Definition 1

The *Cartesian product* (or *cross product*) of A and B, denoted by $A \times B$, is the set

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

1. the elements (a, b) of $A \times B$ are *ordered pairs*
2. for pairs $(a, b), (c, d)$ we have

$$(a, b) = (c, d) \iff a = c \text{ and } b = d$$

Definition 2

The *n-fold product* of sets $A_1, A_2, \ldots, A_n$ is the set of *n-tuples*

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for all } 1 \leq i \leq n\}$$

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$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

$A = \{2, 3, 4\}$
$B = \{4, 5\}$

a) $A \times B = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\}$

b) $B \times A = \{(4, 2), (4, 3), (4, 4), (5, 2), (5, 3), (5, 4)\}$

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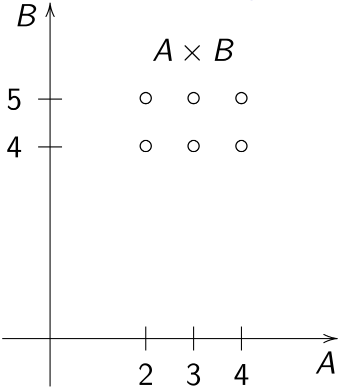

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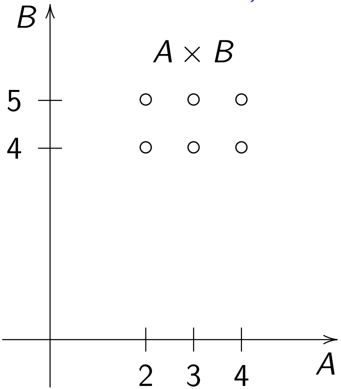

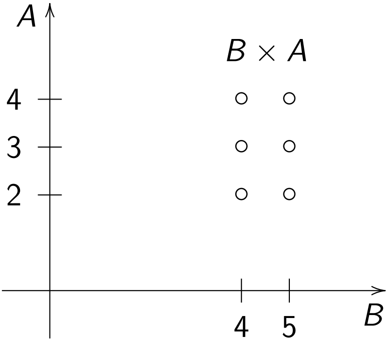

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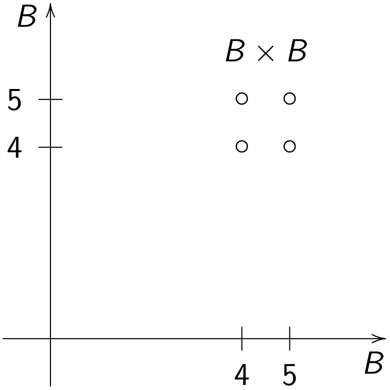

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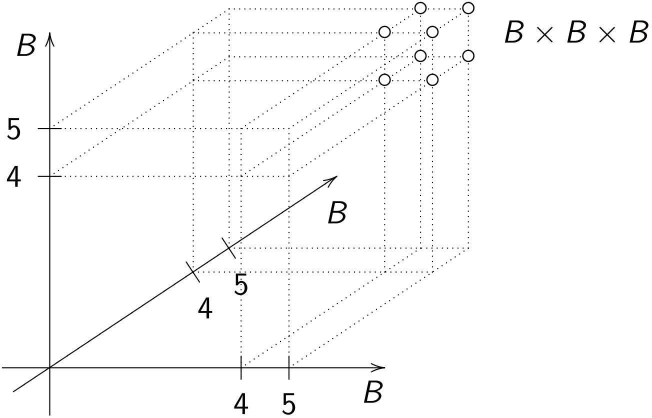
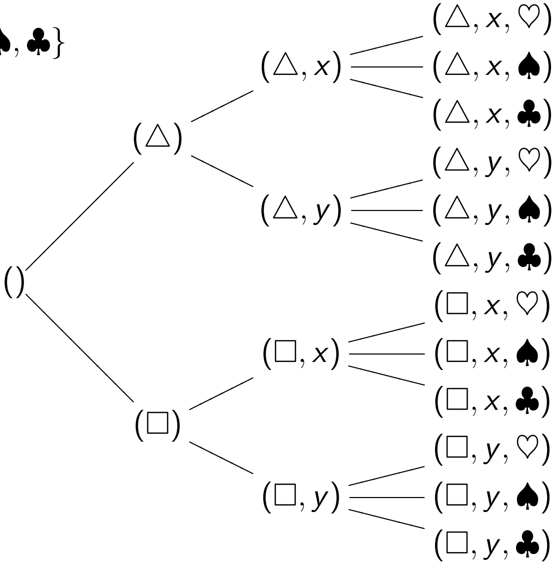


$$\{\triangle, \square\} \times \{x, y\} \times \{\heartsuit, \spadesuit, \clubsuit\}$$

Tree Diagram



Definition 3

A *(binary) relation* from A to B is a subset of $A \times B$.
A *(binary) relation* on A is a subset of $A \times A$.

$A = \{2, 3, 4\}$ and $B = \{4, 5\}$

a) $R_1 = \{(2, 4), (3, 5)\}$
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c) $R_3 = \{(2, 4), (2, 5), (4, 4), (4, 5)\}$
d) $R_4 = \emptyset$

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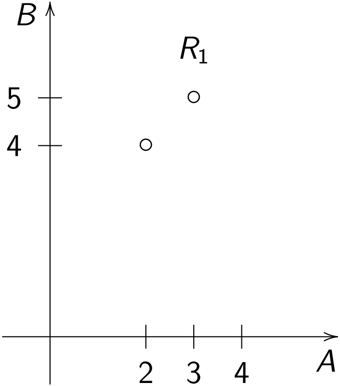

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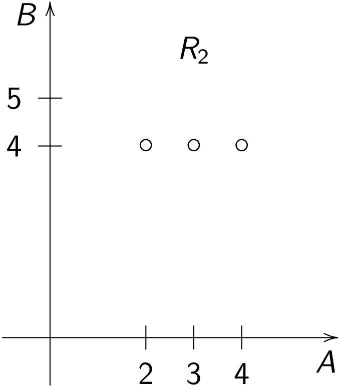

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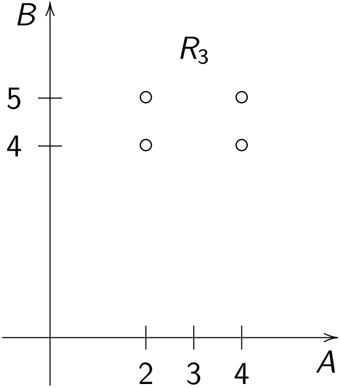

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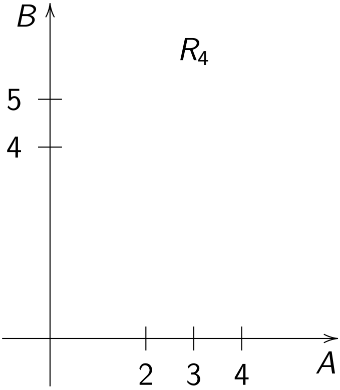

Relation $\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \leq x \leq y \leq 4\} =$
$= \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

Notation

$$(x, y) \in \mathcal{R}$$
$$\Updownarrow$$
$$x\mathcal{R}y$$

(think of $\mathcal{R}$ as $\leq$)

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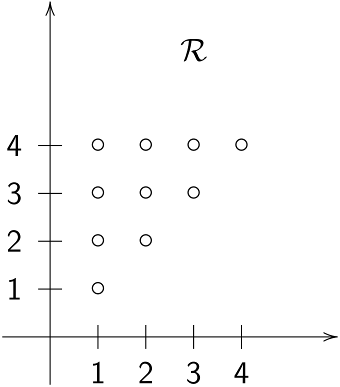

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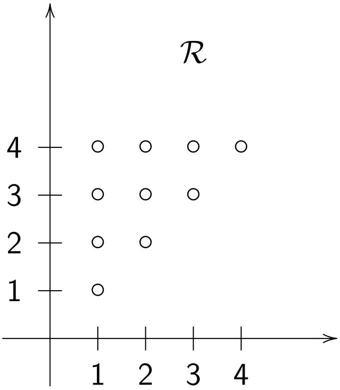

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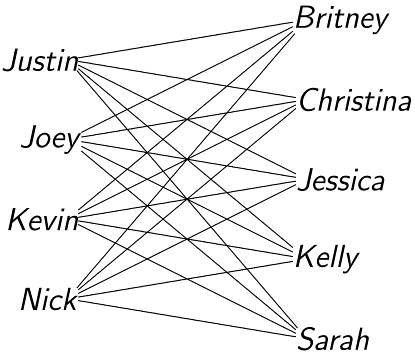
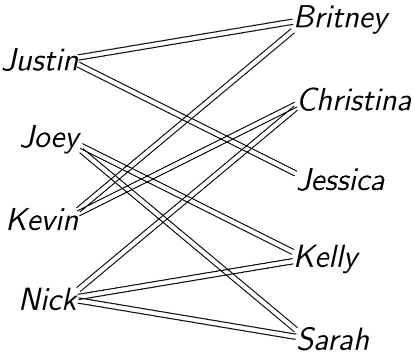
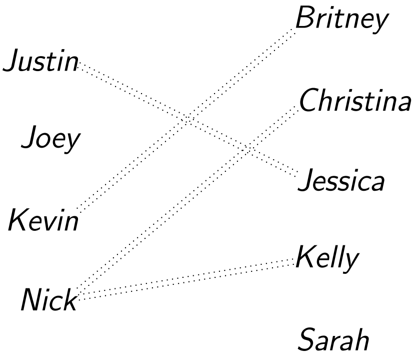
$\{Justin, Joey, Kevin, Nick\} \times \{Britney, Christina, Jessica, Kelly, Sarah\}$



Who dated whom? $\{(Ju, Br), (Ju, Je), (Jo, Ke), (Jo, Sa), (Ke, Br), (Ke, Ch), (Ni, Ch), (Ni, Ke), (Ni, Sa)\}$



Who is dating whom? $\{(Ju, Je), (Ke, Br), (Ni, Ch), (Ni, Ke)\}$



Theorem 4

For any set A, we have $A \times \emptyset = \emptyset$ (and $\emptyset \times A = \emptyset$)

Proof. If $(a, b) \in A \times \emptyset$, then $a \in A$ and $b \in \emptyset$, impossible. □

Theorem 5

For any sets A, B, C

a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Proof. a) $(a, b) \in A \times (B \cap C) \iff a \in A$ and $b \in B \cap C \iff a \in A$ and $b \in B$ and $b \in C \iff (a, b) \in A \times B$ and $(a, b) \in A \times C \iff (a, b) \in (A \times B) \cap (A \times C)$ □

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Observation 6

For any two sets A, B, the number of elements in $A \times B$ is

$$|A \times B| = |A| \cdot |B|$$

Hence there are exactly $|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{|A| \cdot |B|}$ different relations from A to B.

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For any two sets A, B*, the number of elements in* $A \times B$ *is*

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Exercises:

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$

5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| =?$ Answer: 9

b) # of relations from A to B ? Answer: $2^9 = 512$

c) # of relations on A ? Answer: $2^9 = 512$

d) # of relations from A to B that contain $(1, 2)$ and $(1, 5)$? Answer: $2^7 = 128$

e) # of relations from A to B that contain exactly five ordered pairs ? Answer: $\binom{9}{5} = 126$

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5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$

5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| = ?$ Answer: 9

b) # of relations from A to B ? Answer: $2^9 = 512$

c) # of relations on A ? Answer: $2^9 = 512$

d) # of relations from A to B that contain $(1, 2)$ and $(1, 5)$? Answer: $2^7 = 128$

e) # of relations from A to B that contain exactly five ordered pairs ? Answer: $\binom{9}{5} = 126$

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