1. [60 pts] Suppose we add a new concept-forming operator to the DL language defined in Section 9.2. \([\text{SUBIMAGE } r \ s]\) denotes the set of individuals whose image under \(r\) is a subset of its image under \(s\), where the image of an individual \(x\) under a relation \(R\) is the set of individuals related to \(x\) by \(R\), i.e., \(\{y|R(x, y)\}\). We also add a new role-forming operator \([\text{COMPOSE } r_1 \ldots r_n]\) which denotes the composition \(r_1 \circ \ldots \circ r_n\) of the \(r_i\)'s (you may assume that \(n > 1\)).

(a) Present a formal semantics in the style of Section 9.3.1 for the new operators.

(b) Use this semantics to show which subsumption relations exist among the following pairs of concepts (i.e., show whether \(d \subseteq e\) and whether \(e \subseteq d\)):

i. \(d = [\text{AND } [\text{ALL } r [\text{SUBIMAGE } s [\text{COMPOSE } t u]]] [\text{SUBIMAGE } v [\text{COMPOSE } r s]]]\) and \(e = [\text{SUBIMAGE } v [\text{COMPOSE } r t u]]\).

ii. \(d = [\text{SUBIMAGE } [\text{COMPOSE } r s] [\text{COMPOSE } t u]]\) and \(e = [\text{SUBIMAGE } r [\text{COMPOSE } s t u]]\).

2. [30 pts] Consider the following example.

Jason (who is fifteen years old) wants to go to a concert with his friends Patrick and Martha. Jason’s mother will give him permission to go to a concert if she thinks that at least one of the people accompanying him is responsible. She thinks that generally people over eighteen years old are responsible. Also, she remembers that either Patrick or Martha is over eighteen, but she is not sure which one.

One way to formalize this example would be with a default theory \(\Delta = (\mathcal{F}, \mathcal{D})\), where \(\mathcal{F}\) contains the following facts:

\[
\text{Accompanies}(p) \\
\text{Accompanies}(m) \\
(\exists x. \text{Accompanies}(x) \land \text{Responsible}(x)) \supseteq \text{Permission} \\
\text{Over18}(m) \equiv \neg \text{Over18}(p),
\]

and \(\mathcal{D}\) contains a single default rule:

\[
\frac{\text{Over18}(x) \land \text{Responsible}(x)}{\text{Responsible}(x)}.
\]

However, another way to formalize this example would be with \(\Delta' = (\mathcal{F}', \mathcal{D}')\), where \(\mathcal{F}'\) contains:

\[
\text{Accompanies}(p) \\
\text{Accompanies}(m),
\]

and \(\mathcal{D}'\) contains the following rules:
Accompanies(x) ∧ Responsible(x) :

−Over18(m) → Over18(p) 
−Over18(p) → Over18(m)

Over18(x) : Responsible(x) → Responsible(x)

(a) Determine whether ∆ and ∆′ have the same extensions.
(b) Suppose that Jason’s mother learns that Patrick is not responsible. If we add ¬Responsible(p) to the facts of ∆ and ∆′, how does this affect the extensions?

3. [50 pts] Consider the following statements:

Canadians are typically Anglophones.
James is a Canadian.
Jean-Luc is not an Anglophone.
Kathryn is a Canadian.
Benjamin is a Canadian or Christopher is a Canadian.

For the following questions, you may assume that all names denote distinct individuals.

(a) Represent the above statements in default logic. What conclusions can be drawn?
(b) Now, suppose we represent the default with the following rule in default logic:

⟨ TRUE ⇒ (Canadian(x) ⊃ Anglophone(x)) ⟩.

What conclusions can be drawn in this case?
(c) Represent the above statements in first-order logic with an abnormality predicate. What conclusions can be drawn using minimal entailment?
(d) Using autoepistemic logic, we could represent the default in two ways:

i. (strong default) ∀x(Canadian(x) ∧ ¬B¬Anglophone(x) ⊃ Anglophone(x))
ii. (weak default) ∀x(BCanadian(x) ∧ ¬B¬Anglophone(x) ⊃ Anglophone(x))

Using propositional versions of these defaults and only the facts in the statements above, show that the strong and weak defaults lead to different conclusions. Does either correspond to the solutions you obtained using default logic or minimal entailment?

4. [60 pts] Consider the following example:

Headaches and blurred vision may be the result of sitting too close to a monitor. Headaches may also be caused by bad posture. Headaches and blurred vision may cause nausea. Headaches may also lead to blurred vision.

(a) Represent the causal links in a belief network. Let H stand for “headaches”, B for “blurred vision”, S for “sitting too close to a monitor”, P for “bad posture”, and N for “nausea”.
(b) Write the formula for the full joint probability distribution over all 5 variables.

(c) Give an example of an independence assumption that is implicit in this network.

(d) Suppose the following probabilities are given:

\[
\begin{align*}
& P(H | S, P) = 0.8 & & P(H | \overline{S}, P) = 0.4 \\
& P(H | S, \overline{P}) = 0.6 & & P(H | \overline{S}, \overline{P}) = 0.02 \\
& P(B | S, H) = 0.4 & & P(B | \overline{S}, H) = 0.3 \\
& P(B | S, \overline{H}) = 0.2 & & P(B | \overline{S}, \overline{H}) = 0.01 \\
& P(S) = 0.1 \\
& P(P) = 0.2 \\
& P(N | H, B) = 0.9 & & P(N | \overline{H}, B) = 0.3 \\
& P(N | H, \overline{B}) = 0.5 & & P(N | \overline{H}, \overline{B}) = 0.7
\end{align*}
\]

Furthermore, assume that some patient is suffering from headaches but not from nausea. Calculate joint probabilities for the 8 remaining possibilities (that is, according to whether \(S, B, P\) are true or false).

(e) According to the numbers above, the \textit{a priori} probability that the patient suffers from bad posture is 0.2. Given that the patient is suffering from headaches but not from nausea, are we now more or less inclined to believe the hypothesis? Explain.