CS 2429
Communication Complexity
ASSIGNMENT # 3
Due: December 4, 2009

1. In class we proved the first degree-discrepancy theorem showing that the sign degree of a boolean function is d if and only if there exists a distribution μ such that $E_{x \in \mu}[f(x) \cdot \chi_S(x)] = 0$ for all S, $|S| < d$. Prove the second degree-discrepancy theorem, duality of approximation degree:

   Theorem (Duality of approximation degree)

   Fix $\epsilon > 0$. Let $f : \{-1,1\}^n \rightarrow \{-1,1\}$, $deg_\epsilon(f) = d$. Then there exists $g : \{-1,1\}^n \rightarrow \{-1,1\}$ and a distribution μ over $\{-1,1\}^n$ such that: (1) $E_{x \in \mu}[g(x) \cdot \chi_S(x)] = 0$ for all S, $|S| < d$; and (2) $corr_\mu(f, g) > \epsilon$.

2. In class we saw how to prove that the Generalized Inner Product function can be solved with k players using $O(klogn + n/2^k)$ bits. Show how to improve this to get complexity $O(\log n + n/2^k)$.

3. There are many more applications of communication complexity than what we had time to cover in class. Explore a new application of your choosing. Read a paper and write a short (1-3 page) note explaining the application area and how it is connected to communication complexity. State one main theorem that can be proven with the use of communication complexity, and present a brief outline of how communication complexity is used to prove the theorem. You should pick an area that you are interested in, and one that may be relevant to your own research. Some suggested application areas if you can't think of anything are: streaming algorithms, static data structures, dynamic data structures, game theory, mechanism design, other circuit or branching program lower bounds, other proof complexity lower bounds, quantum complexity theory, cryptography.