- Cartesian (cross) product $A \times B = \{(a, b) \mid a \in A, b \in B\}$
- Relation R from A to B, $R \subseteq A \times B$
- Function $f : A \rightarrow B$, a relation $f \subseteq A \times B$ with $\forall a \; \exists! b \; (a, b) \in f$
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$$f(C) = \{b \in B \mid b = f(a), a \in C\} = \{f(a) \mid a \in C\}$$

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Let $f : A \to B$ *and* $A_1, A_2 \subseteq A$, *then*

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c) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ *if* f *is one-to-one*

Proof.

a) $b \in f(A_1 \cup A_2) \iff b = f(a)$ for $a \in A_1 \cup A_2 \iff b = f(a)$ for $a \in A_1$ or $a \in A_2 \iff b \in f(A_1)$ or $b \in f(A_2) \iff b \in f(A_1) \cup f(A_2)$

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Definition 3

For $f : A \to B$ and $C \subseteq A$, let $f|_C : C \to B$ be the function defined as $f|_C(a) = f(a)$ for $a \in C$. We call $f|_C$ the *restriction* of f to C, and call f an *extention* of $f|_C$ to A.

Let $A = \{1, 2, 3, 4\}$ and (note: $A \subsetneq \mathbb{Z} \subsetneq \mathbb{R}$)
let $f : A \to \mathbb{R}$ be defined as $f = \{(1, -4), (2, -1), (3, 2), (4, 5)\}$,
let $g : \mathbb{Z} \to \mathbb{R}$ be defined as $g(x) = 3x - 7$ for $x \in \mathbb{Z}$ and
let $h : \mathbb{R} \to \mathbb{R}$ be defined as $h(x) = 3x - 7$ for $x \in \mathbb{R}$.

a) g is an extention of f (from A) to $\mathbb{Z}$
b) f is the restriction of g (from $\mathbb{Z}$) to A
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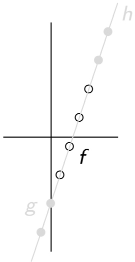

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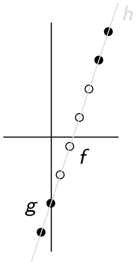

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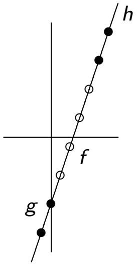

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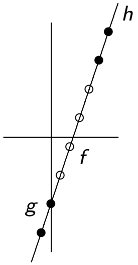

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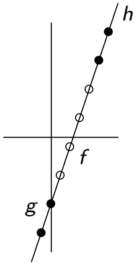

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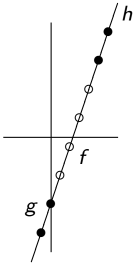

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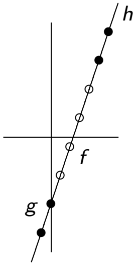

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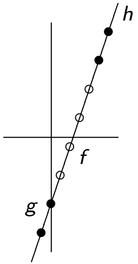

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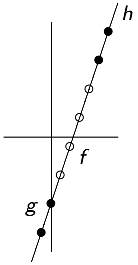

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Theorem 4

For finite sets A, B with $|A| = n$ and $|B| = m$, the number of onto functions from A to B is

$$s(n,m) = \sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n$$

Proof. By induction on n. For $n = 1$ easy (note $0^0 = 1$), let $n > 1$

$s(n,m) = m \cdot (s(n-1,m) + s(n-1,m-1)) =$

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Exercises.

5.2.3 Let $A = \{1, 2, 3, 4\}$ and let $B = \{x, y, z\}$.

a) # of functions from A to B?

b) # of one-to-one functions from A to B

c) # of onto functions from A to B

d) # of functions from B to A?

e) # of one-to-one functions from B to A

f) # of onto functions from B to A

g) # of functions from A to B with $f(1) = x$

h) # of functions from A to B with $f(1) = f(2) = x$

i) # of functions from A to B with $f(1) = x$ and $f(2) = y$

Exercises.

5.2.3 Let $A = \{1, 2, 3, 4\}$ and let $B = \{x, y, z\}$.

a) # of functions from A to B? Answer: $3^4 = 81$

b) # of one-to-one functions from A to B

c) # of onto functions from A to B

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i) # of functions from A to B with $f(1) = x$ and $f(2) = y$

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d) # of functions from B to A? Answer: $4^3 = 64$

e) # of one-to-one functions from B to A Answer: $P(4, 3) = 24$

f) # of onto functions from B to A

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Answer: $4^3 - 4 \cdot 3^3 + 6 \cdot 2^3 - 4 = 0$

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Answer: $4^3 - 4 \cdot 3^3 + 6 \cdot 2^3 - 4 = 0$

g) # of functions from A to B with $f(1) = x$ Answer: $3^3 = 27$

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g) # of functions from A to B with $f(1) = x$ Answer: $3^3 = 27$

h) # of functions from A to B with $f(1) = f(2) = x$

Answer: $3^2 = 9$

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Theorem 5

The number of ways of distributing n distinct objects into m containers where no container is empty is $S(n,m) = \frac{1}{m!} \cdot s(n,m)$

Proof. Each distribution defines a surjective mapping from $\{1,\dots,n\}$ to $\{1,\dots m\}$, and there are $m!$ different ways of numbering the containers. □

Example. 5.3.8 Suppose we have seven differently coloured balls and four containers I, II, III, IV:

a) # ways to distribute balls

b) # ways to distribute balls so that no container is empty

c) # ways to distribute balls so that no container is empty and the blue ball is in II

d) a) and b) if we remove labels from the containers

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Example. 5.3.8 Suppose we have seven differently coloured balls and four containers I, II, III, IV:

a) # ways to distribute balls Answer: $4^7 = 16384$

b) # ways to distribute balls so that no container is empty

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a) # ways to distribute balls — Answer: $4^7 = 16384$

b) # ways to distribute balls so that no container is empty
Answer: $s(7, 4) = 4! \cdot 350 = 8400$

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d) a) and b) if we remove labels from the containers
Answer: a) $S(7,1) + S(7,2) + S(7,3) + S(7,4) = 715$; b) 350

Exercises

5.3.13 How many factorisations of 156,009 into

a) two factors greater than one

b) two or more factors greater than one

c) *a*) and *b*) for any $n = p_1 \cdot \ldots \cdot p_k$ where all p_i are different

5.3.16 Ten candidates $C_1, \ldots, C_{10}$ for senior class president

a) How many possible outcomes of the election (no ties)?

b) How many possible outcomes with possible ties?

c) *b*) where exactly three candidates tie for first place?

d) *a*), *b*) and *c*) where C_3 in first place?

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5.3.13 How many factorisations of $156{,}009 = 3 \cdot 7 \cdot 17 \cdot 19 \cdot 23$ into

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Answer: $S(5,2) + S(5,3) + S(5,4) + S(5,5) = 51$

c) a) and b) for any $n = p_1 \cdot \ldots \cdot p_k$ where all p_i are different

a) $S(k,2)$ b) $S(k,2) + S(k,3) + \ldots + S(k,k) = \sum_{i=2}^{k} S(k,i)$

5.3.16 Ten candidates $C_1, \ldots, C_{10}$ for senior class president

a) How many possible outcomes of the election (no ties)? 10!

b) How many possible outcomes with possible ties?

c) b) where exactly three candidates tie for first place?

d) a), b) and c) where C_3 in first place?

Exercises

5.3.13 How many factorisations of $156{,}009 = 3 \cdot 7 \cdot 17 \cdot 19 \cdot 23$ into

a) two factors greater than one — Answer: $S(5,2) = 15$

b) two or more factors greater than one

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Answer: $s(10,1) + s(10,2) + \ldots + s(10,10) = 102,247,563$

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Answer: $\binom{10}{3} \sum_{i=1}^{7} s(7,i) = 120 \cdot 47,293 = 5,675,160$

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d) *a*), *b*) and *c*) where C_3 in first place?

Answer: *a*) $9!$ *b*) $\sum_{i=1}^{9} s(9,i)$ *c*) $\binom{9}{2} \sum_{i=1}^{7} s(7,i)$