CS137 Discrete Mathematics and its Applications 2

Coursework 3

Due by Monday 25 February 2013 at 12noon
Submit with an appropriate coversheet to a collection box in CS0.06

Attempt to solve ALL SEVEN of the following problems.
Submit a solution to THREE of the seven problems, ONE from EACH GROUP.

Group 1

1. (a) Draw all pairwise non-isomorphic connected graphs with 4 edges.
   (b) A cycle is Hamiltonian if it contains all vertices of the graph. Find an Euler circuit
       and a Hamiltonian cycle in the following graphs. If it does not exist, explain why.

2. A walk with endpoints $u, v$ is a $uv$-walk. A path with endpoints $u, v$ is a $uv$-path.
   Let $G$ be a graph. Prove that
   (a) $G$ contains a $uv$-walk if and only if $G$ contains a $uv$-path.
   (b) $G$ contains a closed walk of odd length if and only if $G$ contains a cycle of odd length.

Group 2

1. Reconstruct the trees from their Prüfer codes: $(3, 3, 5, 5, 6, 6), (1, 5, 1, 5, 9, 8, 2), (1, 5, 2, 2, 1, 5, 5)$
   Let $i$ be a positive integer. What tree has the Prüfer code
   (a) $(i, i, \ldots, i)$ ?
   (b) $(i - 2, i - 3, \ldots, 1)$ ?

2. Let $T$ be a tree with $n$ vertices, $k$ leaves, and no vertex of degree 2.
   (a) Prove that $k \geq (n + 2)/2$.
   (b) What does $T$ look like if $k = (n + 2)/2$?
3. Let $G$ be a graph with $n$ vertices, $m$ edges, and $k$ connected components. Prove that

$$n - k \leq m \leq \binom{n - k + 1}{2}$$

**Group 3**

1. Let $G$ be a graph $m$ edges and chromatic number $\chi(G) = k$. Prove that $m \geq \binom{k}{2}$.

2. Let $G$ be a graph with $n \geq 11$ vertices. Prove that $G$ and $\overline{G}$ cannot be both planar.
   (Hint: use Euler’s formula)