CS 2401
Complexity Theory
ASSIGNMENT # 2
DUE DATE: Thursday, October 26, 2006

(1.) (Decision versus Search) Let L be a language in NP. Let $R(x, y)$ be an associated polynomial-time algorithm such that for any x, x is in L if and only if there is a y such that $R(x, y) = 1$. The search problem associated with L, $Search_f$, takes as input x. If $x \in L$, then $Search_f$ outputs a y such that $R(x, y) = 1$. Otherwise, if $x \notin L$, then $Search_f$ outputs "no witness". Prove that $P = NP$ if and only if for every $L \in NP$, $Search_f$ is polynomial-time computable.

(2.) Prove that IP is contained in $PSPACE$.

(3.) Use the same ideas in the proof that $\#SAT$ is contained in IP to show that TQBF is contained in IP (and thus $IP = PSPACE$).

(4.) a. Show that $AM[2] \subseteq BP \cdot NP$.

b. Show that if $UNSAT \in BP \cdot NP$, then $PH = \Sigma_3^p$.

c. Conclude that if Graph Isomorphism is NP-complete under polynomial-time reductions, then $PH = \Sigma_3^p$. (That is, the polynomial hierarchy collapses to the third level.)

(5.) Suppose we define a logspace hierarchy analogous to the polynomial time hierarchy. Does this hierarchy collapse by Immerman's theorem? Prove your answer.