CSC 438S/2404S **Test #1** **Oct 23, 1998**

Name ______________________ Student No. ______________________

NO AIDS ALLOWED

Answer ALL questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

[10] 1. Prove or disprove each of the following, using any method discussed in class.

(a) $\forall x(Px \vee Qx) \models \forall xPx \vee \forall xQx$

(b) $\forall x(\exists yPy \vee Qx) \models \exists yPy \vee \forall xQx$

[5] 2. Disprove the following:

$$\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow Pxz), \forall x \forall y(Pxy \rightarrow \exists z(Pxz \wedge Pzy)) \models \exists x Pxx$$

[5] 3. Prove from the definition of $\models$ that

$$\alpha, \neg\alpha \models \beta$$

for any formulas α and β.

[10] 4. Let Φ be a set of sentences over a language which includes the binary predicate symbol $<$. If σ is a model for Φ and u and v are two elements in the universe of σ, then we say that u and v are *incomparable* if neither $\langle u, v\rangle \in <^\sigma$ nor $\langle v, u\rangle \in <^\sigma$. Suppose that for each k there is a model σ_k for Φ whose universe has a subset S_k of size k such that any two distinct elements in S_k are incomparable. Show that Φ has a model σ whose universe has an infinite subset S such that any two distinct elements of S are incomparable. HINT: Introduce new constants $c_1, c_2, ...$