Typing and ML

CSC324
Winter 2007
Sheila McIlraith

Acknowledgement:
The material in these notes is derived from a variety of sources, including:
Elements of ML Programming (Ullman),
Concepts in Programming Languages (Mitchell)
and the notes of Wael Aboelsaddat, Tony Bonner, Eric Joanis, Gerald Penn, and Suzanne Stevenson.

Typing

"A name for a set of values and some operations which can be performed on that set of values."

"A collection of computational entities that share some common property."
E.g.,
reals
integers
strings
int → bool
(int → int) → bool

What constitutes a type is language dependent.

Uses/Merits

Program organization and documentation
- Separate types for separate concepts
- Indicate intended use of declared identifiers

Identify and prevent errors
- Compile-time or run-time checking can prevent meaningless computation such as
  5 + true - Charlotte

Support optimization
- Compiler can generate better code if it knows what's in each variable, e.g., short integers require fewer bits.
- Access record component by known offset

Type errors

Definition
- A **type error** occurs when execution of program is not faithful to the intended semantics, i.e., the programmer's intended interpretation.

Hardware errors
- function call y() where y is not a function
- may cause jump to instruction that does not contain a legal op code

Unintended semantics
- int_add(3, 4.5)
- not a hardware error but the bits representing 4.5 will be interpreted as an integer

Type Safety & Type Checking

- A programming language is *type* safe if no program is allowed to violate its type distinctions.
  - Scheme, ML and Java are type safe.
  - C and C++ are not.
- The process of verifying and enforcing the constraints of types is called *type checking*.
- Type checking can either occur at compile-time (static) or at run-time (dynamic).

Compile- vs. Run-time

- Scheme: run-time (dynamic) type checking
  (car x) checks first to make sure x is a list
- ML and Java: compile-time (static) type checking
  f(x) must have f: A → B and x:A

Trade-off:
- Both prevent type errors
- Run-time checking slows down execution
- Compile-time checking restricts program flexibility
  E.g., Scheme list elements can have diff. types,
  ML lists elements must have the same type
- Static typing can make programming more difficult, initially. It's harder to get things to compile, and

Type Checking vs. Type Inference

Standard Type Checking:

```
int  f(int x) { return x+1;};
int  g(int y)  {return f(y+1)*2;};
```

- Look at body of each function and use declared types to check for agreement.

Type Inference:
- Looks at code without type info and figures out what types could have been declared.
- ML is designed to make type inference tractable.
- A cool algorithm!
- Widely regarded as an important language innovation.
- ML type inference gives you some idea of how other static analysis algorithms might work. It uses constraint satisfaction techniques.

Type Inference

This is type inference:
E.g. A3 := B4 + 1;
Q: What type is A3 and B4 ?
A: Must be integer
E.g. if test then …
Q: What type is test ?
A: Must be Boolean

Sound type system: a type system in which all types can always be inferred in any valid program.

ML's Type Inference Algorithm (Mitchell):
1. Assign a type to the expression and each subexpression by using the known type of a symbol of a type variable.
2. Generate a set of constraints on types by using the parse tree of the expression.
3. Solve these constraints by using unification, which is a substitution-based algorithm for solving systems of equations.

ML

Developed at Edinburgh (early '80s) as Meta-Language for a program verification system
- Now a general purpose language
- There are two basic dialects of ML
  - Standard ML (1991) & ML 2000
  - Caml (including Objective Caml, or OCaml)

A pure functional language
- Based on *typed lambda calculus*
- Grew out of frustration with Lisp!
- Major programs can be written w/o variables

Widely accepted
- reasonable performance (claimed)
- can be compiled
- syntax not as arcane as LISP (nor as simple…)

ML: Main Features

Functional Language
HOFs, recursion strongly encouraged, etc.
Combination of Lisp and Algol features
Strong, static typing w/ type inference
Quite a fancy type system!
Polymorphism
a function can take arguments of various types
Abstract & recursive data types
supported through an elegant type system,
the ability to construct new types, and
constructs that restrict access to objects of a given type through a fixed set of ops defined for that type.
Pattern matching
Function as a template
Exception handling
Allow you to handle errors/exception
Elaborate module system
Most highly developed of any language

ML: Tutorial Review

(see your tutorial notes for complete details)

SML environment basics

Each ML expression has a type associated w/ it.
- Interpreter builds the type expression
- Cannot mix types in expressions
- Must explicitly coerce/type-case
  e.g. real(2) + 3.0 : real

Data types (w/ operators):
Basic: unit, bool, integer, real, string
Constructors : list, tuple, array, record, function
operators infix, can be overloaded.

Read-eval-print
- Compiler infers type before compiling & executing.

E.g.,
```
- (5+3)-2;
> val it = 6 : int
- If 5>3 then "Bob" else "Carol";
>val it="Bob" : string
```


Patterns & Declarations

Patterns can be used in place of variables
<pat> ::= <id>|<tuple>|<cons>|<record>|…

Value declaration (general form):
val <pat> = <exp>

E.g. (Declarations),
- val myTuple = ("Jen","Brad");
*val myTuple = ("Jen","Brad") : string * string*

- val(x,y) = myTuple;
val x = "Jen" : string
val y = "Brad" : string

- val myList = [1,2,3,4];
val myList = [1,2,3,4] : int list

- val x::rest = myList;
val x = 1 : int
val rest = [2,3,4] : int list

Declarations

ML has let too!

Local declarations:
- let val x = 2+3 in x*4 end;
val it = 20 : int

```
- let
   val m=3        (* ; is optional *)
   val n=m*m
in
  m+n
end;
```
val it = 12 : int

Pattern Matching

Pattern matching is powerful:
- Allows programmers to see the arguments
- No more heads and tails (cars/cdrs)

Tupple pattern matching
-val v=((2, "Test"),(3.2,#"A"));
*val v = ((2,"Test"),(3.2,#"A")) : (int * string) * (real * char)*

-val ((i,s),(r,c))=v;
val i = 2 : int
val s = "Test" : string
val r = 3.2 : real
val c = #"A" : char

-val (p1,p2)=v;val p1 = (2,"Test") : int * string
*val p2 = (3.2,#"A") : real * char*

-val (_,(r,_))=v; (*_ ("don't care") matches anything!*)
val r = 3.2 : real

Pattern Matching

Record pattern matching
-type stInfo={name:string, id:int, gpa:real};
type stInfo = {gpa:real, id:int, name:string}

-val st1:stInfo={name="jen", id=123, gpa=4.0};
val st1 = {gpa=4.0,id=123,name="jen"} : stInfo

-val {name=N, gpa=G, id=_}=st1; (* order doesn't matter! *)
val G = 4.0 : real
val N = "jen" : string

-val {gpa,id, name}=st1; (* this is an abbreviation in ML *)
val gpa = 4.0 : real
val id = 123 : int
val name = "jen" : string

-val {name,...}=st1l; (* to specify subset of fields *)
val name = "jen" : string

Functions

Like Scheme there are:
- Defined functions
- Anonymous functions
- Recursive functions
- Higher-order functions
- And you can pass functions as parameters, and return them as values.

Unlike Scheme,
- we call these things "functions" not "procedures"

---

f: $A \to B$ means
for every $x \in A$,

f(x) = { some element $y=f(x) \in B$ / run forever / terminate by raising an exception }

A function maps a type to another one: accepts only **one** argument.

What if we need multiple arguments?

Function Declarations

Function Declaration
Single clause definition
fun <fname> (<pat>) =<exp>;

Function arguments (patterns) don't always need parentheses, but it doesn't hurt to use them

Examples:

- fun fahrToCelsius t = (t -32) * 5 div 9;
val fahrToCelsius = fn : int -> int

- fun foo L = (1 + hd L) :: (tl L);
val foo = fn : int list -> int list

- fun quotrem (x,y) = ((x div y), (x mod y));
*val quotrem = fn : int * int → int * int*

Function Declarations

Multiple-clause definition
fun <fname> (<pat1>) = <exp1>
| <fname> (<pat2>) = <exp2>
| …
| <fname> (<patn>) = <expn>

Lazy: The first pattern that matches the actual parameter will be chosen.

Examples:

-fun sum (x,y)= x+y;
*val sum = fn: int*int -> int*

-sum (2,3);
val it = 5 : int

-fun len (nil) = 0 (*nil or [ ] Also we can drop ()*)
| len (h::rest) = 1+len(rest); (* () is necessary!*)
val len= fn: 'a list -> int

-len ([5]);
val it = 1: int
-len ["Alice", "John"];
val it = 2: int

Function Declarations

Watch out!

- val z=4;
val z = 4 : int

-fun sumz (x,y)= x+y+z;
*val sumz = fn: int*int -> int*

-sumz (2,3);
val it = 9 : int

- val z=7;
val z = 7 : int

-sumz (2,3);
val it = 9 : int

No variable can occur twice in a pattern

- fun eq(x,x)=true
| eq(x,y)=false;
Error: duplicate variable in pattern(s)

If the pattern doesn't exhaust all possible values, we get a warning.

Function Declarations

Example:

- fun listsum L = if (null L) then 0
else (hd L) + listsum (tl L);
val listsum = fn : int list -> int

- listsum [1,2,3];
val it = 6 : int

Better:
- fun listsum [] = 0
| listsum L = (hd L) + listsum (tl L);

Best
- fun listsum [] = 0
| listsum (h::t) = h + listsum t;

Anonymous Functions

fn <pat> => <expr>
This is just like a Scheme lambda expression
(lambda (<pat>) (exp))

Examples:

-(fn(x,y)=> x+y) (2,3);
val it = 5 : int

-val mysum= fn (x,y)=> x+y;
*val mysum = fn : int * int -> int*

-mysum(2,3)
val it = 5 : int

The following declarations are identical:

- fun f(n) = 2*n;
val f = fn : int -> int

- val f = fn n => 2*n;
val f = fn : int -> int

Anonymous Functions

What is this doing?

- fun foo (m, n) =
if m > n then [ ] else m :: foo(m+1, n);
*val foo = fn : int * int -> int list*

- foo(1,6);

Recursive Functions

Examples:

- fun append(nil, ys) = ys
| append(x::xs,ys) = x :: append(xs,ys);
*val append = fn : 'a list * 'a list -> 'a list*

- fun reverse nil = nil
| reverse(x::xs) = append((reverse xs),[x]);
val reverse = fn : 'a list -> 'a list

There is a more efficient reverse….

Mutual Recursion

The following is wrong:
fun even 0 = true
| even x = odd (x-1); (*wrong: odd not defined*)

The following is correct, using mutual recursion:
fun even 0 = true
| even x = odd (x-1)
and odd 0 = false
| odd x = even (x-1);

Important Issues

1. Function application is left-associative.
   Use brackets as necessary.
   abs square x + y * abs z; *means*
   (abs square) x + (y * (abs z))
   Our error: operator and operand don't agree

2. The combination of tuples, functions, infix ops, type constructors can be syntactically tricky when defining/calling functions!

Eg. length 2::[1,3] *is wrong: it means*
(length 2) :: [1,3].
Correct formulation?:

Eg.
```
fun f1 nil=0 |
    f1 h::t= 1+f1 t;
```
Error: infix operator "::" used without "op" in fun dec
Error: clauses don't all have same no. of patterns
Correct formulations?:

Important Issues (cont.)

The syntax becomes more complex when considering the following short notation:

In ML
```
fun sum x y=x+y;
```
is short for
```
fun sum x= (fn y=>x+y);
```
So, its type is:
```
fn : int -> (int -> int)
```

Similarly,
```
fun sum3 x y z = x+y+z
```
is short for:
```
fun sum3 x =
      (fn y =>
        (fn z => x+y+z));
```
So it's type is:
```
fn : int -> int -> int -> int
```


Exception Handling

Recall that many functions are *partial* – they are only defined for a subset of the function's domain type. Other values in the domain type may be treated as ***exceptions.*** (E.g., division by zero)

Exceptions are a control construct. They provide a **structured form of jump to exit** a construct such as a function invocation or a block.

Terminate part of computation:
- Jump out of construct
- Pass data as part of jump
- Return to most recent site set up to handle exception
- Unnecessary activation records may be deallocated
  (May need to free heap space, other resources)

Often used for unusual or exceptional condition, but not necessarily

ML Example

```
exception Determinant;  (* declare exception name *)
fun invert (M) =      (* function to invert matrix *)
      …
      if …
          then raise Determinant    (* exit if Det=0 *)
          else …
 end;
...
invert (myMatrix) handle Determinant => … ;
```

Value for expression if determinant of myMatrix is 0

Exception Handling

Many languages have exception mechanisms:
Ada, C++, Java, ML, PL/1, etc.

Why do we need exceptions in a strongly typed language?
- only checks types of parameters, not their values
- In languages w/o exception handling, when an exception occurs, control goes to the OS and the program is terminated, or special code must be written to handle exceptions (e.g., pass special parameter or use return value of procedure to indicate status of program, etc.)
- In contrast, with exception handling, programs can "fix the problem" and continue, if desirable.

Some uses: *(see examples in Mitchell 8.2)*
error handling (when no reasoable value can be returned
- efficiency

Two types of exceptions in ML:
- Built-in exceptions
- User-defined exceptions

Built-In Exceptions

Classical example: division by 0.
- 5 div 3;   (* type of function div is int *)
val it = 1 : int

- 5 div 0;   (* an exception is raised *)
uncaught exception divide by zero
raised at: <file stdIn>

Other examples of built-in exceptions:
- hd([3,5,9]);
val it = 3 : int

- hd(nil: int list);
uncaught exception Empty
raised at: boot/list.sml:36.38-36.43

These both raise exceptions and stop, but exceptions can also be *handled*, a value assigned and computation continued.

User-Defined Exceptions

1. Declare an Exception to establish exception handler
   exception ⟨exception-nm⟩ of ⟨type-expression⟩
   gives name of exception and (optionally) type of data passed when raised

2. Raise Exception
   raise ⟨exception-nm⟩ ⟨parameters⟩
   raises an exception and passes data to handler

Example
- exception NegArg of int;   (* declare an exception *)
exception NegArg of int

- fun fact N = if N=0 then 1
=   else if N>0 then N*fact(N-1)
=   else raise NegArg(N); (* raise excptn *)
val fact = fn : int -> int

- fact(5);
val it = 120 : int

- fact(~5);
uncaught exception NegArg
raised at: stdIn:20.30-20.39

User-Defined Except'ns (cont.)

3. Exception Handling
<expression>
handle <exception1> => <expression1>
| <exception2> => <expression2>
…
| <exceptionn> => <expressionn>

- If no exceptions are raised then return the value of <expression>
- If <exceptioni> is raised, then return the value of <expressioni>

(See more general form + explanation in Mitchell)

Exception Handling

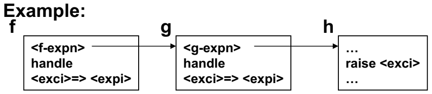
Example: N! / (M! (N-M)!)

```
- fun comb (N,M) =
=    if N < 0 then raise Negative(N)
=    else if M < 0 then raise Negative(M)
=         else if M > N then raise TooBig(M)
=              else
=                  fact(N) div (fact(M) * fact(N-M));
val comb = fn : int * int -> int

- fun mycomb (N,M) =  comb (N,M)
=         handle Negative(X) => ~1
=              |     TooBig(M) => 0;
val mycomb = fn : int * int -> int

- mycomb(12,5);
val it = 792 : int

- mycomb(~12,5);
val it = ~1 : int

- mycomb (5,12);
val it = 0 : int
```


Exceptions & Scoping

Exceptions are handled according to **dynamic scoping**. Otherwise ML uses static scoping.
(More on this in a later unit of the course, but here is an illustrative example)

Example:




```
- exception e1 and e3 and e3;
-  fun h(1) = raise e1
   |  h(2) = raise e2
   |  h(3) = raise e3
   |  h(_) = "ok";

- fun g(N) = h(N)
   handle e2 => "error g2"
     |       e3 => "error g3";

- fun f(N) = g(N)
   handle e1 => "error f1"
     |       e2 => "error f2";
```

```
- f(4);
val it = "ok" : string
- f(3);
val it = "error g3" : string
- f(2);
val it = "error g2" : string
- f(1);
val it = "error f1" : string
- f(0);
val it = "ok" : string
```


(edited out the interpreter responses)

Dynamic Scoping of Handlers

General dynamic scoping rule:
- Jump to most recently established handler on run-time stack

Dynamic scoping is not an accident:
- User knows how to handler error
- Author of library function does not

Typing of Exceptions

Typing of raise ⟨exception⟩
- Recall definition of typing
  - Expression e has type t if normal termination of e

  produces value of type t
- Raising exception is not normal termination
  - Example: 1 + raise X

Typing of handle ⟨exception⟩ => ⟨value⟩
- Converts exception to normal termination
- Need type agreement
- Examples
  - 1 + ((raise X) handle X => e)

    *** Type of e must be int
  - 1 + (e_1 handle X => e_2)

    *** Type of e_1, e_2 must be int