

**CS 438/2404**  
**Computability and Logic**  
**ASSIGNMENT # 4**  
**Due: December 2**

1. Prove using diagonalization that there is no one-to-one function from the set of all subsets of the natural numbers to the natural numbers.
2. Prove that a theory  $\Sigma$  is consistent if and only if  $\Sigma$  has a model.
3. Exercise 12, page 84 of the notes. Prove that there is an  $\exists\Delta_0$  sentence  $A$  such that  $\neg A \in TA$  but  $PA$  cannot prove  $\neg A$ .
4. Classify the following language as one of: (i) recursive, (ii) r.e. but not recursive, (iii) not r.e. Prove your answer. (Prove directly. Do not use Rice's theorem or the S-m-n theorem.)
  - a. Let  $\mathcal{L}$  be the set of all numbers  $x$  such that 10 is in the range of  $\{x\}_1$ .
  - b. Let  $\mathcal{L}$  be the set of all  $x$  such that  $\{x\}_1$  halts on infinitely many inputs.
5. Prove that if  $\Sigma$  is any sound theory (that is,  $\Sigma \subseteq \mathcal{TA}$ ), then  $\Sigma$  is undecidable. Hint: You may want to use the fact that every r.e. set is weakly representable in the theory  $\mathcal{RA}$  by a  $\exists\Delta_0$  formula.