CSC438F/2404F **Problem Set 6** **Fall 1999**

Due: Monday, December 6

1. Show that $\mathbf{PA} \vdash \forall x \forall y \forall z (x \cdot (y + z) = x \cdot y + x \cdot z)$. You may assume that **PA** proves the associative law of addition (see the top of page 82). Write out explicitly any induction axiom (page 81) that you need, including all quantifiers. Indicate each use of any of the axioms P1,...,P6.

2. Prove the three corollaries, page 88 of the Notes.

3. Let $\neg\mathbf{RA} = \{A \mid \mathbf{RA} \vdash \neg A\}$. Thus $\neg\mathbf{RA}$ is the set of sentences which **RA** proves false. Prove that **RA** and $\neg$**RA** are recursively inseparable. That is, prove that there is no recursive set S of sentences such that

$$\mathbf{RA} \subseteq S \text{ and } \neg\mathbf{RA} \subseteq \bar{S}$$

where $\bar{S} = \{A \in \Phi_0 \mid A \notin S\}$. (Note that S need not be a theory.)

Hint: Study the proofs of Tarski's Theorem and of Theorem 3 (page 85). Assume that there is a recursive set S satisfying the indicated conditions. Formulate a sentence asserting "I am not in S", and obtain a contradiction.