CS 2429
Communication Complexity
ASSIGNMENT # 1
Due: October 20, 2009

1. Let f be a boolean function on $X \times Y$. prove that if all of the rows of M_f are distinct, then $D(f) \geq \log\log|X|$. Prove that $D(f) \leq rank(f) + 1$.

2. $MED(x, y)$ is defined to be the median of the multiset $x \cup y$. Using binary search, one can show that $D(MED) = O(\log^2 n)$. Given an $O(\log n)$-bit protocol for MED.

3. Define $CC(M, k)$ to be the following problem. Given the matrix $M(f)$ of a boolean function, and a number k, decide if $C(f) \leq k$.

   (a) Show that CC is in EXP.

   (b) (*Extra Credit*) Determine the exact complexity of CC. Is it complete for some complexity class? This is an open problem. A very recent related paper by Kushelevitz and Weinreb, STOC 2009, studies this problem.

4. $GT(x, y)$, the greater-than function, is 1 if and only if $x > y$. What is the communication comlexity of the greater-than function?

   (a) Prove a lower bound of n on the deterministic communication complexity of GT.

   (b) Given an upper bound of $O(\log^2 n)$ on the randomized complexity.

   (c) (*Extra Credit*) Prove a matching upper and lower bound of $O(\log n)$ on the randomized complexity of the greater-than function.

5. In class we proved that the randomized communication complexity of IP (the inner product function) is $n/2$. Prove $R_{1/2-\epsilon}(IP) \geq n - O(\log(1/\epsilon))$.