementary equivalence for FOL).

Example I: Definability of ‘holes’

Assume $\mathfrak{M}^h_\Sigma$ and $\mathfrak{M}_\Sigma$ to be models of the ontology T_Σ.



(a) $\mathfrak{M}_\Sigma$ (b) $\mathfrak{M}^h_\Sigma$

Definability

Can we discriminate $\mathfrak{M}^h_\Sigma$ from $\mathfrak{M}_\Sigma$ by some sentence Φ_h s.t. $\mathfrak{M}^h_\Sigma \models \Phi_h$ but $\mathfrak{M}_\Sigma \not\models \Phi_h$?

Is the concept of ‘hole’ **definable** in T_Σ?

$\Rightarrow \Phi_h$ would then be a sentence defining a ‘hole’.

Example I: Qualities relating to ‘holes’

- Most properties describing regions qualitatively also apply to holes: Topology, mereology, morphology, dimension, ...
- **New: relation to hosting body**: Topological, mereological, dimension, etc.

(a) Holeless (b) Tangential (c) Interior (d) Scattered

Discriminating these configurations allows us to answer real-world queries!

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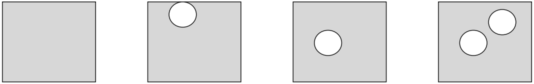
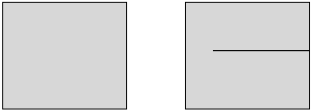


Example II: Definability of ‘cracks’

Assume $\mathfrak{M}^{\mathfrak{c}}_{\Sigma}$ and $\mathfrak{M}_{\Sigma}$ to be models of the ontology T_{Σ}.



(a) $\mathfrak{M}_{\Sigma}$ (b) $\mathfrak{M}^{\mathfrak{c}}_{\Sigma}$

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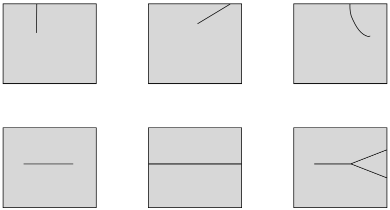
Is the concept of ‘crack’ **definable** in T_{Σ}?

$\Rightarrow$ Φ_c would then be a sentence defining a ‘crack’.

Example II: Qualities relating to ‘cracks’

‘Crack’ $\cong$ just a ‘hole’ of a lower dimension

- Topology: connection to host, e.g. tangential, interior, separating
- Mereology: parts of a crack, e.g. branching crack
- Morphology: curvature, congruence
- Orientation: relative to hosting body, e.g. perpendicular, parallel



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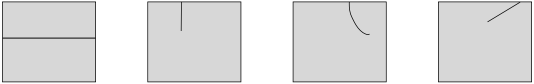

Approaches for comparing expressiveness

Defining a benchmark problem: Begin with the intended concept

1. Give some (formal) definition capturing its intended semantics:
   x is a 'Tangential Hole' iff it is a 'negative' proper n-dimensional part of some n-dimensional object y; x is completely in y and the boundaries of x and y share at least one point.
2. Check whether that is definable in T_i and/or T_j

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Benchmarking (against an existing ontology): Begin with an ontology

1. Take a concept definable in T_i
   x is a 'Tangential hole' $THole(x) \equiv TPart(x, y) \wedge ProperHole(x)$
2. Is 'Tangential hole' $THole(x)$ expressible in the language of T_j ?
   $\Rightarrow$ Definable Interpretation

Task: Categorize QSR benchmarking problems

Each (set of) benchmarking problem(s) needs to include

1. Definition of terms and their semantics
   $\Rightarrow$ implicitely defines the required qualities
2. Relevant queries (benchmarking queries)
3. Models that need to be distinguished

Categorize problems by **expressiveness/qualities required for ...**

1. a **single benchmarking problem**
   e.g. distinguishing an interior hole from a tangential hole
2. a **benchmarking domain**
   e.g. detecting mereotopological properties of holes

Formal tools for comparing expressiveness

Tradeoff between expressiveness and complexity

- unfair to compare two formalism of different expressiveness
- more expressive ontologies, i.e. those that can distinguish more concepts, are usually computationally more expensive

$\Rightarrow$ Performance benchmarking is not enough.

$\Rightarrow$ **Need (formal!) benchmarking of expressiveness.**

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Proposal: Definability as tool to compare expressiveness

- Formal notion of 'Definability'
- Comparing definability of spatial concepts within ontologies

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