CHAPTER 5

Dangerous Exponentials

In this book we will explore a few key foundational concepts that will help you gain a better understanding of what lies ahead. None of them is more important than exponential growth. **Exponential growth** holds the honorary position as the "fourth E" in this story. Understanding the ways in which our lives are surrounded and shaped by exponential growth is a necessary part of our ability to effectively anticipate and proactively prepare for our future.

When the president comes on television and says that our highest priority is "returning the economy to a path of growth," what he is really saying is that our top priority is returning the economy to a path of *exponential* growth. Exponential growth is the only type of growth that is expected and required by our economy.

Examples of exponential growth in your life extend well beyond the economy. We are literally surrounded by examples of exponential growth. The human population has been growing exponentially for thousands of years; consequently, so has humans' use of resources. This decade there will be exponentially more retail outlets, reams of paper produced, cars on the road, units of energy burned, money created, and food consumed than last decade.

Exponential growth dominates and defines everything that is happening —and that will happen—regarding the economy, energy, and resources of all kinds, which is why you should pay particular attention to this chapter. As soon as you understand exponential growth and can connect it to the other three Es, then you, too, will appreciate why the future will be radically different from the past.

If exponential growth is so ubiquitous and surrounds us at every turn, why is it not completely obvious to everyone? Why do we need to discuss it at all? The reason is that we're all accustomed to thinking linearly, and exponential growth is nonlinear. We think in straight lines, but exponentials are curved. Here is an example: Suppose I gave you two chalkboard erasers, and asked you to hold them at arm's length and then move them together at a constant (linear) rate of speed. You would do pretty well at this task, as would most people.

Now let's repeat the same experiment, but this time we'll replace the erasers with two powerful magnets. As you move them together, the first part of the journey will progress in a nice, constant fashion, just like with the erasers. But at a certain point—BANG!—the magnets will suddenly draw themselves together and wreck your deliberately even speed. (Let's hope your fingers were out of the way.)

We could run this experiment a hundred times, and you would never be able to get your body to achieve the same linear control with the magnets as with the erasers. That is because our brains and bodies are wired to process linear forces, and magnets do not exert constant (or linear) force over distance because their force of attraction increases exponentially as they get closer.

Despite our natural affinity for straight lines and constant forces, we *can* still achieve a useful understanding of exponential growth and why it is important. That is what we're going to do in this chapter.

Exponential growth is not unnatural, but the idea of *perpetual* exponential growth is. We have no models of perpetual exponential growth in the physical world to which we can turn for observation and study. For example, microorganisms in a culture will increase exponentially, but only until an essential nutrient is exhausted, and at that point, the population crashes. Viruses will reproduce and then spread exponentially throughout a population, but they will eventually burn out as their hosts either develop immunity or die off. Nuclear chain reactions caused by neutrons cascading through fissile material are exponential, at least until the resulting explosion forces the material too far apart for the reaction to be sustained.

One thing that we lack here on earth, however, is an example of something growing exponentially *forever*. Exponential growth is always self-limiting and is usually relatively short in duration. Nothing can grow forever, yet somehow that's exactly what we expect *and* require of our economy. But we will explore more about why that's the case in a bit.

The Concept of Exponential Growth

What do we mean when we say that something is "growing exponentially"?

To begin with, let's define "growth." When we say that something is growing, we're saying that it's getting larger. Children grow by eating and adding mass, equities grow in price, and the economy grows by producing and consuming more goods. Ponds get deeper, trees grow taller, and profits expand. Within these examples of growth, we can identify two types.

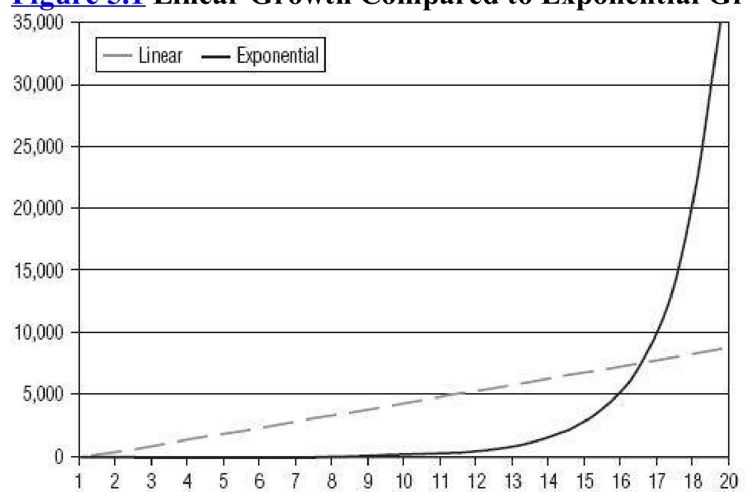
The first type is what we would call "linear growth." *Linear* means adding (or subtracting) the same amount each time. The sequence 1, 2, 3, 4, 5, 6, 7 is an example of linear (or arithmetic) growth in which the same number is reliably added to the series at every step. If we add one each time, or five, or forty-two, or even a million, it won't change the fact that this kind of growth is linear. If the amount being added is constant, then it represents linear growth.

The other type of growth is known as "geometric" or exponential growth, and it is notable for constantly *increasing* the amount of whatever is being added each time to the series. One example is the sequence 1, 2, 4, 8, 16, 32, 64, in which the last number in the series is multiplied by two (or increased by 100 percent) at every step. The amount that gets added in each period is both dependent upon and a little bit larger than the prior amount. In the sequence example given, we see a case where the growth rate is 100 percent. So 2 becomes 4, and 4 becomes 8, and so on. But it doesn't have to grow by 100 percent to be exponential; it could be any other constant percentage and it would still fit the definition.

Now let's take a closer look at exponential growth so that we can all be clear about what it is and how it relates to our collective future. The chart below illustrates exponential growth—a chart pattern that is often called a "hockey stick."

In Figure 5.1, we're graphing an amount of something over time. It could be the number of yeast grown in a flask of freshly squeezed grape juice every 10 minutes, or it could be the number of McDonald's hamburgers sold each year. It doesn't really matter what it is or what's driving the growth; all that is required to create a line on a graph that looks like the curve seen in Figure 5.1 is that whatever is being measured must grow by some percentage over each increment of time. That's it. Any percentage will do: 50 percent, 25 percent, 10 percent, or even 1 percent. It doesn't matter: 10 percent more yeast per hour, 5 percent more hamburgers per year, and 0.25 percent interest on your savings account will all result in a line on a chart that looks like a hockey stick.

Figure 5.1 Linear Growth Compared to Exponential Growth



Linear growth is the dotted line; exponential growth is the solid line. The units on both axes are arbitrary; amount is on the vertical (or Y) axis and time is on the horizontal (or X) axis.

Looking at the figure a bit more closely, we observe that the curved line on the chart begins on the left with a flat part, seems to turn a corner (at what we might call the elbow), and then has a steep part.

A more subtle interpretation of Figure 5.1 reveals that once an exponential function turns the corner, even though the *percentage rate* of growth might remain constant (and low!), the *amounts* do not. They pile up faster and faster. For example, imagine that a long-ago ancestor of yours put a single penny into an interest-bearing bank account for you some 2,000 years ago and it earned just 2 percent interest the whole time. The difference in your account balance between years 0 and 1 would be just two one-hundredths of a cent. Two thousand years later, your account balance would have grown to more than $1.5 quadrillion dollars (more than 20 times all the money in the world in 2010) and the difference in your account between the years 1999 and 2000 alone would have been more than $31 trillion dollars. Where the amount added was two one-hundredths of a cent at the beginning, it was roughly equivalent to half of all the money in the entire world at the end. That's a rather dramatic demonstration of how the amounts vary over time, but it gets the point across.

Now let's look at an exponential chart of something with which you are intimately familiar that has historically grown at roughly 1 percent per year. It is a chart of world population; the solid part is historical data and the dotted line is the most recent UN projection of population growth for just the next 42 years.[1]

Again I want to draw your attention to the fact that the chart has a flat part, then a corner that gets turned, and then a steep part. By now, it is quite possible that any mathematicians reading this are hopping up and down because of what they might view to be an enormous error on my part.

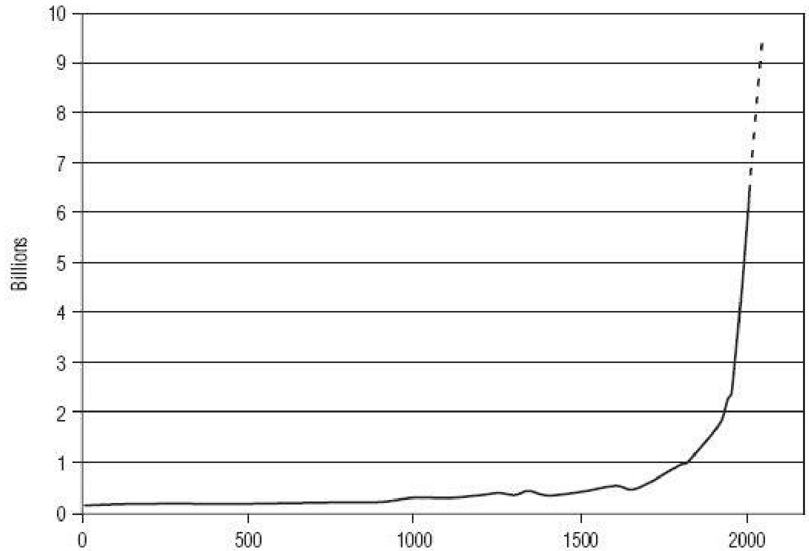
A first point of departure is that where mathematicians have been trained to define exponential growth in terms of the *rate* of change, we're going to concentrate here on the *amount* of change. Both are valid, it's just that *rates* are easier to express as a formula and *amounts* are easier for most people to intuitively grasp. So we're going to focus on amounts, even though this is not where classical mathematicians would train their logical eyes.

Unlike the *rate* of change, the *amount* of change is not constant in exponential growth; it grows larger and larger with every passing unit of time. For our purposes, it is more important that we appreciate what exponential growth demands in terms of physical amounts than whatever intellectual gems are contained within the rate of growth.

A second point of contention that I expect most mathematicians would vigorously dispute is the idea that there's a turn-the-corner stage in an exponential chart. In fact, they're right. It turns out that the point where an exponential chart appears to turn the corner is an artifact of how we draw the left-hand scale. An exponential chart is indeed turning the corner at any and every point along its trajectory. Where that point happens to *appear* on our charts is simply a function of how we scale the vertical axis.

For example, if we take our population chart above, and instead of setting the left axis at 10 billion we set it at one billion (Figure 5.3), we see that the line disappears entirely off the chart somewhere around 1850. We can't see the part after that because it is now way above the top of the chart frame, but in this version of the chart we note that the turn-the-corner event appears to happen around 1900. Instead of having this conversation about turning the corner with population growth right now, it appears as though we really should have had it back in 1900.

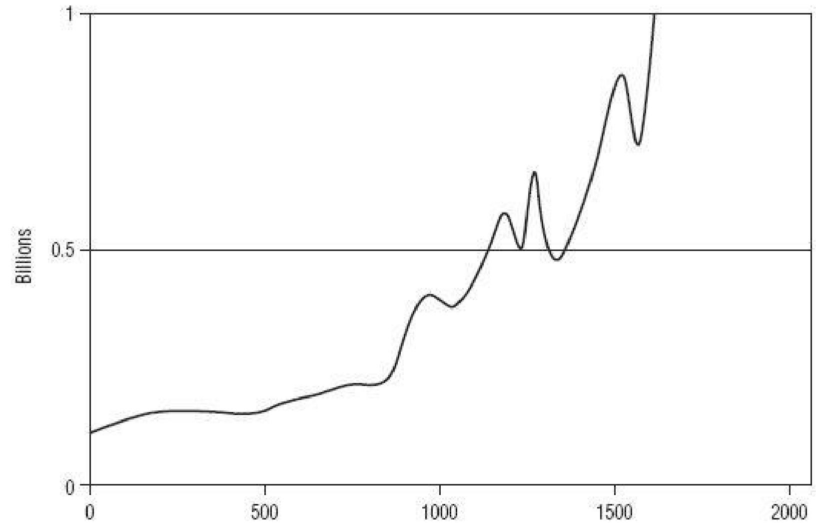
Figure 5.2 World Population



The solid line is historical; the dotted line represents the UN projection.

Source: U.S. Census Bureau Historical Estimates[2] *& U.N. 2004 Projections.*[3]

Figure 5.3 World Population

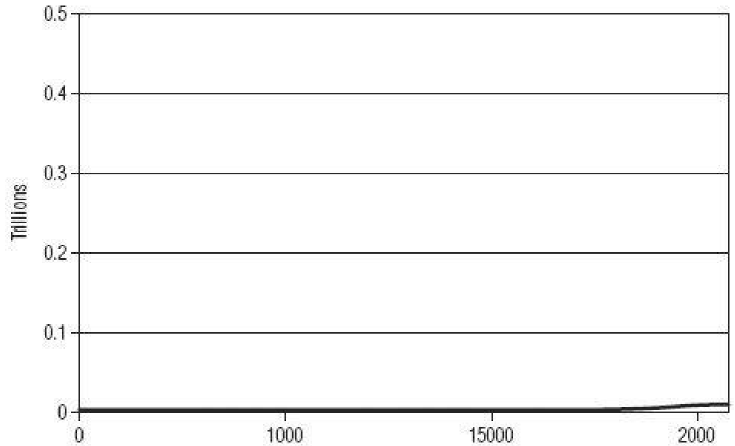


Same world population chart as Figure 5.2, but with left axis set at one billion.

Source: U.S. Census Bureau Historical Estimates.

Similarly, if we scale our left axis to, say, one trillion (Figure 5.4), the corner disappears entirely and the entire line becomes flat. We can't see its curve anymore. But it is still there; it has just been suppressed by our management of the left axis. No more population problem! Right?

Figure 5.4 World Population



Same world population chart as Figure 5.2, but expressed in trillions.

Source: U.S. Census Bureau Historical Estimates.

So the turn-the-corner moment is really just a product of how we draw our chart. Does it mean that the turn-the-corner stage is a worthless artifact and that we can forget all about it? No, far from it. It is very real and vitally important. Let me explain why.

Where the turn-the-corner stage becomes enormously meaningful and important is when you can reasonably set a boundary—that is, *fix* the left axis to a defined limit—because you know how much of something you have. When you do this, the shape of the chart tells you important things about how much time you have left and what the future will hold. For example, if we were studying yeast growth, we might start with a flask that holds one liter of grape juice, a quantity that we already know can only support so many yeast cells. With this defined limit, we can accurately calculate when an introduced population of yeast will crest and then crash.

Similarly, if we happen to know the carrying capacity of the earth for human beings, then we can "fix the left axis" and make some important observations about what the future might bring and how much time remains to stabilize things. Without fossil fuels to assist with agricultural production, the total carrying capacity of the earth for humans is thought to be somewhat less than the current 6.8 billion and possibly as low as one billion.[4]

Even if these carrying-capacity calculations prove to be pessimistic and we could set the left axis for sustainable human population at 10 billion (although I've not read any scientific analyses that would support such a number), we would still discover that population has turned the corner and that we're no longer on the flat portion of the curve but the steep portion. This means that you and I happen to live in a very different world with entirely different challenges and opportunities than the people who came before us. We live at a time when most people alive will hopefully witness the transition of human population from exponentially expanding to "not growing." I say hopefully, because the alternative is to overshoot and collapse, just like our friends, the yeast in the flask.

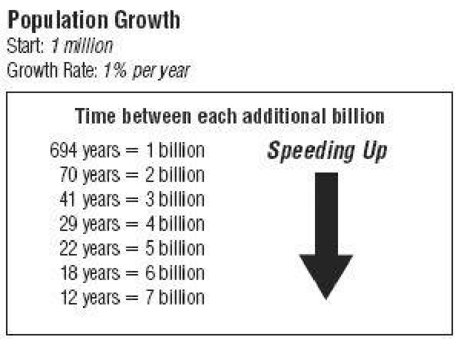
Speeding Up

A critical concept that I want you to take away from this discussion about exponential growth is that of "speeding up."

It doesn't matter how you prefer to approach this concept. You can either think of speeding up in terms of how the amounts accelerate in size over each unit of time, *or* you can think about how the amount of time shrinks between each fixed amount that is added. It's more stuff with each unit of time or less time between each unit of stuff. Either way you prefer to think of it, you'll come away with a sense of speeding up.

To illustrate this idea using population, if we started with one million people on the planet and set their growth rate to a relatively tame rate of 1 percent per year (it is actually higher than that), we would find that it would take 694 years for world population to grow from one million to one billion people.

Figure 5.5 Population Growth Example



Note how time "speeds up" by shrinking between each new billion people added to the total population.

But we would reach a world population of 2 billion people after only 100 more years, while the third billion would require just 41 more years. Then 29 years, then 22, and then finally only 18 years, to bring us to a total of 6 billion people. Each additional billion-people mark on our graph took a shorter and shorter amount of time to achieve. The time between each billion shrank each time, meaning that each billion came sooner and sooner, faster and faster. That's what I mean by speeding up.

Speeding up is a critical feature of exponential growth—things just go faster and faster, especially toward the end.

Making It Real

Using an example loosely adapted from a magnificent paper by Dr. Albert Bartlett,[5] let me illustrate the power of compounding for you.

Suppose I had a magic eye dropper and I placed a single drop of water in the middle of your left hand. The magic part is that this drop of water will double in size every minute. At first nothing seems to be happening, but by the end of a minute, that tiny drop is now the size of two tiny drops. After another minute, you now have a little pool of water sitting in your hand that is slightly smaller in diameter than a dime. After six minutes, you have a blob of water that would fill a thimble.

Now imagine that you're in the largest stadium you've ever seen or been in—perhaps Fenway Park, the Astrodome, or Wembley Stadium. Suppose we take our magic eye dropper to that enormous structure, and right at 12:00 pm in the afternoon, we place a magic drop way down in the middle of the field.

To make this even more interesting, suppose that the park is watertight and that you're handcuffed to one of the very highest bleacher seats. My question to you is this: *How long do you have to escape from the handcuffs?* When would the park be completely filled? Do you have days? Weeks? Months? Years? How long before the park is overflowing?

The answer is this: You have until exactly 12:50 pm *on that same day*—just 50 minutes—to figure out how you're going to escape from your handcuffs. In only 50 minutes, our modest little drop of water has managed to completely fill the stadium. But wait, you say, how can I be sure which stadium you picked? Perhaps the one you picked is 100 percent larger than the one I used to calculate this example (Fenway Park). Wouldn't that completely change the answer? Yes, it would—by one minute. Every minute, our magic water doubles, so even if your selected stadium happens to be 100 percent larger or 50 percent smaller than the one I used to calculate these answers, the outcome only shifts by a single minute.

Now let me ask you a far more important question: *At what time of the day would your stadium still be 97 percent empty space (and how many of you would realize the severity of your predicament)?* Take a guess.

The answer is that at 12:45 pm—only five minutes earlier—your park is only 3 percent full of water and 97 percent remains free of water. If at 12:45, you were still handcuffed to your bleacher seat patiently waiting for help to arrive, confident that plenty of time remained because the field was only covered with about 5 feet of water, you would actually have been in a very dire situation.

And that right there illustrates one of the key features of compound growth and one of the principal things that I want you take away from this chapter. With exponential growth in a fixed container, events progress much more rapidly toward the end than they do at the beginning. We sat in our seats for 45 minutes and nothing much seemed to be happening. But then, over the course of five minutes—whoosh!—the whole place was full of water. Forty-five minutes to fill 3 percent; only five more minutes to fill the remaining 97 percent. It took every year of human history from the dawn of time until 1960 to reach a world population of 3 billion people, and only 40 additional years to add the next 3 billion people.

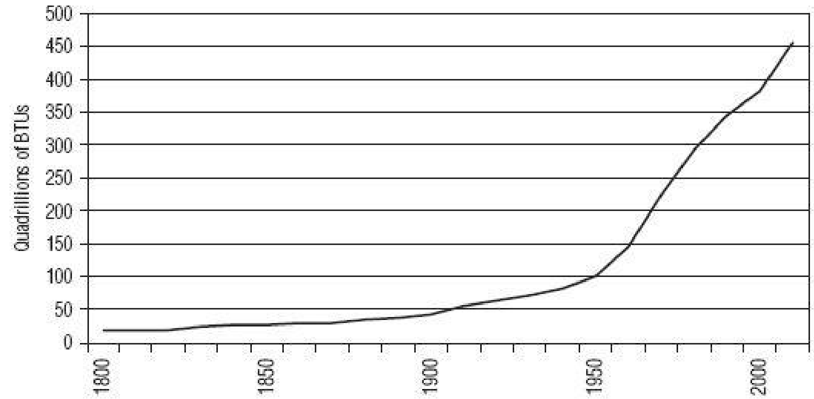
With this understanding, you will begin to understand the urgency I feel —there's simply not a lot of maneuvering room once you hop on the vertical portion of a compound graph. Time gets short.

Surrounded by Exponentials

Dr. Albert Bartlett once said that ". . . the greatest shortcoming of the human race is the inability to understand the exponential function."[6] He is absolutely right. We are literally surrounded by examples of exponential growth that we have created for ourselves, yet very few people recognize this or understand the implications. You now know one implication: speeding up.

Figure 5.6 shows total global energy consumption over the past 200 years. It is plainly obvious that energy use has been growing nonlinearly; the line on the chart looks like one of our hockey sticks. Can energy consumption grow exponentially forever, or is there some sort of a limit, a defined capacity to the energy stadium, that would cause us to fix the left axis on this chart?

Figure 5.6 Total Energy Consumption

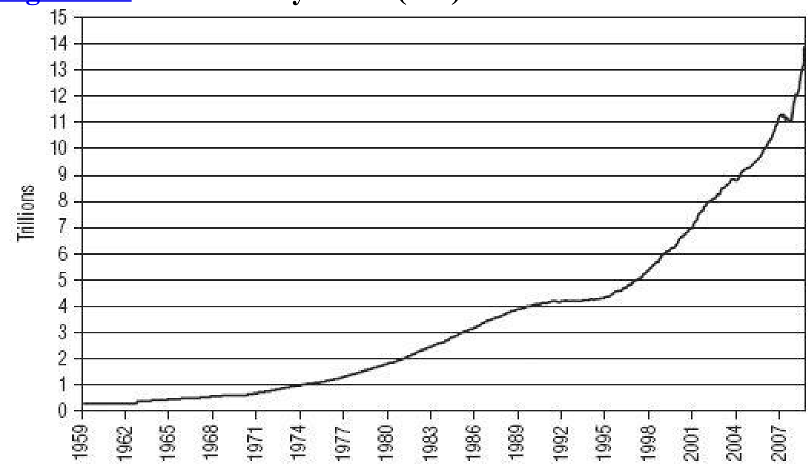


This chart includes energy from all sources: hydrocarbons, nuclear, biomass, and hydroelectric.

Source: Vaclav Smil, Energy Transitions.

On the following page is another exponential chart—the U.S. money supply, which has been compounding at incredible rates ranging between 5 and 18 percent per year (Figure 5.7).

Figure 5.7 Total Money Stock (M3)



This was the widest measure of money before its reporting was discontinued by the Federal Reserve. M3 money included cash, checking and savings accounts, time deposits, and

Eurodollars.
Source: Federal Reserve.

These are just a few examples. We could review hundreds of separate charts of things as diverse as the length of paved roads in the world, species loss, water use, retail outlets, miles traveled, or widgets sold, and we'd see the same sorts of charts with lines that curve sharply up from left to right.

The point here is that you are literally surrounded by examples of exponential growth found in the realms of the economy, energy, and the environment. Far from being a rare exception, they are the norm, and because they dominate your experience and will shape the future, you need to pay attention to them.

The Rule of 70

As I said before, anything that is growing by some percentage is growing exponentially. Another handy way to think about this is to be able to quickly calculate how long it will take for something to double in size. For example, if you are earning 5 percent on an investment, the question would be, *How long will it be before a $1,000 investment has doubled in size to $2,000?* The answer is surprisingly easy to determine using something called the "Rule of 70."[1]

To calculate how long it will be before something doubles, all we need to do is divide the percentage rate of growth into the number 70. So if our investment were growing at 5 percent *per year*, then it would double in 14 years (70 divided by 5 equals 14). Similarly if something is growing by 5 percent *per month*, then it will double in 14 months.

How long before something growing at 10 percent per year will double? Easy; 70 divided by 10 is 7, so the answer is 7 years.

Here's a trick question: *Suppose something has been growing at 10 percent per year for 28 years. How much has it grown?* Some people intuitively guess eight times larger (2 + 2 + 2 + 2 = 8), or four separate doublings over each of the four 7 year periods in the example, but the answer is sixteen, because each doubling builds off of the last (2 ⇒ 4 ⇒ 8 ⇒ 16). Two doubles to four, which doubles to eight, which doubles to sixteen, which is twice as large as intuition might suggest.

Here's where we might use that knowledge in real life. You might have read about the fact that China's energy consumption grew at a rate of slightly more than 8 percent between 2000 and 2009, which perhaps sounds somewhat tame. Using the Rule of 70, however, we discover that China is doubling the amount of energy it uses roughly every 9 years, as was confirmed by the International Energy Agency (IEA) in 2010.[7] After 9 years of 8 percent growth, you're not using just a little bit more energy, but 100 percent more. If a country has 500 coal-fired electricity plants today, after 9 years of 8 percent growth it will need 1000 such plants.

If this seems rather dramatic and nontrivial to you, you're right. Time for another trick question about doublings: *Which is larger in size, the amount of energy China used over just the past 9 years (its most recent doubling time), or the amount of energy China has used throughout* ***all*** *of history?* The intuitive answer is that the total amount of energy consumed throughout China's thousands of years of history is far larger than the amount consumed over the past 9 years, but the correct answer is that the most recent doubling is larger than all the prior doublings put together.[8]

This is a general truth about doublings, not China in particular, and applies to anything and everything that has gone through a doubling cycle. To make sense of this preposterous claim, let's use the legend of the mathematician who invented the game of chess for a king. So pleased was the king with this invention that he asked the mathematician to name his reward. The mathematician made a request that seemed modest: to be given a single grain of rice for the first square on the board, two grains for the second square, four grains for the third square, and so on. The king agreed, and foolishly committed to a sum of rice that was approximately 750 times larger than the entire annual worldwide harvest of rice in 2009. That's what 64 doublings will get you.

Note that the first square had one grain of rice placed upon it, while the next square, the first doubling, got two grains. Here on the very first doubling, we can observe that more rice was placed upon the board than was already on the board; two compared to one. That is, the doubling was larger in size than all of the grains that had come before it. And on the next doubling, when we place four grains upon the board, we see that these four grains of rice are more numerous than the three grains (1 + 2) already upon the board from all the prior doublings. And at the next doubling we place eight grains on the board, which is a larger total than the seven that are already upon it (1 + 2 + 4). And so on. In every doubling, we'll find that the most recent doubling is larger in size than all of the prior doublings put together. That's one of the less intuitive but more important features of doublings. Each doubling is larger than all the ones that came before *put together*.

So if your town administrators are targeting, say, 5 percent growth, what they're really saying is that in 14 years time they want to have more than twice as much of everything in the town than it currently has. More than twice as many people, sewage treatment plants, schools, congestion, electrical and water demand, and everything else that a town needs. Not a few more, but *more than twice as many*.

Your Exponential World

The reason we took this departure into discussing exponential growth and doubling times is that you happen to be completely surrounded by examples of exponential growth. And your future, like it or not, will be heavily shaped by their presence.

As you read the rest of this book, it will be helpful to continue to recall these three concepts related to exponential growth and doublings:

1. *Speeding up*. Time really gets compressed toward the end of the exponential phase of growth.

2. *Turning the corner*. This is a very real and extremely important event in systems with limits.

3. *More than double*. Each doubling equals more than all of the prior ones combined.

This information is going to be especially critical when we talk about the idea that our economy, our money system, and all of our associated institutions are fundamentally predicated on exponential growth. As we'll see, it's not just any type of growth that our money system requires, but *exponential* growth.

Up until recently, that has been a fine and workable model, but once we introduce the idea of resource limits into our collective story of growth (in other words, once we know just how big is the stadium in which we're all sitting), we quickly discover some serious flaws in our current narrative. It turns out that the economy does not exist in a vacuum, and it does not have the power to create reality. The economy is really just a reflection of our access to abundant energy and other concentrated resources that we can transform into useful products and services. As long as those resources can continue to be extracted from the earth in ever-increasing quantities, then our economic model is safe and sound. And that is where the trouble in this story begins.

1 Some use "the Rule of 72," which is more accurate in some circumstances, but less easy to calculate in our heads, so we'll stick to 70 for now, as it is perfectly accurate for our purposes.