CS 2429
Communication Complexity
ASSIGNMENT # 2
Due: November 16, 2009

1. If $S(n) \leq n$, show that a space $S(n)$ TM takes at least $\Omega(n^2/S(n))$ steps to decide the language $\{x\#x \mid x \in \{0,1\}^*\}$.

2. Let $f_m(x_1, \ldots, x_m)$ be 1 iff the m-bit string $x_1 \ldots x_m$ contains two consecutive 1's. Prove that $D^{worst}(f_m) = \Theta(m)$.

3. (Complexity of Relations) For any graph G with n vertices, consider the following communication problem. Alice receives a clique C in G and Bob receives an independent set, I. They want to communicate in order to determine $|C \cap I|$. (Note that this number is either 0 or 1.) Prove an $O(\log^2 n)$ upper bound on the communication complexity.

   Extra Credit (Open Problem): Improve your upper bound or prove a lower bound better than $\Omega(\log n)$.

4. For $x, y \in \{0,1\}^n$, let $d(x,y)$ denote the Hamming distance between x and y, that is, the number of indices where x and y differ. Let R be a relation consisting of all triples (x, y, m) such that $|m - d(x,y)| \leq n/3$. That is, computing R is the problem of approximating the Hamming distance between x and y. Prove that $D(R) = \Omega(n)$. (Observe that computing the Hamming distance exactly is as hard as computing the equality function.)

5. (Exercise 12.17 from KN book) The Element Distinctness function (ED) is defined as follows. The input is k integers in the range $0, 1, \ldots, 4k-1$ (thus, each integer is specified by $\log(4k)$ bits.) It outputs 1 iff all k integers are distinct. Prove that any oblivious branching program for ED requires either $\Omega(m \log m)$ length or exponential width, where $m = k(\log 4k)$ is the input length.