CS 2429 - Foundations of Communication Complexity

Lecture #2: 22 September 2009

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Scribe Notes by: Noone

As of now, there are no scribe notes for Lecture 2. If you have good notes, please consider volunteering to make the lecture notes complete!

Here is a brief summary of what was covered in Lecture 2. The material can be found in the Kushilevitz and Nisan book.

First, Theorem 2.11 was stated and the two proofs given in KN were presented. This theorem is stated formally below. Informally, it states that the deterministic communication complexity of any boolean function is at most the product of the nondeterministic and conondeterministic complexities of the function.

Theorem 1. *For every function* $f : X \times Y \to \{0,1\}$, $D(f) = O(N^0(f)N^1(f))$.

Secondly, Theorem 3.14 was presented and proven. It is stated formally below. Informally, it states that the public and private coin model are essentially equivalent. (Any public coin protocol can be simulated by a private coin protocol with at most a logarithmic additive increase in the communication cost.)

Theorem 2. *Let* $f : \{0,1\}^n x \{0,1\}^n \to \{0,1\}$ *be a function. For every* $\delta > 0$ *and every* $\epsilon > 0$, $R_{\epsilon+\delta}(F) \leq R_\epsilon^{pub}(f) + O(\log n + \log \delta^{-1})$.

Lastly, the log rank conjecture was briefly discussed. It is known that $D(f) \geq \log rank(f)$ (this was proven in the last class). The best upper bound known is $D(f) \leq rank(f) + 1$. Thus the gap between these bounds is enorous. The log rank conjecture asserts that $D(f) = (\log rank(f))^{O(1)}$ for every boolean function f.