1. Prove that a theory $\Sigma$ is consistent if and only if $\Sigma$ has a model.

2. (10 points) Prove that a unary function $f$ is recursive iff $\text{graph}(f)$ is r.e. (Recall $\text{graph}(f)$ is the relation $R(x, y) = (y = f(x))$. Note that $f$ may not be total.

3. Are each of the following languages (i) recursive, (ii) r.e. but not recursive, (iii) not r.e. Prove your answer. Do not use the S-m-n theorem.
   (a.) Let $L$ be the set of all numbers $x$ such that $x$ codes a TM program, and 10 is in the range of the function computed by the program.
   (b.) Let $L$ be the set of all numbers $x$ such that $x$ encodes a TM program, and where the program coded by $x$ halts on only finitely many inputs.

4. (5 points) Let $\mathcal{L}$ be a first order language with finitely many function symbols and predicate symbols. Prove that the set of unsatisfiable $\mathcal{L}$ sentences is recursively enumerable.