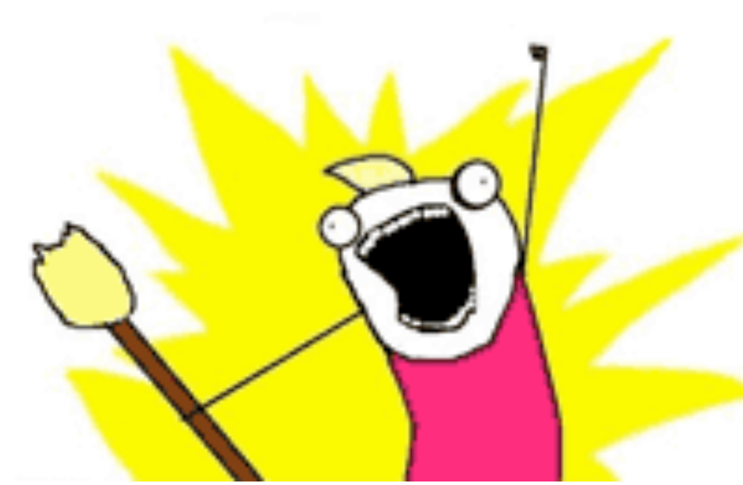
CSC263 Week 3

Announcements

Problem Set 2 is out today!

Due Tuesday (Oct 13)

More challenging so start early!

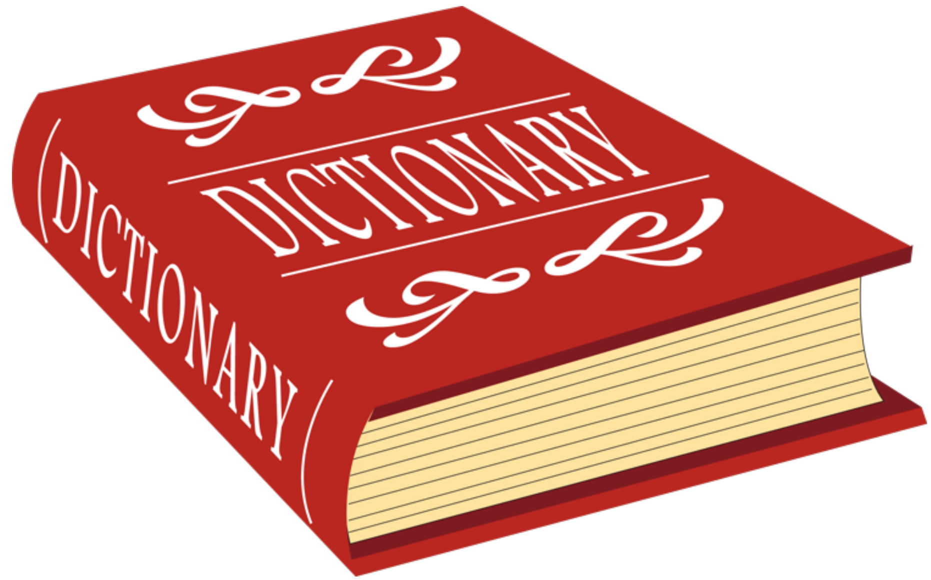



NOT EVERY GROUP PROJECT
DOES 99% OF THE WORK
HAS NO IDEA WHAT'S GOING ON THE WHOLE TIME
SAYS HE'S GOING TO HELP BUT HE'S NOT
DISAPPEAR AT THE VERY BEGINNING AND DOESN'T SHOW UP AGAIN TIL THE VERY END
IN SCHOOL YOU HAVE EVER DONE
WeKnowMemes

This week

- ➔ADT: Dictionary
- ➔Data structure:
  - ◆ Binary search tree (BST)
  - ◆ Balanced BST - AVL tree

Dictionary



What's stored:

➔words

Supported operations

➔Search for a word

➔Insert a word

➔Delete a word

Dictionary, more precisely

What's stored

- A set **S** where each node **x** has a field **x.key**
  (assumption: keys are distinct, unless o.w. specified)

Supported operations

- **Search(S, k)**: return **x** in **S**, s.t., **x.key = k**
  - return NIL if no such **x**
- **Insert(S, x):** insert node **x** into **S**
  - if already exists node **y** with same key , replace **y** with **x**
- **Delete(S, x):** delete a given **node** **x** from **S**

A thing to note: k is a key, x is a node.

More on Delete

Why Delete(S, **x**) instead of Delete(S, **k**)?

Delete(S, **k**) can be implemented by:

1. x = Search(S, k)
2. Delete(S, x)

We want separate different operations, i.e., each operation focuses on only one job.

Implement a Dictionary using simple data structures

40 -> 33 -> 18 -> 65 -> 24 -> 25

Unsorted (doubly) linked list

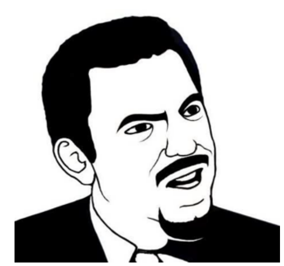
- ➔**Search(S, k)**
  - **O(n)** worst case
  - go through the list to find the key
- ➔**Insert(S, x)**
  - **O(n)** worst case
  - need to check if **x.key** is already in the list
- ➔**Delete(S, x)**
  - **O(1)** worst case
  - Just delete, O(1) in a doubly linked list

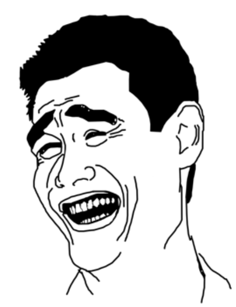
Sorted array [18 , 24 , 25 , 33 , 40 , 65]

- ➔ **Search(S, k)**
  - **O(log n)** worst case
  - binary search!
- ➔ **Insert(S, x)**
  - **O(n)** worst case
  - insert at front, everything has to shift to back
- ➔ **Delete(S, x)**
  - **O(n)** worst case
  - Delete at front, everything has to shift to front

We can do better using smarter data structures, of course

	unsorted list	sorted array	BST	Balanced BST
Search(S, k)	O(n)	O(log n)	O(n)	O(log n)
Insert(S, x)	O(n)	O(n)	O(n)	O(log n)
Delete(S, x)	O(1)	O(n)	O(n)	O(log n)

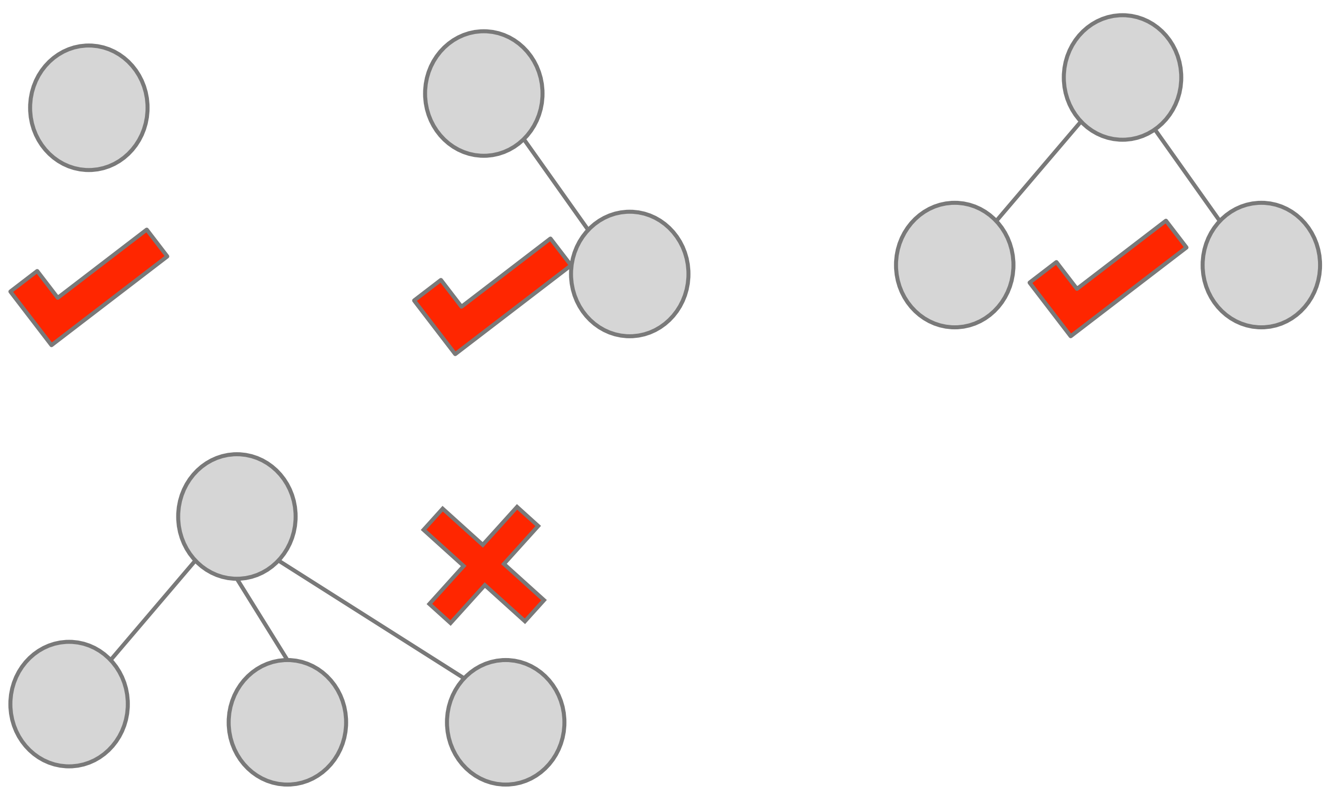




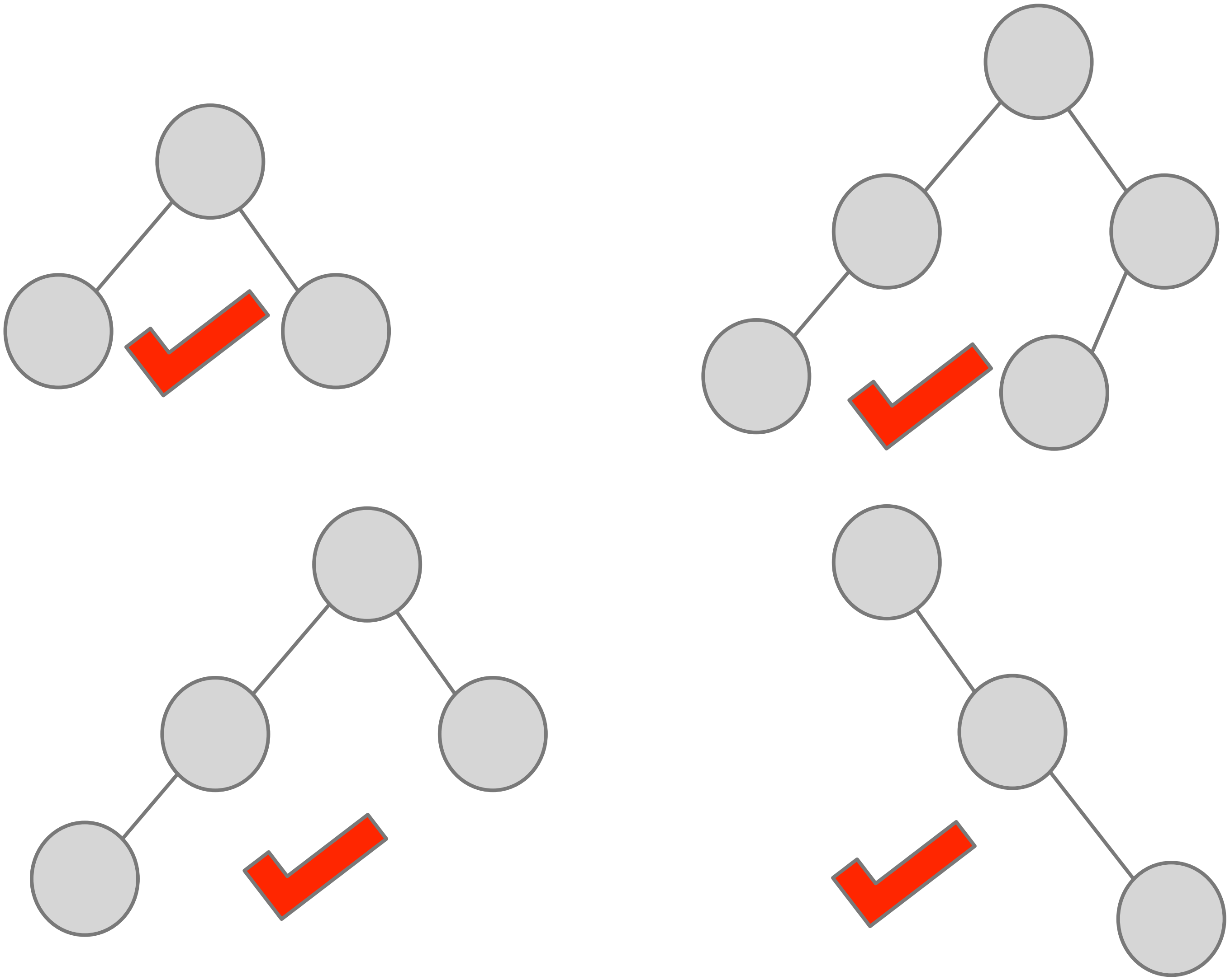
Binary Search Tree

It's a binary tree, like binary heap

Each node has **at most** 2 children

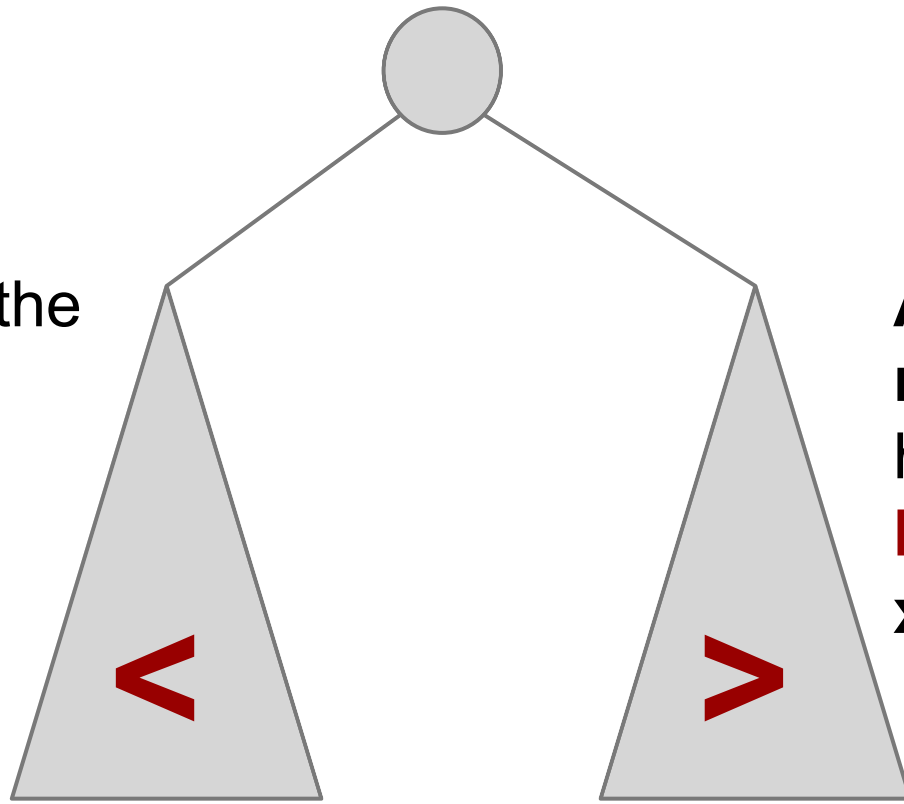


need NOT be nearly-complete, unlike binary heap



It has the BST property

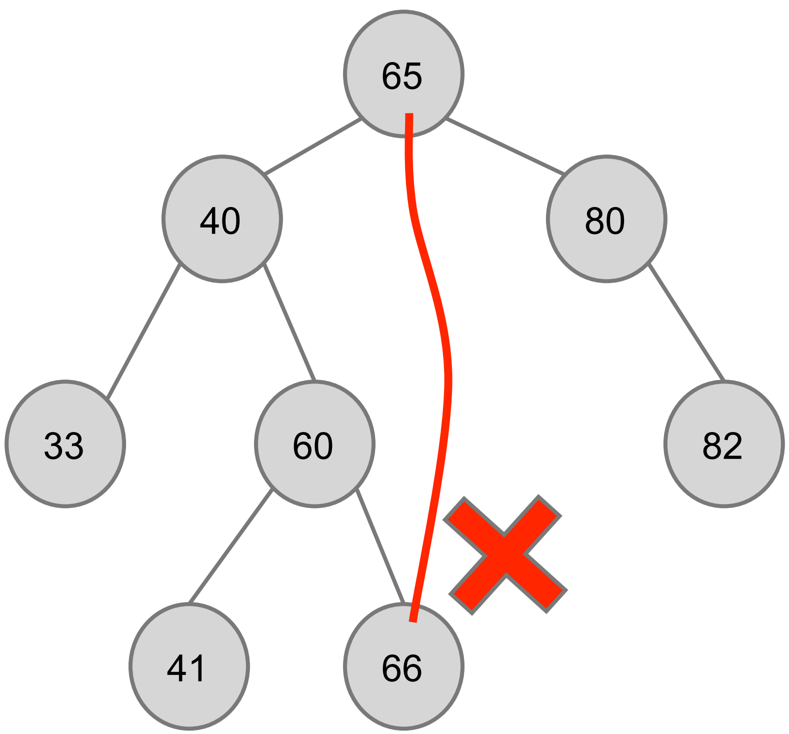
For **every** node **x** in the tree

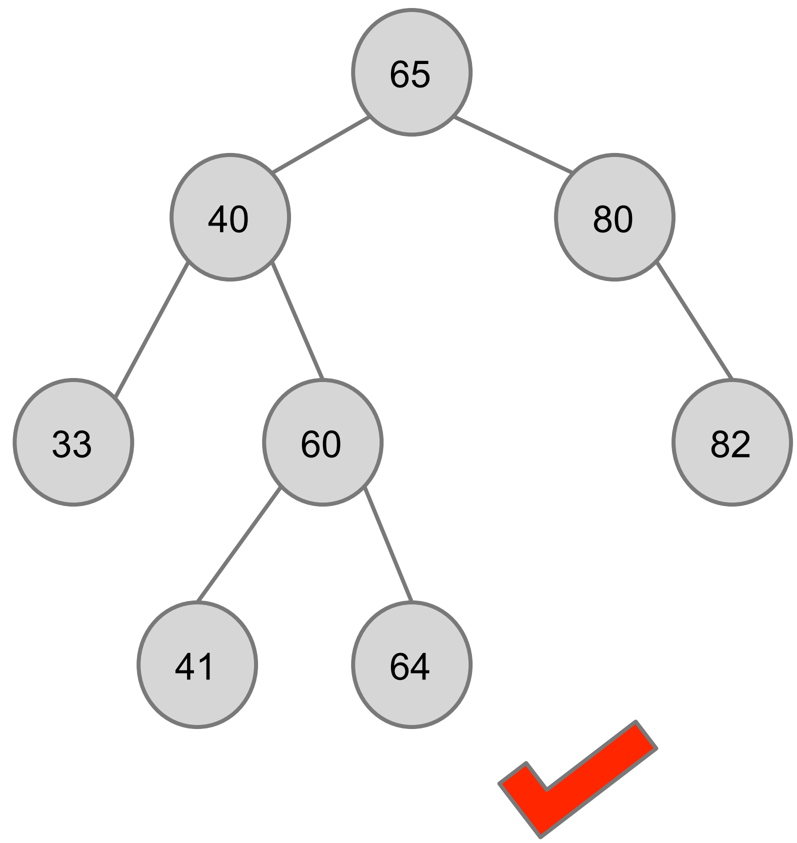


All nodes in the **left** subtree have keys **smaller** than **x.key**

All nodes in the **right** subtree have keys **larger** than **x.key**

BST or NOT?





Because of BST property, we can say that the keys in a BST are sorted.

CSC148 Quiz: How to obtain a sorted list from a BST?

Perform an inorder traversal.

We pass a BST to a function by passing its **root** node.

```
InorderTraversal(x):
# print all keys in BST rooted at x in ascending order

    if x ≠ NIL:
        InorderTraversal(x.left)
        print x.key
        InorderTraversal(x.right)
```

Worst case running time of InorderTraversal:
O(n), because visit each node exactly once.

Operations on a BST

First, information at each node x

- ➔x.key: the key
- ➔x.left: the left child (node)
- ➔x.right: the right child (node)
- ➔x.p: the parent (node)

Operations on a BST

read-only operations

- ➔TreeSearch(root, k)
- ➔TreeMinimum(x) / TreeMaximum(x)
- ➔Successor(x) / Predecessor(x)

modifying operations

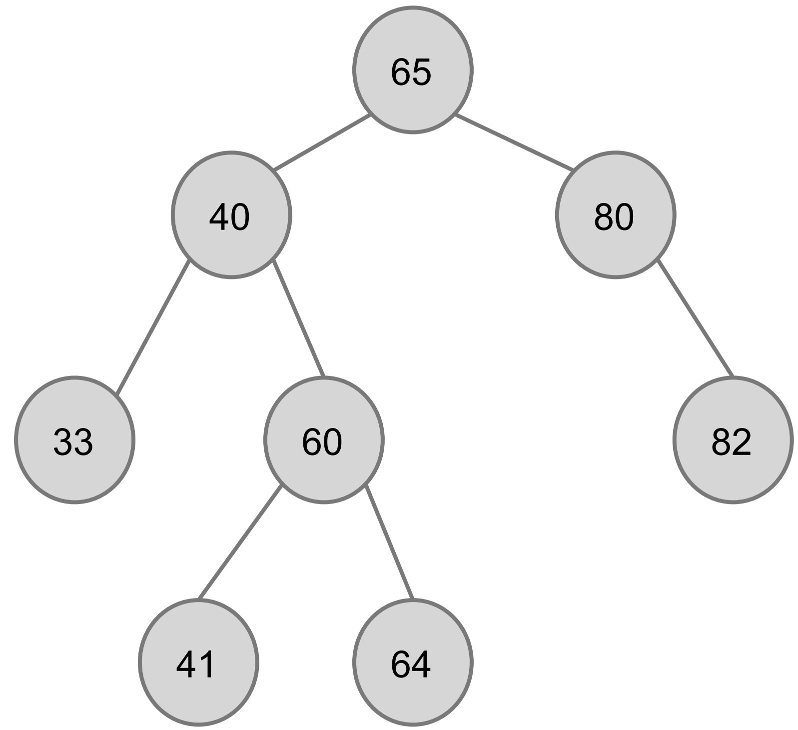
- ➔TreeInsert(root, x)
- ➔TreeDelete(root, x)

TreeSearch(root, k)

Search the BST rooted at root, return the node with key k; return NIL if not exist.

TreeSearch(root, k)

- ➔ start from root
- ➔ if **k** is **smaller** than the key of the current node, go **left**
- ➔ if **k** is **larger** than the key of the current node, go **right**
- ➔ if equal, **found**
- ➔ if going to **NIL**, **not found**



TreeSearch(root, k): Pseudo-code

```
TreeSearch(root, k):

   if root = NIL or k = root.key:
      return root
   if k < root.key:
      return TreeSearch(root.left, k)
   else:
      return TreeSearch(root.right, k)
```

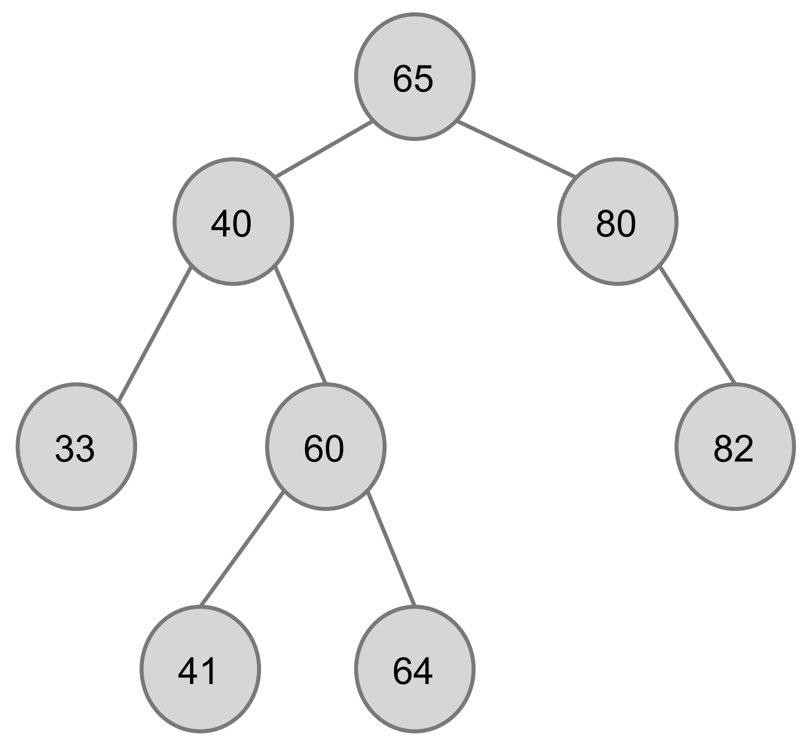
Worst case running time:
O(h), height of tree, going at most from root to leaf

TreeMinimum(x)

Return the node with the minimum key of the tree rooted at x

TreeMinimum(x)

- ➔ start from root
- ➔ keep going to the left, until cannot go anymore
- ➔ return that final node



TreeMinimum(x): pseudo-code

```
TreeMinimum(x):

   while x.left ≠ NIL:
      x ← x.left
   return x
```

Worst case running time:

O(h), height of tree, going at most from root to leaf

TreeMaximum(x) is exactly the same, except that it goes to the right instead of to the left.

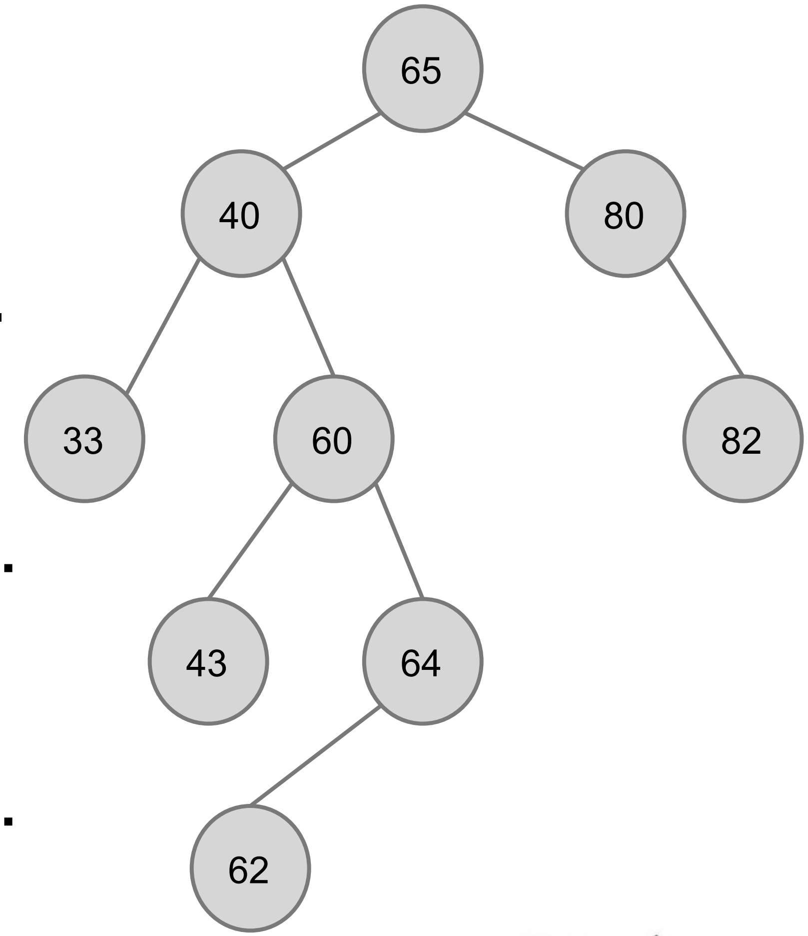
Successor(x)

Find the node which is the successor of x in the sorted list obtained by inorder traversal

or, node with the smallest key larger than x

Successor(x)

- ➔The successor of 33 is...
  - ◆40
- ➔The successor of 40 is…
  - ◆43
- ➔The successor of 64 is…
  - ◆65
- ➔The successor of 65 is …
  - ◆80



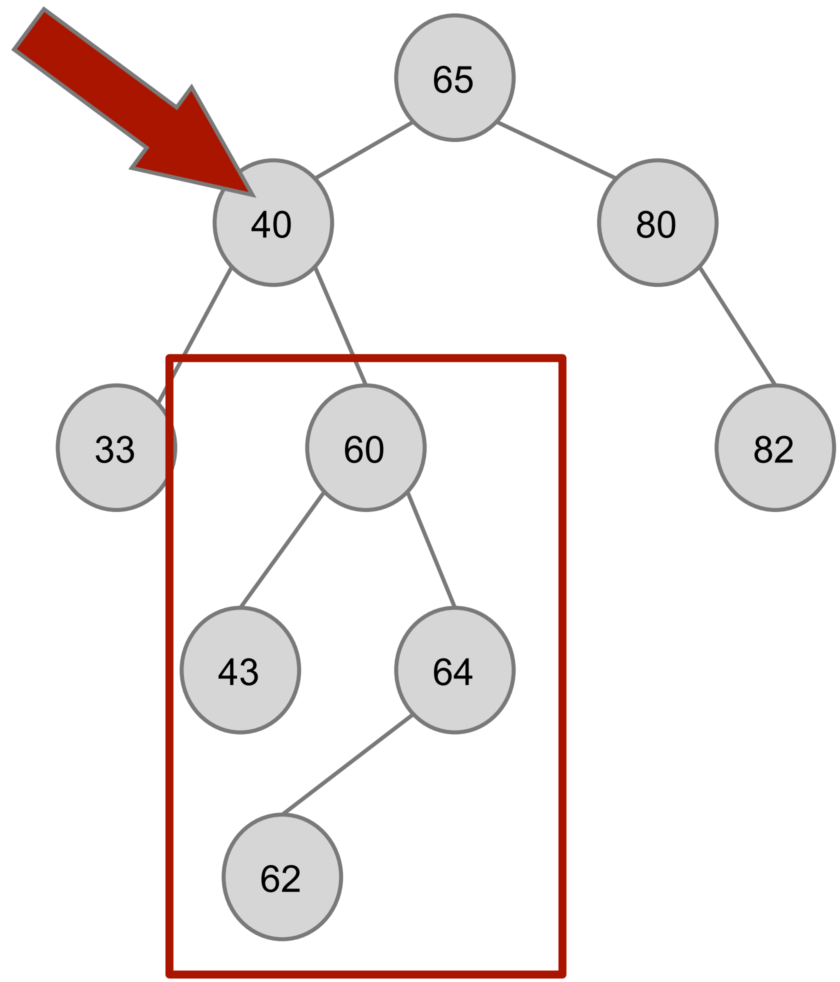
Successor(x): Organize into two cases

- ➔x has a right child
- ➔x does not have a right child

x has a right child

Successor(x) must be in x's **right subtree** (the nodes **right after x** in the inorder traversal)

It's the **minimum** of x's right subtree, i.e., TreeMinimum(x.right)



The first (smallest) node among what's right after x.

x does not have a right child

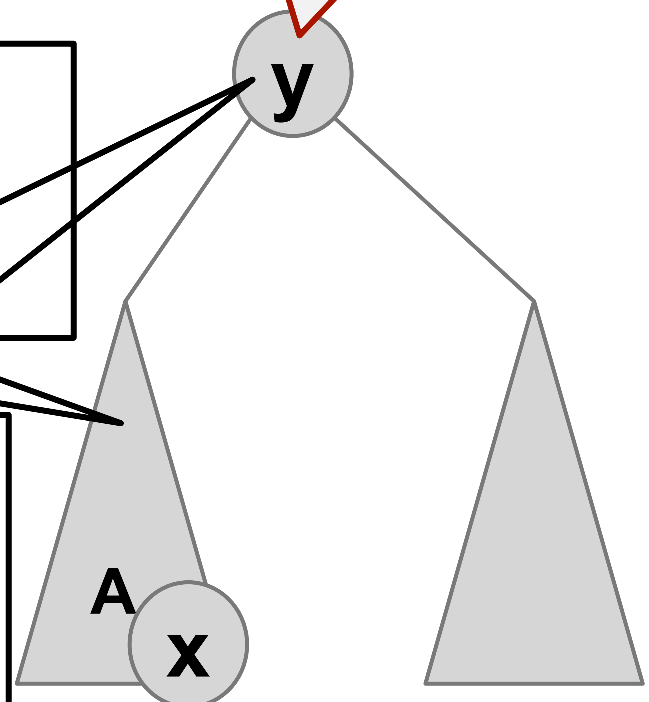
Find this guy!

Consider the **inorder traversal**
(left subtree -> root -> right subtree)

x is the **last one** visited in some **left subtree A**
(because no right child)

The successor **y** of x is the **lowest ancestor** of **x** whose **left subtree** contains **x**

(**y** is visited right after finishing subtree **A** in inorder traversal)



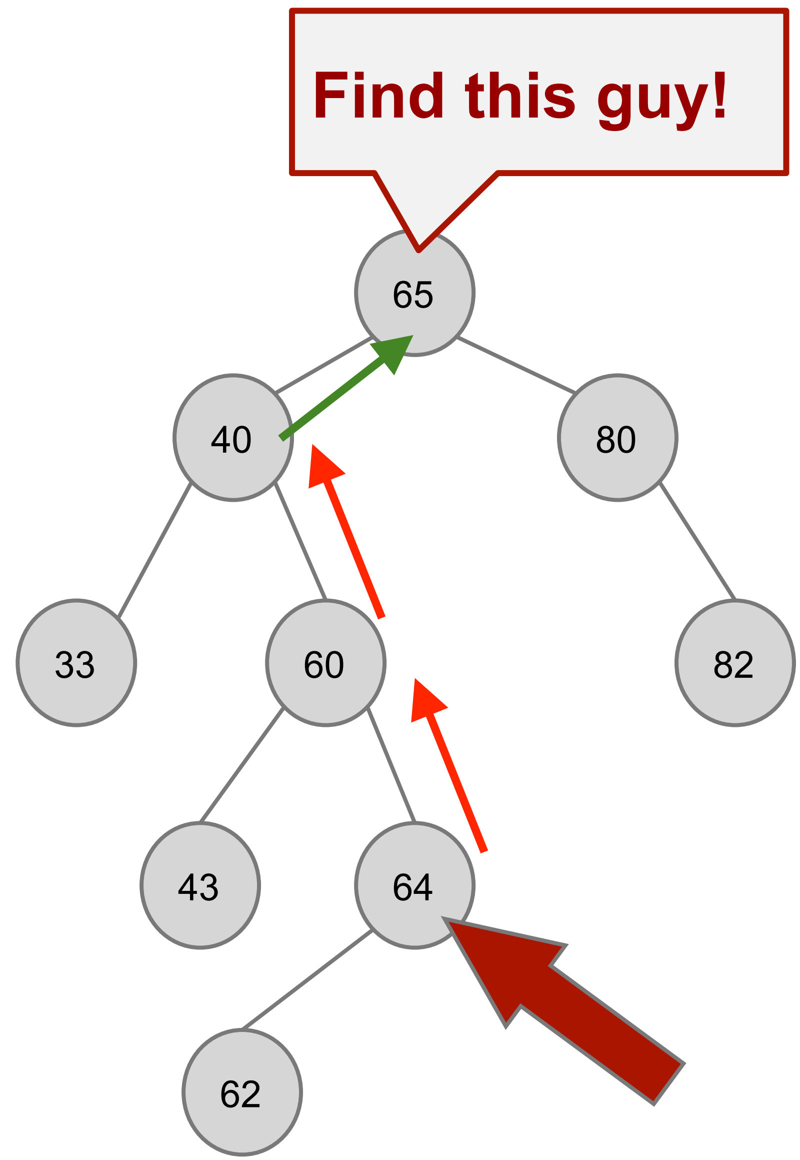
x does not have a right child

How to find:

➔go up to **x.p**

➔if **x** is a **right** child of x.p, keep going up

➔if x is a **left** child of **x.p**, stop, **x.p** is **the guy**!



Summarize the two cases of Successor(x)

- ➔If x has a right child
  - ◆return TreeMinimum(x.right)

- ➔If x does not have a right child
  - ◆keep going up to x.p while x is a right child, stop when x is a left child, then return x.p
  - ◆if already gone up to the root and still not finding it, return NIL.

Successor(x): pseudo-code

```
Successor(x):
   if x.right ≠ NIL:
      return TreeMinimum(x.right)
   y ← x.p
   while y ≠ NIL and x = y.right: #x is right child
      x = y
      y = y.p # keep going up
   return y
```

Worst case running time
O(h), Case 1: TreeMin is O(log n); Case 2: at most leaf to root

Predecessor(x) works symmetrically the same way as Successor(x)

TreeInsert(root, x)

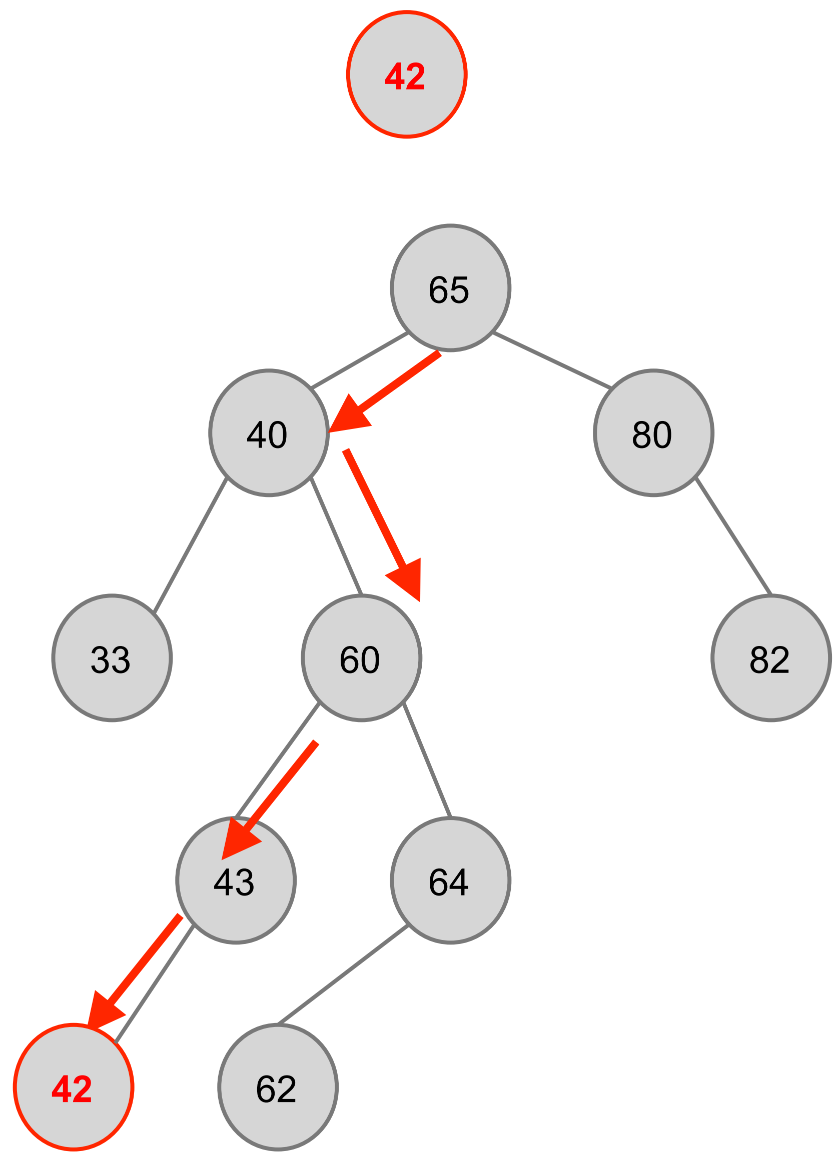
Insert node x into the BST rooted at root
return the new root of the modified tree
if exists y, s.t. y.key = x.key, replace y with x

TreeInsert(root, x)

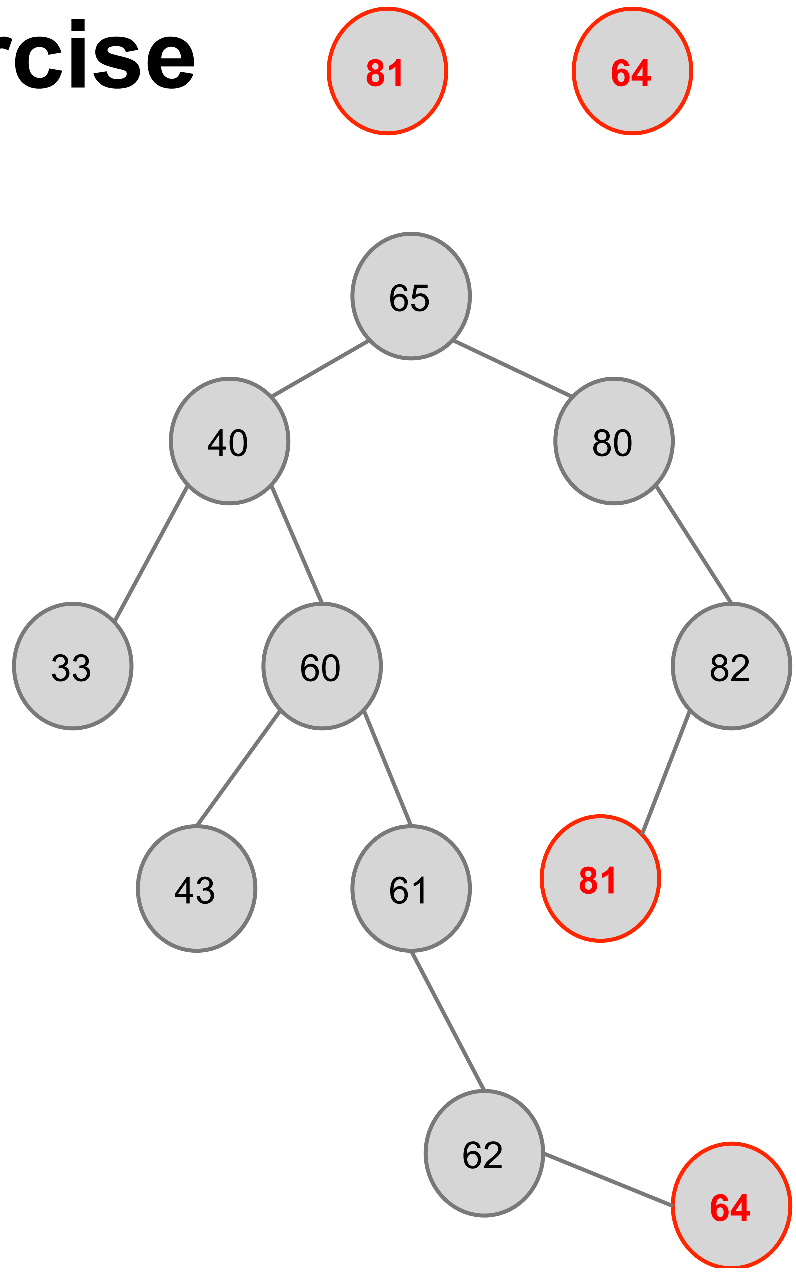
Go down, left and right like what we do in TreeSearch

When next position is NIL, insert there

If find equal key, replace the node

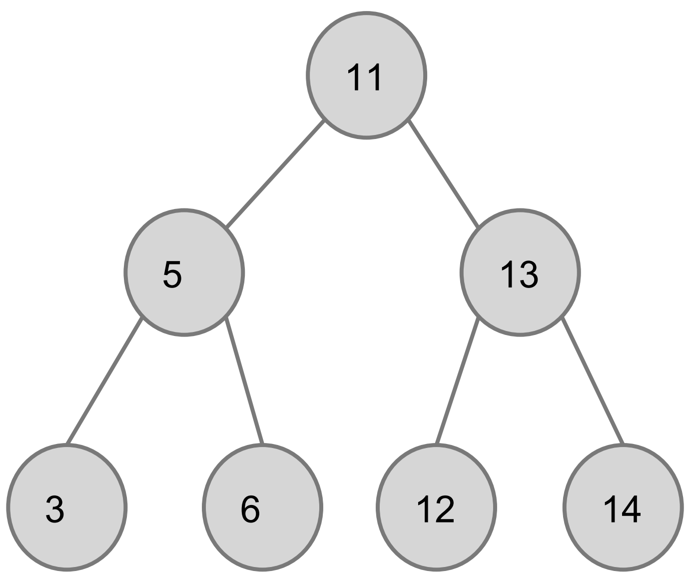


Exercise

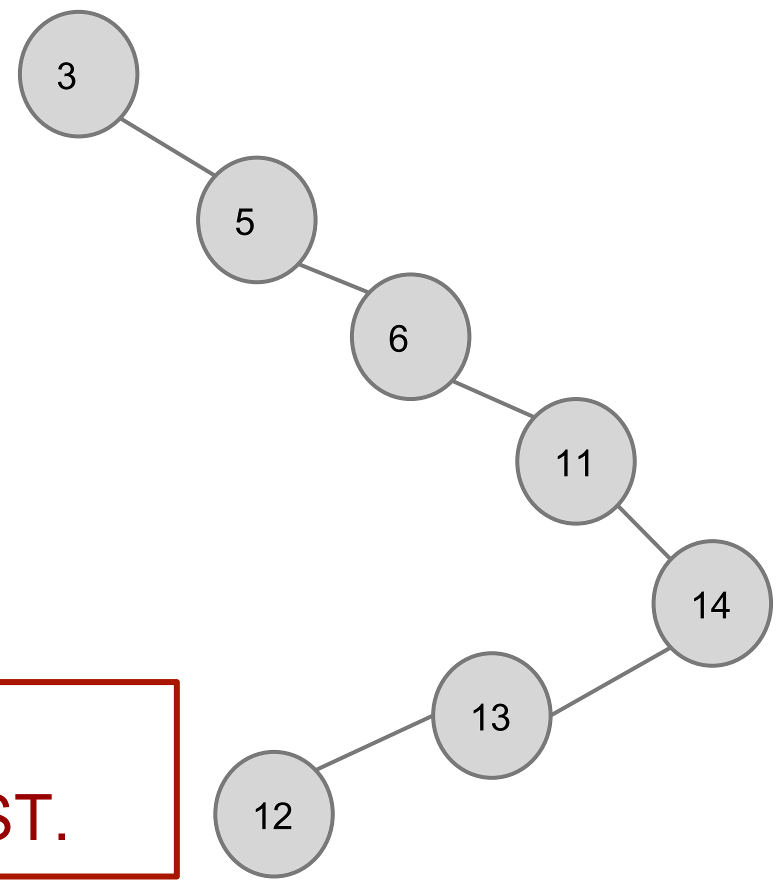


Ex 2: Insert sequence into an empty tree

Insert sequence 1:
11, 5, 13, 12, 6, 3, 14



Insert sequence 2:
3, 5, 6, 11, 14, 13, 12



Different insert sequences result in different “shapes” (heights) of the BST.

TreeInsert(root, x): Pseudo-code

```
TreeInsert(root, x):
#  insert and return the new root
   if root = NIL:
      root ← x
   elif x.key < root.key:
      root.left ← TreeInsert(root.left, x)
   elif x.key > root.key:
      root.right ← TreeInsert(root.right, x)
   else # x.key = root.key:
      replace root with x # update x.left, x.right
   return root
```

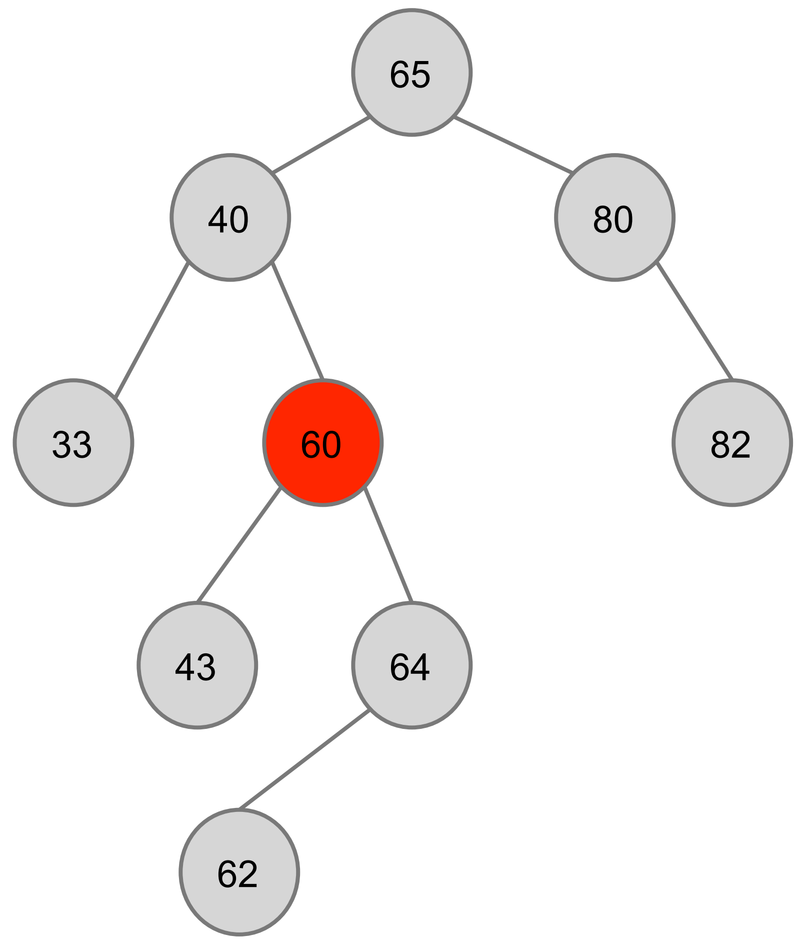
Worst case running time:
O(h)

TreeDelete(root, x)

Delete node x from BST rooted at root while maintaining BST property,
return the new root of the modified tree

What's tricky about deletion?

Tree might be **disconnected** after deleting a node, need to **connect** them back together, while maintaining the **BST property**.



Delete(root, x): Organize into 3 cases

Case 1: **x** has **no** child

Easy

Case 2: **x** has **one** child

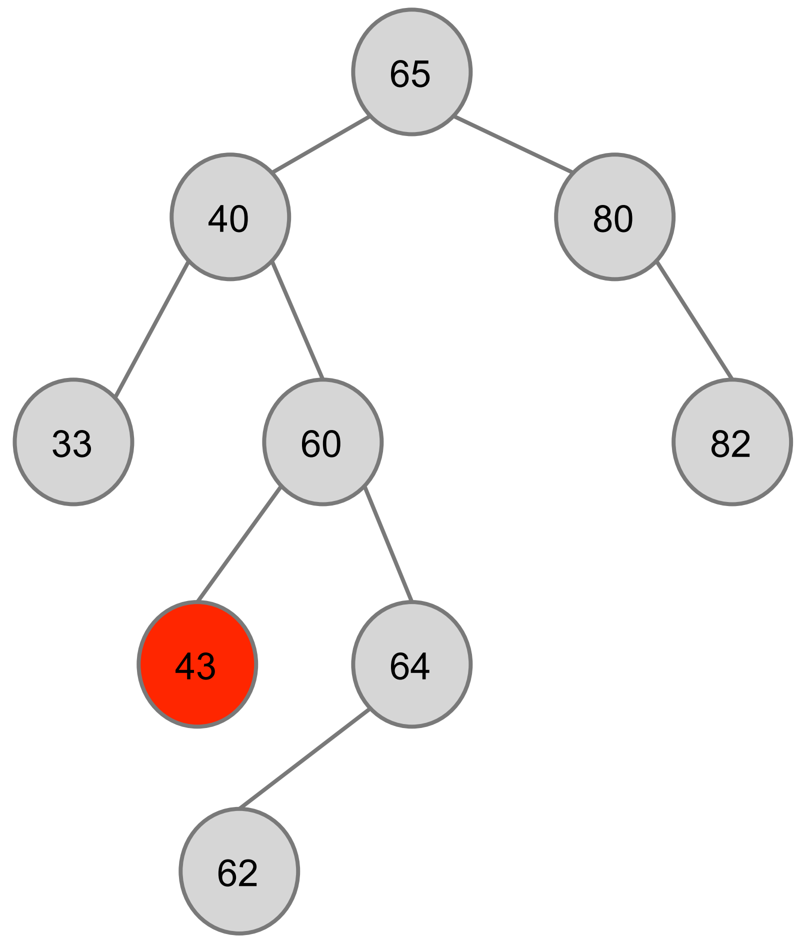
Easy

Case 3: **x** has **two** children

less easy

Case 1: x has no child

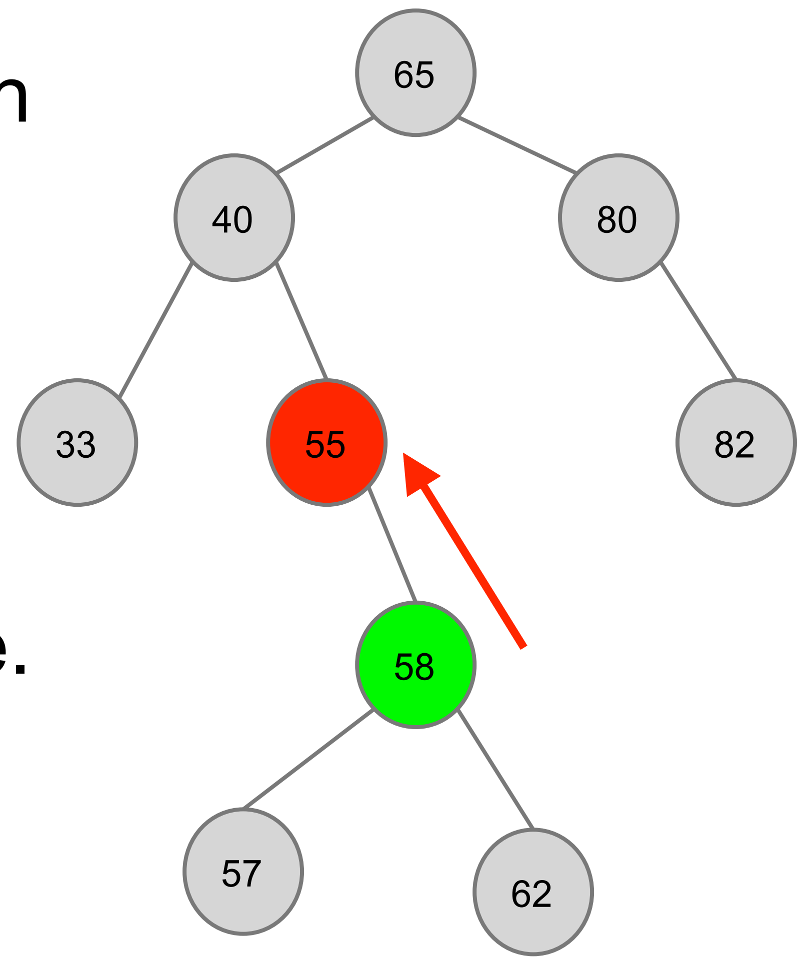
Just delete it, nothing else need to be changed.



Case 2: x has one child

First delete that node, which makes an **open spot**.

Then **promote** **x’s only child** to the spot, together with the only child’s subtree.



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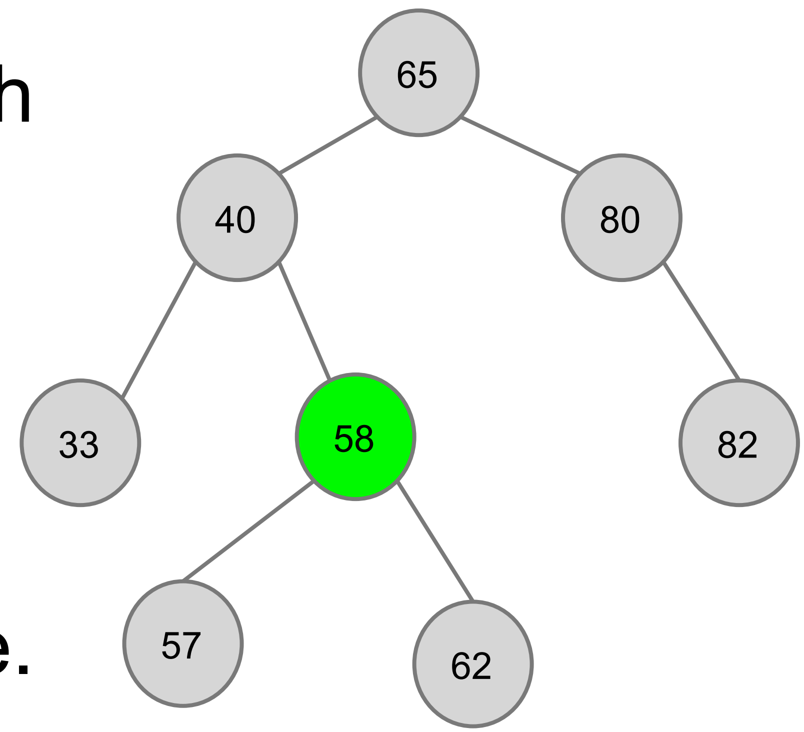
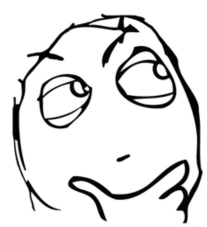


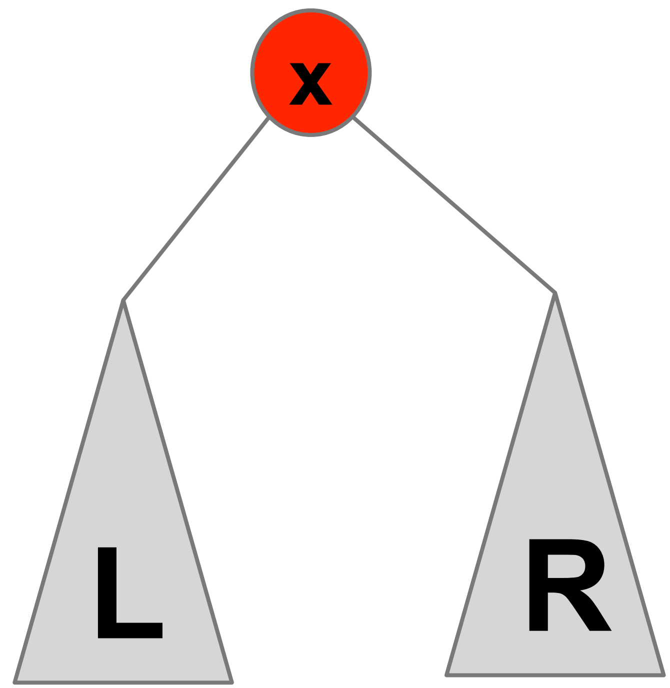
Case 3: x has two children

Delete **x**, which makes an open spot.

A node **y** should fill this spot, such that **L < y < R,**

Who should be y?



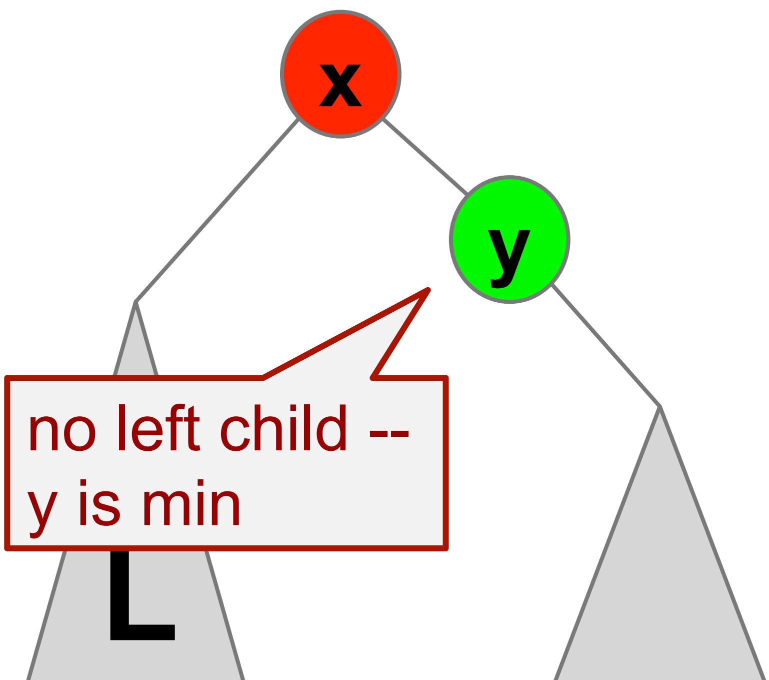


y ← the minimum of R, i.e., Successor(x)

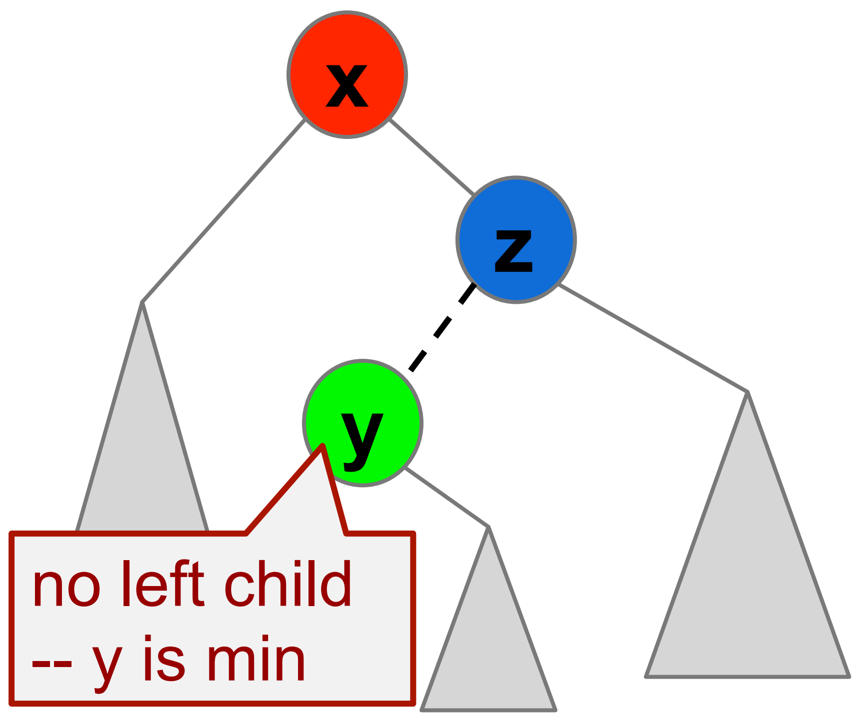
L < y because y is in R, **y < R** because it's minimum

Further divide into two cases

Case 3.1:
y happens to be the right child of **x**

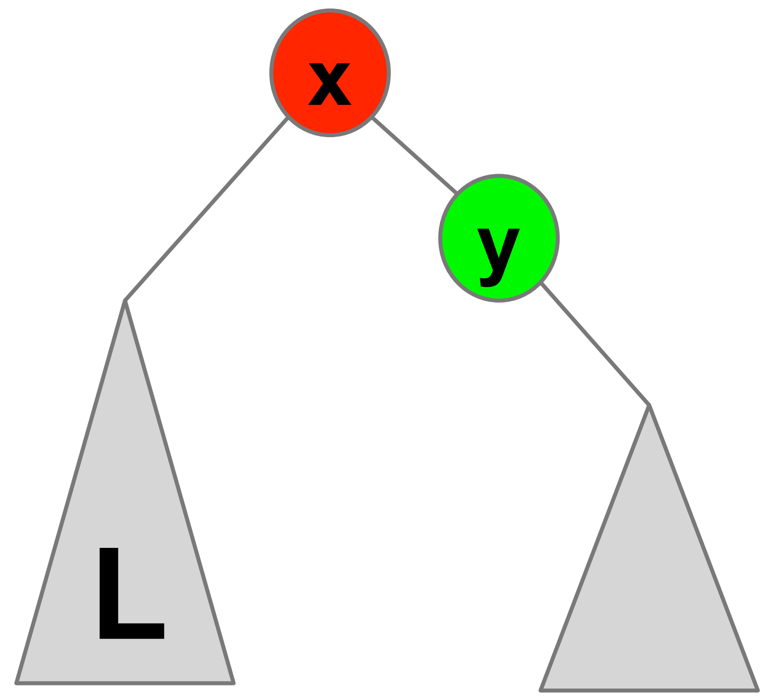


Case 3.2:
y is not the right child of **x**



Case 3.1: y is x’s right child

Easy, just **promote** **y** to **x**’s spot



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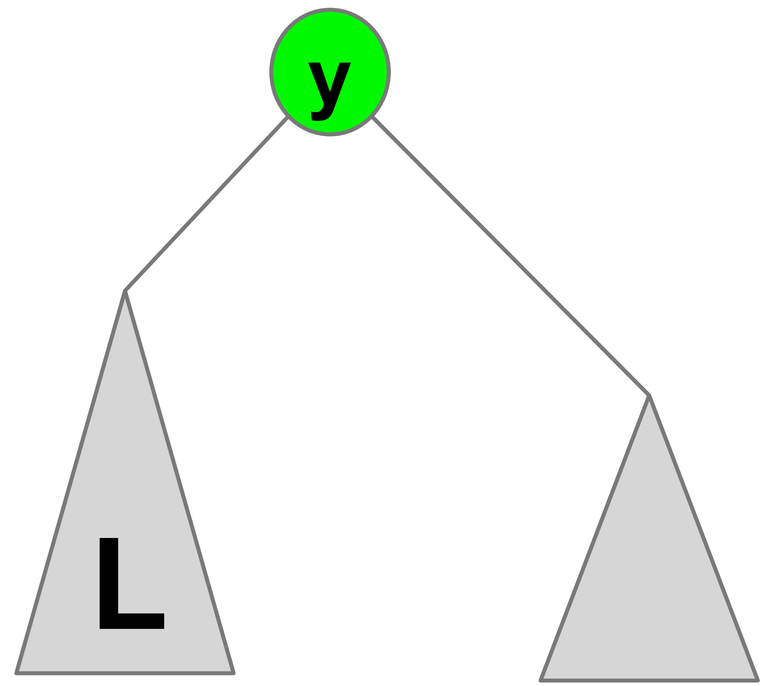
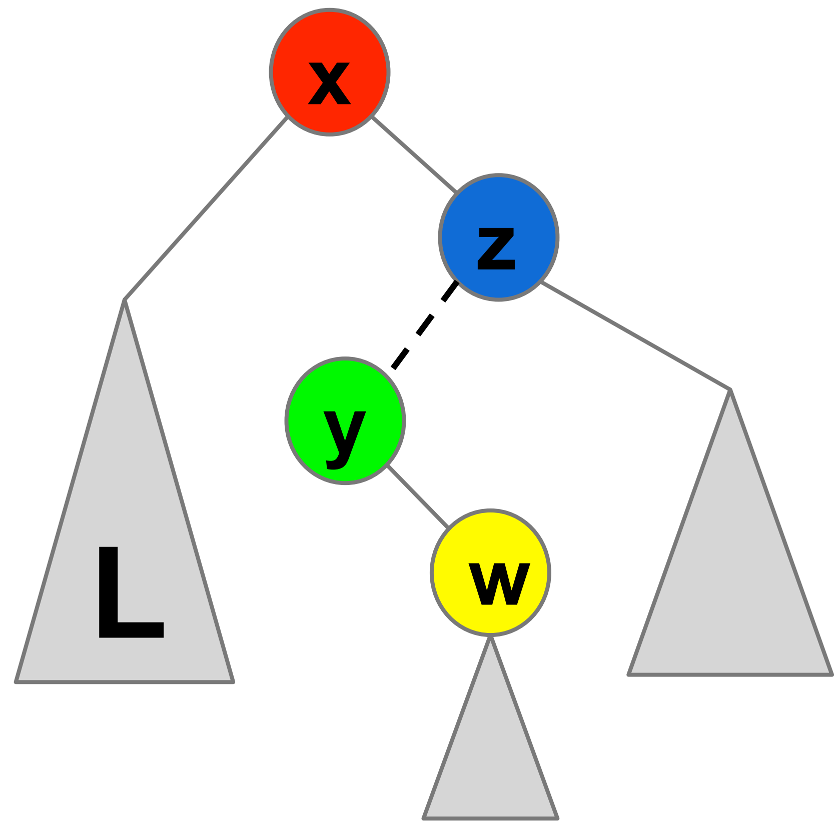


Case 3.2: y is NOT x’s right child

1. Promote **w** to **y**’s spot, **y** becomes free.

Order: y < w < z



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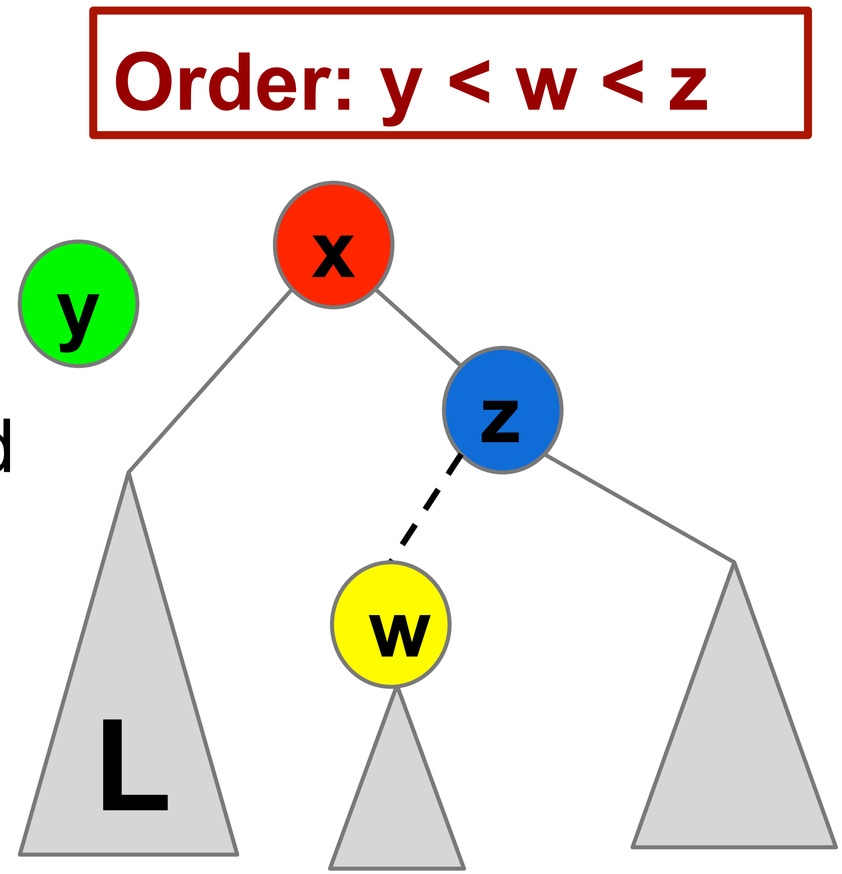
1. Promote **w** to **y**’s spot, **y** becomes free.

2. Make **z** be **y**’s right child (**y** adopts **z**)



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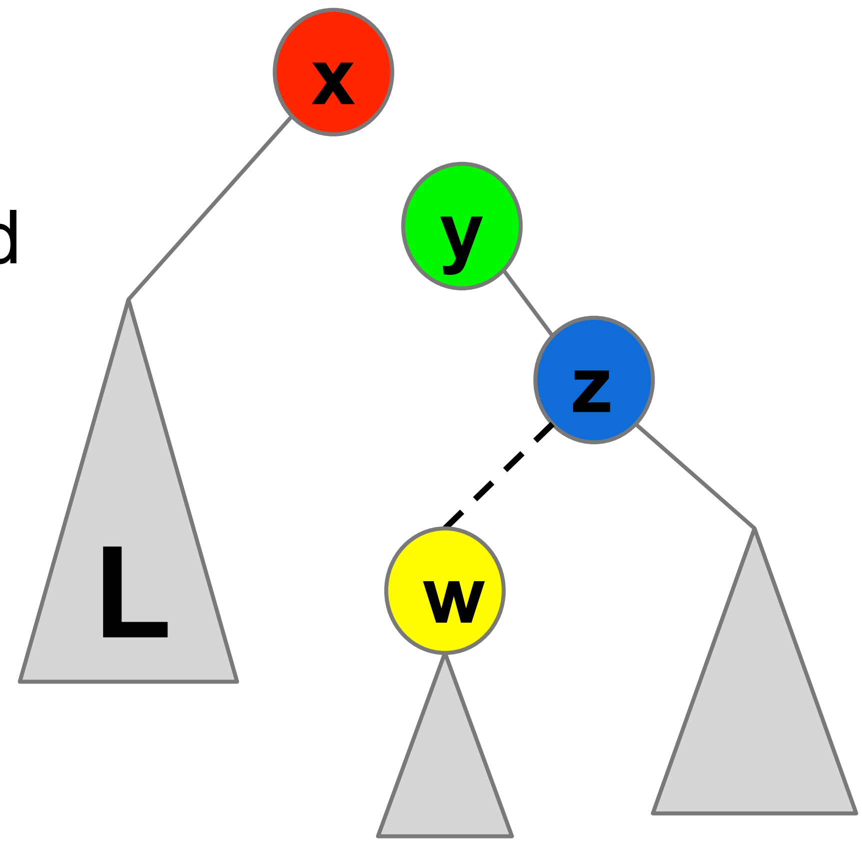

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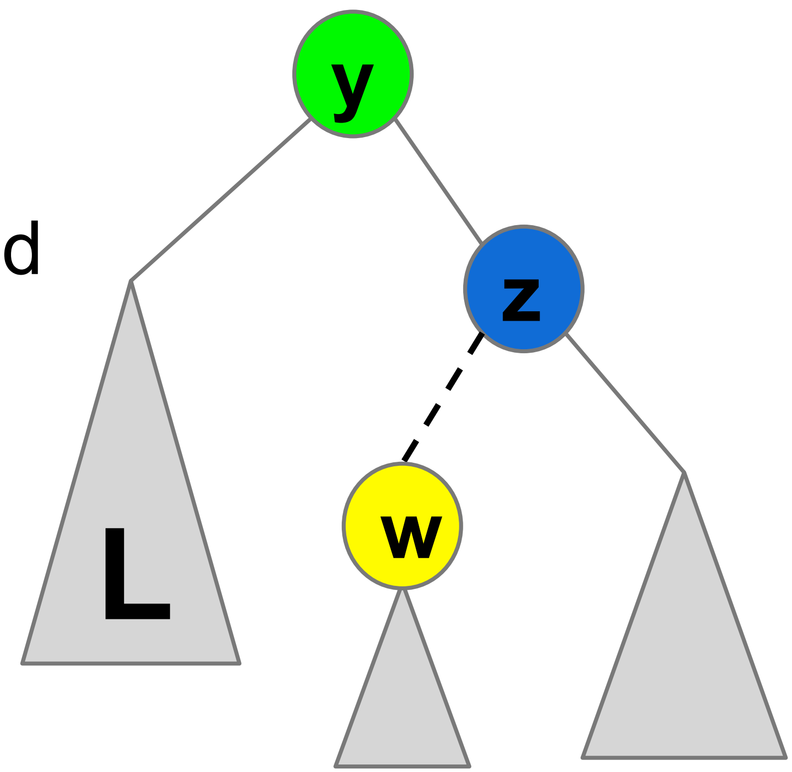


x deleted, BST order maintained, all is good.

Summarize TreeDelete(root, x)

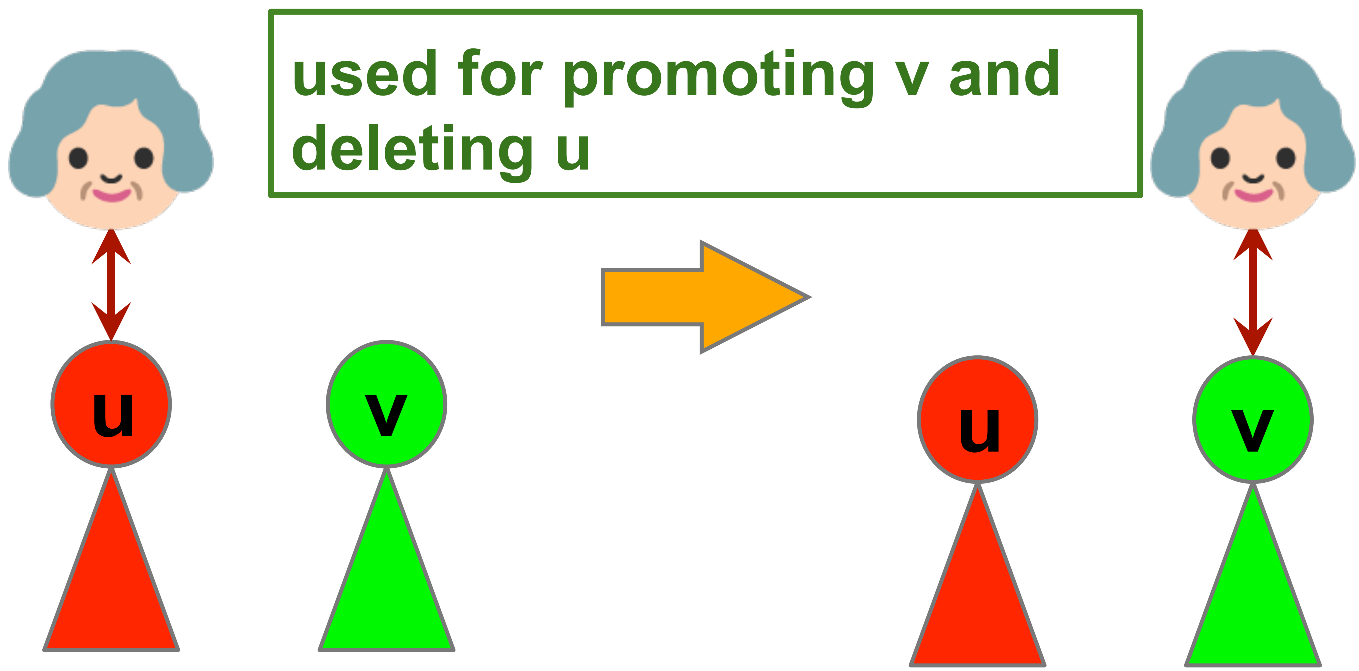
- ➔Case 1: x has no child, **just delete**
- ➔Case 2: x has one child, **promote**
- ➔Case 3: x has two children, y = successor(x)
  - ◆Case 3.1: y is x’s right child, **promote**
  - ◆Case 3.2: y is NOT x’s right child
    - **promote** y’s right child
    - y **adopt** x’s right child
    - **promote** y

TreeDelete(root, x): pseudo-code

Textbook Chapter 12.3

Key: Understand **Transplant(root, u, v)**

 # v takes away u's parent



```
Transplant(root, u, v):
# v takes away u's parent
  if u.p = NIL:  # if u is root
     root ← v    # v replaces u as root
  elif u = u.p.left:# if u is mom's left child
     u.p.left ← v  #mom accepts v as left child
  else:  # if u is mom's right child
     u.p.right ← v  #mom accept v as right child
  if v ≠ NIL:
     v.p ← u.p  # v accepts new mom
                # u can cry now...
```

```
TreeDelete(root, x):
    if x.left = NIL:
        Transplant(root, x, x.right)
    elif x.right = NIL:
        Transplant(root, x, x.left)
    else:
        y ← TreeMinimum(x.right)
        if y.p ≠ x:
            Transplant(root, y, y.right)
            y.right ← x.right
            y.right.p ← y
        Transplant(root, x, y)
        y.left ← x.left
        y.left.p ← y
    return root
```

Case 1 & 2

Case 3

Case 3.2

Promote right child

Promote left child

get successor(x)

y is not right child of x

promote **w**

y adopts **z**

promote y

update pointers

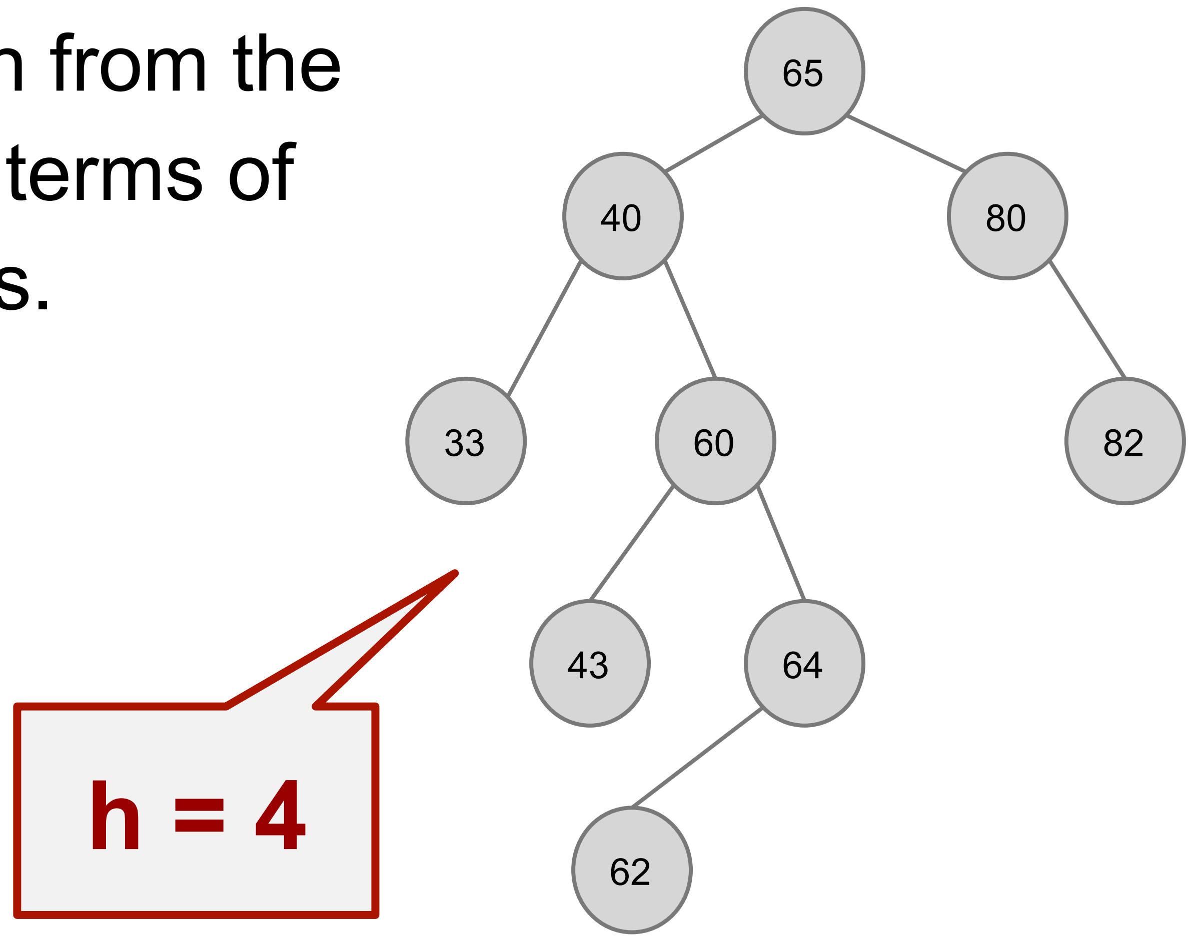
TreeDelete(root, x) worst case running time

O(h) (time spent on TreeMinimum)

Now, about that h
(height of tree)

Definition: height of a tree

The longest path from the root to a leaf, in terms of number of edges.

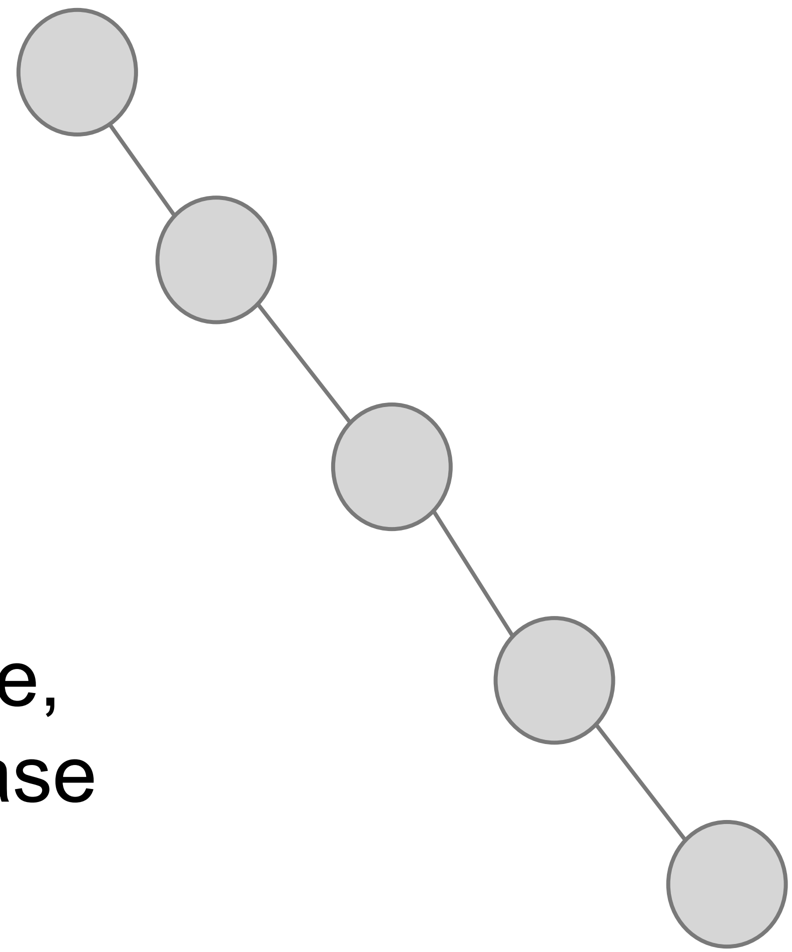


Consider a BST with n nodes, what's the highest it can be?

h = n-1

i.e, in worst case
h $\in$ **Θ(n)**

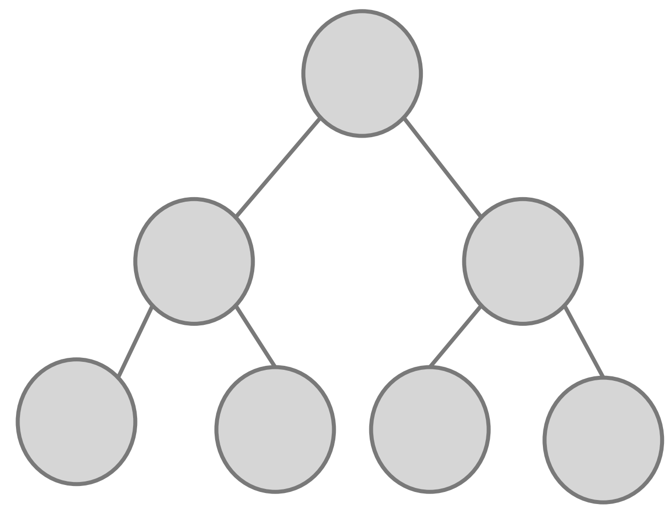
so all the operations we learned with **O(h)** runtime, they are **O(n)** in worst case



So, what's the best case for h ?

In best case, $h \in \Theta(\log n)$

A **Balanced BST** guarantees to have height in $\Theta(\log n)$



Therefore, all the **O(h)** become **O(log n)**

Next week

A Balanced BST called an **AVL tree**