# LOGIC PROGRAMMING AND PROLOG

**Reading:**

- Sebesta, chapter 16

**References:**

- Clocksin and Mellish, Ch. 1-4, 6, 8
- Online Resources (tutorials, SWI page etc.)





## Logic Programming and Prolog

Logic programming languages are not procedural or functional.

- Specify *relations* between objects
  - `larger(3,2)`
  - `father(tom,jane)`
- Separate logic from control:
  - Programmer declares **what** facts and relations are true.
  - System determines **how** to use facts to solve problems.
  - System **instantiates** variables in order to make relations true!
- Computation engine: theorem-proving and recursion (Unification, Resolution, Backward Chaining, Backtracking)
  - Higher-level than imperative languages



## Jumping Right In

Suppose we state these facts:

```
male(albert).              parent(albert,edward).
female(alice).             parent(victoria,edward).
male(edward).              parent(albert,alice).
female(victoria).          parent(victoria,alice).
```

We can then make queries:

```
?- male(albert).
Yes

?- male(victoria).
No

?- female(Person).
Person = alice;
Person = victoria;
No

?- parent(Person, edward).
Person = albert;
Person = victoria;
No

?- parent(Person, edward), female(Person).
Person = victoria;
No
```



We can also state rules, such as this one:

```
sibling(X, Y) :- parent(P, X),
                 parent(P, Y).
```

Then the queries become more interesting:

```
?- sibling(albert, victoria).
No

?- sibling(edward, Sib).
Sib = edward;
Sib = alice;
Sib = edward;
Sib = alice;
No
```



## Prolog vs Scheme

In Scheme, we program with **functions** ("procedures").

- A function's arguments are different from the function's value.
- Give a single Scheme function, we can only ask one kind of question:

  Here are the argument values; tell me what is the function's value.

In Prolog, we program with **relations**.

- There is no bias; all arguments are the same.
- Given a single Prolog predicate, we can ask many kinds of question:

  Here are some of the argument values; tell me what the others have to be in order to make a true statement.



## Logic Programming

- A program consists of facts and rules.
- Running a program means asking queries.
- The language tries to find one way (or more) to prove that the query is true.
- This may have the side effect of freezing variable values.
- The language determines how to do all of this, *not* the program
- How does the language do it? Using unification, resolution, and backtracking.



## The swi Interface on cdf

```
cdf% ls
family.pl

cdf% swi
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.11)
Copyright (c) 1990-2003 University of Amsterdam.
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free so
and you are welcome to redistribute it under certain conditio
Please visit http://www.swi-prolog.org for details.

For help, use ?- help(Topic). or ?- apropos(Word).

?- ['family'].   <------------- load file family.pl
% family compiled 0.00 sec, 5,264 bytes
Yes


?- parent(Person, edward).

Person = albert;   <----------- ";" to get more
Person = victoria;
No


?- parent(Person, edward).

Person = albert   <--------------"a", CR, space to break
Yes
```



```
 ?- trace.
 Yes
 [trace]

 [trace] ?- parent(Person, edward).

 [trace]  ?- parent(Person, edward).
    Call: (7) parent(_G28, edward) ? creep  <- CR to continue
   Exit: (7) parent(albert, edward) ? creep

 Person = albert ;
   Redo: (7) parent(_G28, edward) ? creep
  Exit: (7) parent(victoria, edward) ? creep

Person = victoria

Yes
[debug]  ?-


   (0) Call: parent(_57,edward) ?
   (0) Exit: parent(albert,edward) ?
Person = albert;
   (0) Redo: parent(albert,edward) ?
   (0) Exit: parent(victoria,edward) ?
Person = victoria
Yes
[trace]
```

```
?- notrace.
Yes

?- parent(Person, edward).

Person = albert;
Person = victoria;
No


?- halt.
cdf%
```

```
cdf% pl
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.11) ...

For help, use ?- help(Topic). or ?- apropos(Word).

?- [family].
[family loaded]
Yes

?- parent(Person, edward).

Person = albert;
Person = victoria;
No

--- edit family.P and remove parent(albert,edward). ---

?- ['family'].

% family compiled 0.00 sec, 5,200 bytes
Yes

?- parent(Person, edward).

Person = victoria;
No

?- halt.
cdf%
```



## Some Prolog Syntax

Lexical Rules:

- Variables are capitalized.
- Constants begin with a lower case letter.
- Predicate names begin with a lower case letter.

Simplified Grammar:

```
<clause> ::= <pred> |

             <pred> :- <pred> { , <pred> } .


<pred>   ::= <pname>'(' <term> { , <term> } ')'


<term>   ::= <int> | <atom> | <var>
```

Note: No blank between predicate name and opening bracket.



## Prolog Queries

A query is a proposed fact that is to be proven.

- If the query has no variables, returns yes/no.
- If the query has variables, returns appropriate values of variables (called a substitution).



## Horn Clauses (Rules)

A Horn Clause is: $c \leftarrow h_1 \wedge h_2 \wedge h_3 \wedge \cdots \wedge h_n$

- Antecedents: conjunction of zero or more conditions which are atomic formulae in predicate logic
- Consequent: an atomic formula in predicate logic

Meaning of a Horn clause:

- "The consequent is true if the antecedents are all true"
- $c$ is true if $h_1$, $h_2$, $h_3$, ..., and $h_n$ are all true



## Horn Clause Terminology

- Horn Clause = Clause
- Consequent = Goal = Head
- Antecedents = Subgoals = Tail
- Horn Clause with No Tail = Fact
- Horn Clause with Tail = Rule

In Prolog, a Horn clause

$c \leftarrow h_1 \wedge \cdots \wedge h_n$

is written

c :- $h_1$, , $h_n$.

Syntax elements: ':-' ',' '.'



## Prolog Horn Clause Examples

A Horn clause with no tail:

male(albert).

I.e., a fact: albert is a male dependent on no other conditions

A Horn clause with a tail:

father(albert,edward):-
male(albert), parent(albert,edward).

I.e., a rule: albert is the father of edward if albert is male and albert is a parent of edward's.



## Meaning of Prolog Rules Without Variables

A prolog rule must have this form:

$$c \ :- \ a_1, \ \ a_2, \ \ a_3, \ \ \cdots, \ \ a_n.$$

which means in logic:

$$a_1 \wedge a_2 \wedge a_3 \wedge \cdots \wedge a_n \rightarrow c.$$

**Restrictions**

- There can be zero or more antecedents, but they are conjoined; we cannot disjoin the
- There cannot be more than 1 consequent.

## Bending the Restrictions

**Getting disjoined antecedents**

Example: $a_1 \vee a_2 \vee a_3 \vee \rightarrow c$.

Solution:

**Getting more than 1 consequent, conjoined**

Example: $a_1 \wedge a_2 \wedge a_3 \rightarrow c_1 \wedge c_2$.

Solution:

**Getter more than 1 consequent, disjoined**

Example: $a_1 \wedge a_2 \wedge a_3 \rightarrow c_1 \vee c_2$.

Solution:



## Why Can't We Disjoin Consequents?

Why did the designers of Prolog disallow this?



## Logic Review



## Horn Clauses with Variables

Variables may appear in the antecedents **and** consequent of a Horn clause:

- c($X_1$,...,$X_n$) :- h($X_1$,...,$X_n$).

  "For all values of $X_1$,...,$X_n$, the formula c($X_1$,...,$X_n$) is true if the formula h($X_1$,...,$X_n$) is true"

- c($X_1$,...,$X_n$) :- h($X_1$,...,$X_n$,$Y_1$,...,$Y_k$).

  "For all values of $X_1$,...,$X_n$, the formula c($X_1$,...,$X_n$) is true if there exist values of $Y_1$,...,$Y_k$ such that the formula h($X_1$,...,$X_n$,$Y_1$,.. is true"



## Meaning of Prolog Rules With Variables

Example:

```
isaMother(X) :- female(X), parent(X, Y).
```

Logic:

$parent(X,Y) \wedge female(X) \supset isaMother(X)$.

But this is meaningless without quantifiers for the variables.

**The rule**

A Prolog rule of this form ($n \geq 0, m \leq n, k \geq o$):

$$c(X_1, \cdots X_n)\ :-\ a(X_1, \cdots X_m, Y_1, \cdots Y_k).$$

means:

$\forall X_1, \cdots X_n$

$[\exists Y_1, \cdots Y_k\ \ [a(X_1, \cdots X_m, Y_1, \cdots Y_k) \supset c(X_1, \cdots X_n)]\ \ ]$



**Sample run**

```
cdf% pl
Welcome to SWI-Prolog (Multi-threaded, Version 5.2.11).``
?- ['family'].

Warning: (./family.pl:50):
        Singleton variables: [Y]
% family compiled 0.00 sec, 5,528 bytes

?- isaMother(X).
X = victoria;
X = victoria;
No
```



## Rule Ordering and Unification

1. rule ordering used in search
2. unification requires two instances of the same variable in the same rule to get the same value
3. unification does not require differently named variables to get different values: hence, sibling(edward,edward).
4. all rules searched if requested by ';'



## How Prolog Handles a Query

**Example 1**

Database:

```
1)  male(tom).
2)  male(peter).
3)  male(doug).
4)  female(susan).
5)  male(david).

6)  parent(doug, susan).
7)  parent(tom, william).
8)  parent(doug, david).
9)  parent(doug, tom).

10) grandfather(GP, GC) :- male(GP),
                   parent(GP, X) ,
                   parent(X, GC).
```

Query:

```
| ?- grandfather(X,Y).
```

## Trace it by hand



## Trace it in Prolog

```
[trace]  ?- grandfather(X,Y).
   Call: (7) grandfather(_G28, _G284) ? creep
  Call: (8) male(_G283) ? creep
  Exit: (8) male(tom) ? creep
  Call: (8) parent(tom, _L205) ? creep
  Exit: (8) parent(tom, william) ? creep
  Call: (8) parent(william, _G284) ? creep
  Fail: (8) parent(william, _G284) ? creep
  Redo: (8) male(_G283) ? creep
  Exit: (8) male(peter) ? creep
  Call: (8) parent(peter, _L205) ? creep
  Fail: (8) parent(peter, _L205) ? creep
  Redo: (8) male(_G283) ? creep
  Exit: (8) male(doug) ? creep
  Call: (8) parent(doug, _L205) ? creep
  Exit: (8) parent(doug, susan) ? creep
  Call: (8) parent(susan, _G284) ? creep
  Fail: (8) parent(susan, _G284) ? creep
  Redo: (8) parent(doug, _L205) ? creep
  Exit: (8) parent(doug, david) ? creep
  Call: (8) parent(david, _G284) ? creep
  Fail: (8) parent(david, _G284) ? creep
  Redo: (8) parent(doug, _L205) ? creep
  Exit: (8) parent(doug, tom) ? creep
  Call: (8) parent(tom, _G284) ? creep
  Exit: (8) parent(tom, william) ? creep
  Exit: (7) grandfather(doug, william) ? creep
X = doug
Y = william
Yes
```



## Prolog Search Trees

- Each **node** is an ordered list of goals.
- Each **edge** is labelled with the variable bindings that occurred due to applying a rule. (The binding are in effect throughout the subtree.)
- Each **leaf** represents either success or failure.



## Example 2

Database:

```
1)  male(albert).
2)  female(alice).
3)  male(edward).
4)  female(victoria).
5)  parent(albert,edward).
6)  parent(victoria,edward).
7)  parent(albert,alice).
8)  parent(victoria,alice).
9)  sibling(X, Y) :- parent(P, X), parent(P, Y).
```

Query:

```
?- sibling(alice,Asib).
Asib = edward ;
Asib = alice ;
Asib = edward;
Asib = alice ;
No
?- sibling(Asib, alice).
Asib = edward ;
Asib = edward ;
Asib = alice ;
Asib = alice ;
No
```



## Trace it by hand



## Trace it in Prolog

```
[trace]  ?- sibling(alice,Asib).
   Call: (7) sibling(alice, _G284) ? creep
   Call: (8) parent(_L205, alice) ? creep
   Exit: (8) parent(albert, alice) ? creep
   Call: (8) parent(albert, _G284) ? creep
   Exit: (8) parent(albert, edward) ? creep
   Exit: (7) sibling(alice, edward) ? creep

Asib = edward ;
   Redo: (8) parent(albert, _G284) ? creep
   Exit: (8) parent(albert, alice) ? creep
   Exit: (7) sibling(alice, alice) ? creep

Asib = alice ;
   Redo: (8) parent(_L205, alice) ? creep
   Exit: (8) parent(victoria, alice) ? creep
   Call: (8) parent(victoria, _G284) ? creep
   Exit: (8) parent(victoria, edward) ? creep
   Exit: (7) sibling(alice, edward) ? creep

Asib = edward ;
   Redo: (8) parent(victoria, _G284) ? creep
   Exit: (8) parent(victoria, alice) ? creep
   Exit: (7) sibling(alice, alice) ? creep

Asib = alice ;

No
```



## The Anonymous Variable

If a rule has a variable that appears only once, that variable is called a "singleton variable".

Its value doesn't matter — it doesn't have to match anything elsewhere in the rule.

```
isaMother(X) :- female(X), parent(X, Y).
```

Such a variable consumes resources at run time.

We can replace it with "_", the anonymous variable. It matches anything.

If we don't, Prolog will warn us.



## Procedural Semantics of Prolog

Notice the recursion in this algorithm: "find" calls "find". This reasoning is recursively applied until we reach rules that are facts.

This process is called **Backward Chaining**.

## Logic Programming vs. Prolog

```
cousin(X,Y) :- parent(W,X), sister(W,Z),
               parent(Z,Y).

cousin(X,Y) :- parent(W,X), brother(W,Z),
               parent(Z,Y).

| ?- cousin(X,jane). % a query
```

Rule and Goal Ordering:

- There are two rules for cousin
- Which rule do we try first?
- Each rule for cousin has several subgoals
- Which subgoal do we try first?



## Logic Programming vs. Prolog

**Logic Programming:** *Nondeterministic*

- Arbitrarily choose rule to expand first
- Arbitrarily choose subgoal to to explore first
- Results don't depend on rule and subgoal ordering

**Prolog:** *Deterministic*

- Expand first rule first
- Explore first subgoal first
- Results may depend on rule and subgoal ordering