
2. [40 pts] Chapter 3, Exercise 4 in the textbook.

3. [40 pts] Chapter 4, Exercise 3 in the textbook.

4. [40 pts] In the lectures, we considered some Resolution strategies. We consider two additional strategies here.

   - A unit resolution step is one in which a resolvent is obtained using at least one parent which is a unit clause (i.e., a single literal) or a unit factor\(^1\) of a parent clause.
   - An input resolution step is one in which at least one of the two parent clauses comes from the original set of clauses (i.e., the KB and the negated query).
   - A unit (respectively, input) derivation is one in which every Resolution step is a unit (respectively, input) resolution step.
   - A unit (respectively, input) refutation is a unit (respectively, input) derivation of the empty clause.

(a) While completeness is an important consideration for automated deduction, so is efficiency. At times we may be willing to forego completeness for the sake of efficiency. Show that unit resolution is incomplete. [Hint: exhibit a set of clauses and explain your answer.]

(b) Given a finite set \(S\) of ground clauses, show that there is a unit refutation from \(S\) if and only if there is an input refutation from \(S\). [Hint: use mathematical induction on the number of elements in the Herbrand base of \(S\).]

(c) Given any finite set \(S\) of clauses, use the result you just proved in (4b) to show that there is a unit refutation from \(S\) if and only if there is an input refutation from \(S\). [Hint: You may find the following “lifting lemma” useful: If \(C'_1\) and \(C'_2\) are instances of \(C_1\) and \(C_2\), respectively, and if \(C'\) is a resolvent of \(C'_1\) and \(C'_2\), then there is a resolvent \(C\) of \(C_1\) and \(C_2\) such that \(C'\) is an instance of \(C\).]

(d) Consider combining Resolution strategies to create new strategies. If we combine unit resolution and input resolution into a single strategy that says we can only obtain a resolvent using at least one parent which is a unit input clause or a factor of a unit input clause, does this make it impossible to prove some things that are provable by unit resolution alone? If not, prove that there is no difference.

---

\(^1\)If two (or more) literals of a clause \(C\) have a most general unifier \(\sigma\), then \(C\sigma\) is known as a factor of \(C\). For example, 
\(C = [Q(x), Q(g(y)), \neg R(x)]\) has a most general unifier \(\sigma = \{x/g(y)\}\) giving the factor \([Q(g(y)), \neg R(g(y))]\).