Relations - review

- A *binary relation* on A is a subset of $A \times A$ (set of *ordered* pairs of elements from A)

- Example:

  $A = \{a,b,c,d,e\}$

  $R = \{(a,a),(a,b),(b,b),(b,c),(c,e),(d,a),(d,c),(e,b)\}$

	a	*b*	*c*	*d*	*e*
a	1	1	0	0	0
b	0	1	1	0	0
c	0	0	0	0	1
d	1	0	1	0	0
e	0	1	0	0	0

- A *binary relation* between A and B is a subset of $A \times B$ (a set of pairs (a,b) where $a \in A$ and $b \in B$)

Types of relations

- **reflexive** $(a,a)\in R$ for all $a\in A$
- **symmetric** if $(a,b)\in R$, then $(b,a)\in R$
- **transitive** if $(a,b)\in R$ and $(b,c)\in R$, then $(a,c)\in R$

	a	b	c	d	
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b	0	1	1	0	0
c	0	0	0	0	1
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reflexive ✗

symmetric ??

transitive ??

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reflexive

symmetric

transitive

Question #1

- Number of _____ relations on a set of *n* elements ?

symmetric
reflexive
symmetric and **reflexive**
symmetric and not **reflexive**
irreflexive
asymmetric
antisymmetric

	a	*b*	*c*	*d*	*e*
a					
b					
c			?		
d					
e					

- **irreflexive** $(a,a) \notin R$ for all $a \in A$
- **asymmetric** if $(a,b) \in R$, then $(b,a) \notin R$
- **antisymmetric** if $(a,b) \in R$ and $(\mathrm{b},\mathrm{a}) \in R$, then $a=b$

Question #2

- **equivalence relation = reflexive symmetric transitive**

“equivalence” of objects e.g., “X has the same age as Y”

- **partial order = reflexive antisymmetric transitive**

“order” of objects e.g., “X is a subset of Y”

- **strict partial order = ireflexive asymmetric transitive**

“strict order” of objects e.g., “X is older than Y”

- **total (linear) order = p.o.** + every pair comparable:

$(a,b)\in R$ or $(b,a)\in R$ for all $a,b\in A$

e.g., “X is a subset of Y” is a partial but not a total order

Question #2

- equivalence relation = reflexive symmetric transitive
- partial order = reflexive antisymmetric transitive
- strict partial order = ireflexive asymmetric transitive
- total (linear) order = p.o. + every pair comparable:
  $(a,b) \in R$ or $(b,a) \in R$ for all $a,b \in A$

- If R and S are ______ relations on the same set A:
  - is $R \cap S$ also a _______ relation ?
  - is $R \cup S$ also a _______ relation ?
  - is P $\subseteq$ R also a _______ relation ?
  - is P $\supseteq$ R also a _______ relation ?

reflexive antisymmetric equivalence relation partial order
symmetric ireflexive strict partial order total order

Question #3

- **prove** *Multinomial Theorem*
  - by *induction* on *k* — by *induction* on *n*
  - using *Binomial Theorem*
- **simplify**

(1)
$$\sum_{m=0}^{n} \binom{m}{m-k}$$

(2)
$$\sum_{k=0}^{n} \binom{n-k}{m-k}$$

(3)
$$\sum_{k=0}^{n} k \binom{m-k-1}{m-n-1}$$