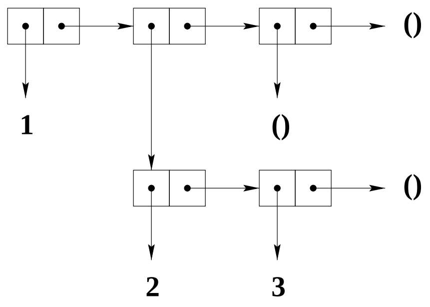
Lists

A simple but powerful general-purpose datatype. (How many datatypes have we seen so far?)

```
(1 #t 1)
()
(1 (2 3) ())
```

Building block: the cons cell.



Note: Sebesta uses NIL. That is LISP notation! In Scheme, we use ().

Things you should know about cons, pairs and lists

The *pair* or *cons cell* is the most fundamental of Scheme's structured object types.

A **list** is a sequence of **pairs**; each pair's cdr is the next pair in the sequence.

The cdr of the last pair in a **proper list** is the empty list. Otherwise the sequence of pairs forms an **improper list**. I.e., an empty list is a proper list, and and any pair whose cdr is a proper list is a proper list.

An improper list is printed in **dotted-pair notation** with a period (dot) preceding the final element of the list. A pair whose cdr is not a list is often called a **dotted pair**

Creating lists

- Quote: `'(1 (2 3) ()) => (1 (2 3) ())`
  or `(quote (1 (2 3) ())) => (1 (2 3) ())`
- `list`: `(list 1 '(2 3) ()) => (1 (2 3) ())`
- Build it, piece by piece:
  ```
  (cons 1 (cons (cons 2 (cons 3 ()))
                (cons () ())))
  ```
- Appending lists:
  `(append lst '(4 5)) => ((1 (2 3) () 4 5))`

cons vs. list: The procedure `cons` actually builds *pairs*, and there is no reason that the cdr of a pair must be a list, as illustrated on the previous page.

The procedure `list` is similar to `cons`, except that it takes an arbitrary number of arguments and always builds a proper list.

E.g., `(list 'a 'b 'c)` → `(a b c)`

Useful predicates

Testing for equality

- `(eq? a b)`: Returns `#t` iff `a` and `b` are the same Scheme object. (Don't use `eq?` with numbers!)
- `(= a b)`: Returns `#t` iff `a` and `b` are numerically equal. Pre: `a` and `b` must evaluate to numbers.
- `(eqv? a b)`: Similar to `eq?`, but works for numbers and characters. More expensive than `eq?`, however.
- `(equal? a b)`: Returns `#t` iff `a` and `b` have the same structure and contents. Thus, `equal?` recursively tests for equality. The most expensive equality predicate.

Recommended Reading:
Dybvig §6.1, 2nd ed. (available online), or
Dybvig §6.2, 3rd ed.

Equality Checking

The `eq?` predicate doesn't work for lists.

Why not?

1. `(cons 'a '())` makes a new list
2. `(cons 'a '())` makes a(nother) new list
3. `eq?` checks if its two args are *the same*
4. `(eq? (cons 'a '()) (cons 'a '()))` evaluates to () (ie, #f)

Lists are stored as pointers to the first element (car) and the rest of the list (cdr).

Symbols are stored uniquely, so `eq?` works on them.

Equality Checking for Lists

For lists, need a comparison procedure to check for the same **structure** in two lists. How might you write such a procedure?

```
(define (equal? x y)
  (or (and (atom? x) (atom? y) (eq? x y))
      (and (not (atom? x)) (not (atom? y))
           (equal? (car x) (car y))
           (equal? (cdr x) (cdr y)))))
```

- `(equal? 'a 'a)` evaluates to `#t`
- `(equal? 'a 'b)` evaluates to ()
- `(equal? '(a) '(a))` evaluates to `#t`
- `(equal? '((a)) '(a))` evaluates to ()

Note there is a built-in predicate procedure **equal?**. Play around with it!

More pre-defined predicates

- `(null? a)`: Returns `#t` iff `a` is the empty list (or `#f`, depending on the implementation).
- `(pair? a)`: Returns `#t` iff `a` is a pair, *i.e.*, a cons cell.
- `(number? a)`: Returns `#t` iff `a` is a number.

Lots more in Dybvig §6.

Code as Data—Eval

Scheme code is simply data that is treated as code. If you build an expression, using any data processing technique, and you want to evaluate it as code, use `eval`:

```
(define a (+ 4 6))
a => 10
(define b '(+ 4 6))
b => (+ 4 6)
(eval b ()) => 10
```

More on this later...

Recursive Procedures: Counting

```
(define (atomcount x)
  (cond ((null? x) 0)
        ((atom? x) 1)
        (else (+ (atomcount (car x))
                 (atomcount (cdr x))))))
```

- `(atomcount '(1 2))` ⇒ 2
- `(atomcount '(1 (2 (3)) (5)))` ⇒ 4:

```
(at '(1 (2 (3)) (5)))
(+ (at 1) (at ((2 (3)) (5))))
(+ 1 (+ (at (2 (3))) (at ((5)))))
(+ 1 (+ (+ (at 2) (at ((3)))) (+ (at (5)) (at ()))))
(+ 1 (+ (+ 1 (+ (at (3)) (at ()))) (+ (+ (at 5) (at ())) 0)))
(+ 1 (+ (+ 1 (+ (+ (at 3) (at ())) 0)) (+ (+ 1 0) 0)))
(+ 1 (+ (+ 1 (+ (+ 1 0) 0)) (+ 1  0)))
(+ 1 (+ (+ 1 (+ 1 0)) 1))
(+ 1 (+ (+ 1 1) 1))
(+ 1 (+ 2 1))
(+ 1 3)
4
```

This is called "car-cdr-recursion.'

Efficiency Issues

Problem: Evaluating the same expression twice.

Example:

```
(define (longest-nonzero x y)
  (cond ((and (null? x) (null? y)) -1)
        ((> (length x) (length y))
         (length x))
        (else (length y))

  ))
```

What can you do if there is no assignment statement?

Efficiency Issues

Solution 1: Bind values to parameters in a helper procedure.

```
(define (maximum x y)
  (cond ((> x y) x)
        (else y)
  ))

(define (longest-nonzero x y)
  (cond ((and (null? x) (null? y))  -1)
        (else
         (maximum (length x) (length y)))
  ))
```

Note: There is a built-in `max` function.

Note 2: Helper procedures are an important and useful tool!

Efficiency Issues

Solution 2: Use a `let` or `let*` construct, that binds variables to expression results.

```
(let ((var1 expr1)
      ...
      (varn exprn))
 <vars are defined and can be used here>)
(let* ((var1 expr1)
        ...
       (varn exprn))
 <vars are defined and can be used here>)
```


Polymorphic and Monomorphic Functions

- *Polymorphic* functions can be applied to arguments of many forms
- The function `length` is polymorphic: it works on lists of numbers, lists of symbols, lists of lists, lists of anything
- The function `square` is monomorphic: it only works on numbers

Higher-Order Procedures

Procedures as input values:

```
(define (all-num lst)
  (or (null? lst)
      (and (number? (car lst))
           (all-num (cdr lst))))
  )
(define (all-num-f f lst)
  (cond ((all-num lst) (f lst))
        (else 'error))
  )
1 ]=> (all-num-f abs-list '(1 -2 3))
;Value 1: (1 2 3)

1 ]=> (all-num-f abs-list '(1 a))
;Value: error
```


Higher-Order Procedures

Procedures as returned values:

```
(define (plus-list x)
  (cond ((number? x)
         (lambda (y) (+ (sum-n x) y)))
        ((list? x)
         (lambda (y) (+ (sum-list x) y)))
        (else (lambda (x) x))
  ))
1 ]=> ((plus-list 3) 4)
;Value: 10

1 ]=> ((plus-list '(1 3 5)) 5)
;Value: 14
```


Built-In Higher-Order Procedures: `map`

```
(define (map f l)
  (cond ((null? l) '())
        (else (cons (f (car l))
                    (map f (cdr l))))
  ))
```

- `map` takes two arguments: a function and a list
- `map` builds a new list whose elements are the result of applying the function to each element of the (old) list

Higher-order Procedures: `map`

- Example:

  `(map abs '(-1 2 -3 4))` ⇒
  `(1 2 3 4)`

  `(map (lambda (x) (+ 1 x)) '(-1 2 -3))` ⇒
  `(0 3 -2)`

- Actually, the built-in `map` can take more than two arguments:

  `(map cons '(a b c) '((1) (2) (3)))` ⇒
  `((a 1) (b 2) (c 3))`

What's Wrong Here??

```
1 ]=>
(define (atomcount s)
  (cond ((null? s) 0)
        ((atom? s) 1)
        (else (+ (map atomcount s)))
  ))
;Value: atomcount
1 ]=> (atomcount '(a b))

;The object (1 1), passed as an argument
;to +, is not the correct type.
...
2 error>
```

Why doesn't this work?

Using `eval` to Correct the Problem

```
(define (atomcount s)
 (cond ((null? s) 0)
       ((atom? s) 1)
       (else
        (eval
         (cons '+ (map atomcount s)) '()))
))
1 ]=> (atomcount '(a b))

;Value: 2

1 ]=> (atomcount '((1) (2 3 (4)) ((((5))))))

;Value: 5
```


Limitations of Using `eval`

BUT: `eval` only works in the current definition of `atomcount` because numbers evaluate to themselves.

```
1 ]=> (+ 1 2 3)
;Value: 6

1 ]=> (cons '+ '(1 2 3))
;Value 12: (+ 1 2 3)

1 ]=> (eval (cons '+ '(1 2 3)) '())
;Value: 6
```


Using `eval` to Evaluate Expressions

```
1 ]=> (append '(a) '(b))
;Value 13: (a b)
1 ]=> (cons 'append '((a) (b)))
;Value 14: (append (a) (b))

1 ]=> (eval (cons 'append '((a) (b))) '())
;Unbound variable: b
...
1 ]=> (cons 'append '( '(a) '(b) ))
;Value 15: (append (quote (a)) (quote (b)))

1 ]=> (eval
         (cons 'append '( '(a) '(b))) '())
;Value 16: (a b)
```

Too complicated!!

Applying Procedures with `apply`

```
1 ]=> (apply + '(1 2 3))
;Value: 6
1 ]=> (apply append '((a) (b)))
;Value 5: (a b)

1 ]=>
(define (atomcount s)
 (cond ((null? s) 0)
       ((atom? s) 1)
       (else
        (apply + (map atomcount s)))))

;Value: atomcount
1 ]=>  (atomcount '(a (b) c))
;Value: 3
```


Higher-order Procedures: `reduce`

```
(define (reduce op l id)
 (if (null? l)
     id
     (op (car l)
         (reduce op (cdr l) id))
))
```

A binary $\mapsto$ n-ary procedure.

The `reduce` procedure takes a binary operation and applies it right-associatively to a list of an arbitrary number of arguments.

NOTE: `reduce` is not equivalent to `apply`.

Higher-order Procedures: `reduce`

`(reduce + '(1 2 3) 0)` $\Rightarrow$ 6:

```
(reduce + '(1 2 3) 0)
(+ 1 (reduce + '(2 3) 0))
(+ 1 (+ 2 (reduce + '(3) 0)))
(+ 1 (+ 2 (+ 3 (reduce + '() 0))))
(+ 1 (+ 2 (+ 3 0)))
6
```

Note: `(+ 1 2 3)` $\Rightarrow$ 6

`(reduce / '(24 6 2) 1)` $\Rightarrow$ 8:

```
(reduce / '(24 6 2) 1)
(/ 24 (reduce / '(6 2) 1))
(/ 24 (/ 6 (reduce / '(2) 1)))
(/ 24 (/ 6 (/ 2 (reduce / '() 1))))
(/ 24 (/ 6 (/ 2 1)))
8
```

Note: `(/ 24 6 2)` $\Rightarrow$ 2

Higher-order Procedures: `reduce`

Given `union`, which takes two lists representing sets and returns their union:

```
1 ]=> (apply union '((1 3)(2 3 4)))
;Value 21: (1 2 3 4)

1 ]=> (apply union '((1 3)(2 3)(4 5)))
;The procedure #[compound-procedure union]
;has been called with 3 arguments;
;it requires exactly 2 arguments.

1 ]=> (reduce union '((1 3)(2 3)(4 5)) '())
;Value 22: (1 2 3 4 5)
```

Question: How would you have to change `reduce` to be able to take `intersection` as its function argument?