CSC2542
Representations
for Classical Planning

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Winter 2009

Acknowledgements

Some the slides used in this course are modifications of Dana Nau's lecture slides for the textbook *Automated Planning,* licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License: http://creativecommons.org/licenses/by-nc-sa/2.0/

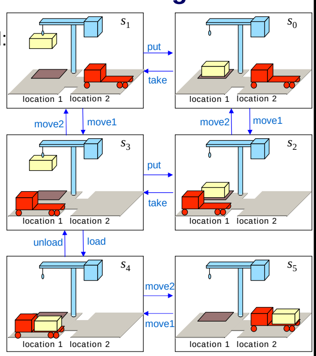
Other slides are modifications of slides developed by Malte Helmert, Bernhard Nebel, and Jussi Rintanen.

I have also used some material prepared by P@trick Haslum and Rao Kambhampati.

I would like to gratefully acknowledge the contributions of these researchers, and thank them for generously permitting me to use aspects of their presentation material.

Quick Review of Classical Planning

8 restrictive assumptions req'd:
- A0: Finite
- A1: Fully observable
- A2: Deterministic
- A3: Static
- A4: Attainment goals
- A5: Sequential plans
- A6: Implicit time
- A7: Offline planning



Representation: Motivation for Approach

Default view:
- represent state explicitly
- represent actions as a transition system (e.g., as an incidence matrix)

Problem:
- explicit graph corresponding to transition system is huge
- direct manipulation of transition system is cumbersome

Solution:

Provide **compact representation** of transition system & induced graph
1. Explicate the structure of the "states"
   - e.g., states specified in terms of state variables
2. Represent actions not as transition system/incidence matrices but as functions (e.g., operators) specified in terms of the state variables
   - An action is applicable to a state when some state variables have certain values. When applicable, it will change the values of certain (other) state variables
3. To plan,
   - Just give the initial state
   - Use the operators to generate the other states as needed

Why is this more compact?

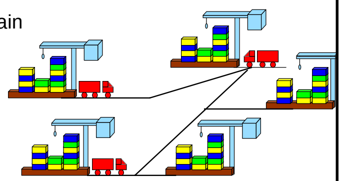
Why is this more compact than an explicit transition system?
- In an explicit transition system, actions are represented as state-to-state transitions. Ech action will be represented by an incidence matrix of size |S|x|S|
- In the proposed model, actions are represented only in terms of state variables whose values they care about, and whose value they affect. *(It exploits the structure of the problem!)*
- Consider a state space of 1024 states. It can be represented by $\log_2 1024=10$ state variables. If an action needs variable v1 to be true and makes v7 to be false, it can be represented by just 2 bits (instead of a 1024x1024 matrix)
  - Of course, if the action has a complicated mapping from states to states, in the worst case the action rep will be just as large
  - The assumption being made here is that the actions will have effects on a small number of state variables.

Outline

- Representation schemes
  1. Classical representation
  2. Set-theoretic representation
  3. State-variable representation
- Examples: DWR and the Blocks World
- Comparisons

1. Classical Representation

- Start with a *function-free* first-order language
  - Finitely many predicate symbols and constant symbols, but *no* function symbols
- Example: the DWR domain
  - Locations: l1, l2, …
  - Containers: c1, c2, …
  - Piles: p1, p2, …
  - Robot carts: r1, r2, …
  - Cranes: k1, k2, …



Classical Representation

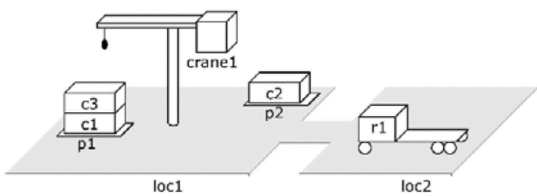
- *Atom*: predicate symbol and args
  - Use these to represent both fixed and dynamic ("fluent") relations

adjacent(*l*,*l'*)	attached(*p*,*l*)	belong(*k*,*l*)
occupied(*l*)	at(*r*,*l*)	
loaded(*r*,*c*)	unloaded(*r*)	
holding(*k*,*c*)	empty(*k*)	
in(*c*,*p*)	on(*c*,*c'*)	
top(*c*,*p*)	top(pallet,*p*)	

- *Ground* expression: contains no variable symbols - e.g., in(c1,p3)
- *Unground* expression: at least one variable symbol - e.g., in(c1,*x*)
- *Substitution*: $\theta = \{x_1 \leftarrow v_1,\ x_2 \leftarrow v_2,\ \ldots,\ x_n \leftarrow v_n\}$
  - Each x_i is a variable symbol; each v_i is a term
- *Instance* of *e*: result of applying a substitution θ to *e*
  - Replace variables of *e* simultaneously, not sequentially

States

- *State*: a set *s* of ground atoms
  - The atoms represent the things that are true in one of Σ's states
  - Only finitely many ground atoms, so only finitely many possible states



{attached(p1,loc1), in(c1,p1), in(c3,p1), top(c3,p1), on(c3,c1), on(c1,pallet), attached(p2,loc1), in(c2,p2), top(c2,p2), on(c2,pallet), belong(crane1,loc1), empty(crane1), adjacent(loc1,loc2), adjacent(loc2,loc1), at(r1,loc2), occupied(loc2), unloaded(r1)}.

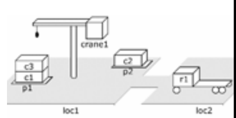
Operators

- *Operator*: a triple *o*=(name(*o*), precond(*o*), effects(*o*))
  - name(*o*) is a syntactic expression of the form $n(x_1,\ldots,x_k)$
    - *n*: *operator symbol* - must be unique for each operator
    - $x_1,\ldots,x_k$: variable symbols (parameters)
      - must include every variable symbol in *o*
  - precond(*o*): *preconditions*
    - literals that must be true in order to use the operator
  - effects(*o*): *effects*
    - literals the operator will make true

take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p), on(c,d)
effects: holding(k,c), $\neg$empty(k), $\neg$in(c,p), $\neg$top(c,p), $\neg$on(c,d), top(d,p)

Actions

- *Action*: ground instance (via substitution) of an operator



take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p), on(c,d)
effects: holding(k,c), $\neg$empty(k), $\neg$in(c,p), $\neg$top(c,p), $\neg$on(c,d), top(d,p)

take(crane1,loc1,c3,c1,p1)
;; crane crane1 at location loc1 takes c3 off c1 in pile p1
precond: belong(crane1,loc1), attached(p1,loc1), empty(crane1), top(c3,p1), on(c3,c1)
effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1), ¬top(c3,p1), ¬on(c3,c1), top(c1,p1)

Notation

- Let *a* be an operator or action. Then
  - $\text{precond}^+(a)$ = {atoms that appear positively in *a*'s preconditions}
  - $\text{precond}^-(a)$ = {atoms that appear negatively in *a*'s preconditions}
  - $\text{effects}^+(a)$ = {atoms that appear positively in *a*'s effects}
  - $\text{effects}^-(a)$ = {atoms that appear negatively in *a*'s effects}

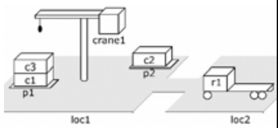
E.g.,

take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p), on(c,d)
effects: holding(k,c), $\neg$empty(k), $\neg$in(c,p), $\neg$top(c,p), $\neg$on(c,d), top(d,p)

- effects^+(take(*k*,*l*,*c*,*d*,*p*) = {holding(*k*,*c*), top(*d*,*p*)}
- effects^-(take(*k*,*l*,*c*,*d*,*p*) = {empty(*k*), in(*c*,*p*), top(*c*,*p*), on(*c*,*d*)}

Applicability

- An action *a* is *applicable* to a state *s* if *s* satisfies precond(*a*),
  - i.e., if precond$^+$(a) $\subseteq s$ and precond$^-$(a) $\cap\, s = \varnothing$
- Here are an action and a state that it's applicable to:

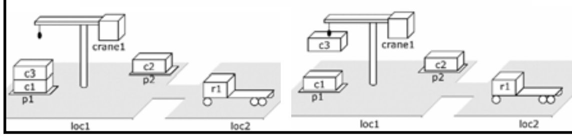


take(crane1,loc1,c3,c1,p1)
;; crane crane1 at location loc1 takes c3 off c1 in pile p1
precond: belong(crane1,loc1), attached(p1,loc1), empty(crane1), top(c3,p1), on(c3,c1)
effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1), ¬top(c3,p1), ¬on(c3,c1), top(c1,p1)

Result of Performing an Action

- If *a* is applicable to *s*, the result of performing it is
  $\gamma(s,a) = (s - \text{effects}^-(a)) \cup \text{effects}^+(a)$
  - Delete negative effects, and add positive ones

take(crane1,loc1,c3,c1,p1)
;; crane crane1 at location loc1 takes c3 off c1 in pile p1
precond: belong(crane1,loc1), attached(p1,loc1), empty(crane1), top(c3,p1), on(c3,c1)
effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1), ¬top(c3,p1), ¬on(c3,c1), top(c1,p1)



Operators for the DWR Domain

move(r, l, m)
;; robot r moves from location l to location m
precond: adjacent(l, m), at(r, l), ¬occupied(m)
effects: at(r, m), occupied(m), ¬occupied(l), ¬at(r, l)

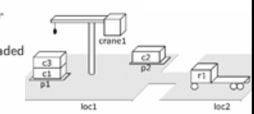
load(k, l, c, r)
;; crane k at location l loads container c onto robot r
precond: belong(k, l), holding(k, c), at(r, l), unloaded(r)
effects: empty(k), ¬holding(k, c), loaded(r, c), ¬unloaded(r)

unload(k, l, c, r)
;; crane k at location l takes container c from robot r
precond: belong(k, l), at(r, l), loaded(r, c), empty(k)
effects: ¬empty(k), holding(k, c), unloaded(r), ¬loaded

put(k, l, c, d, p)
;; crane k at location l puts c onto d in pile p
precond: belong(k, l), attached(p, l), holding(k, c), top(d, p)
effects: ¬holding(k, c), empty(k), in(c, p), top(c, p), on(c, d), ¬top(d, p)

take(k, l, c, d, p)
;; crane k at location l takes c off of d in pile p
precond: belong(k, l), attached(p, l), empty(k), top(c, p), on(c, d)
effects: holding(k, c), ¬empty(k), ¬in(c, p), ¬top(c, p), ¬on(c, d), top(d, p)

- Planning domain: language & operators
- Operators corresponds to a set of state-transition systems



Planning Problems

Given a planning domain (language *L*, operators *O*)

- *Statement* of a planning problem: a triple $P=(O,s_0,g)$
  - *O* is the collection of operators
  - s_0 is a state (the initial state)
  - *g* is a set of literals (the goal formula)
- The actual *planning problem*: $\mathcal{P} = (\Sigma, s_0, g)$
  - s_0 and g are as above
  - $\Sigma = (S,A,\gamma)$ is a state-transition system
  - S = {all sets of ground atoms in L}
  - A = {all ground instances of operators in O}
  - γ = state-transition function determined by the operators

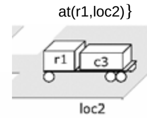
Plans and Solutions

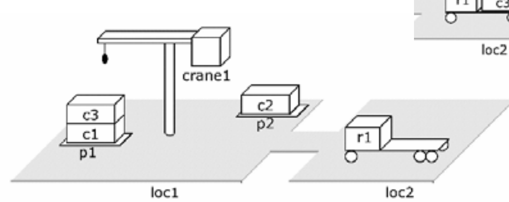
- *Plan*: any sequence of actions $\sigma = \langle a_1, a_2, \dots, a_n\rangle$ such that each a_i is a ground instance of an operator in *O*
- The plan is a *solution* for $P=(O,s_0,g)$ if it is executable and achieves *g*
  - i.e., if there are states $s_0, s_1, \dots, s_n$ such that
    - $\gamma(s_0,a_1) = s_1$
    - $\gamma(s_1,a_2) = s_2$
    - …
    - $\gamma(s_{n-1},a_n) = s_n$
    - s_n satisfies g

Example

- Let $P_1 = (O, s_1, g_1)$, where
  - *O* is the set of operators given earlier

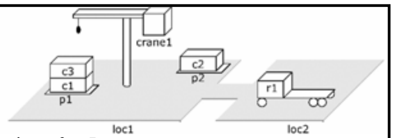
g_1={loaded(r1,c3), at(r1,loc2)}



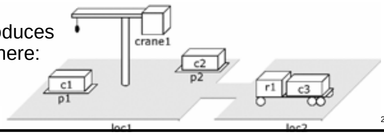


s_1={attached(p1,loc1), in(c1,p1), in(c3,p1), top(c3,p1), on(c3,c1), on(c1,pallet), attached(p2,loc1), in(c2,p2), top(c2,p2), on(c2,pallet), belong(crane1,loc1), empty(crane1), adjacent(loc1,loc2), adjacent(loc2,loc1), at(r1,loc2), occupied(loc2), unloaded(r1)}.

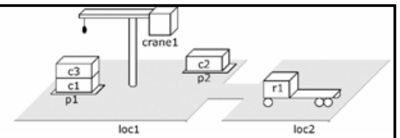
Example (cont.)



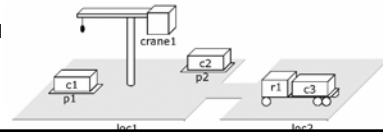
- Here are three solutions for P_1:
  - ⟨take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), move(r1,loc1,loc2), move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
  - ⟨take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
  - ⟨move(r1,loc2,loc1), take(crane1,loc1,c3,c1,p1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
- Each of them produces the state shown here:



Example (cont.)

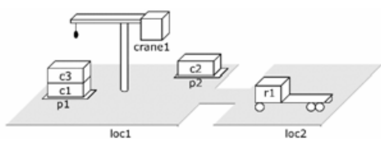


- First is *redundant*: can remove actions and still have a solution
  - ⟨take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), move(r1,loc1,loc2), move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
  - ⟨take(crane1,loc1,c3,c1,p1), move(r1,loc2,loc1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
  - ⟨move(r1,loc2,loc1), take(crane1,loc1,c3,c1,p1), load(crane1,loc1,c3,r1), move(r1,loc1,loc2)⟩
- The 2nd and 3rd are *irredundant* and *shortest*



2. Set-Theoretic Representation

Like classical representation, but restricted to propositional logic



- States:
  - Instead of a collection of ground atoms …

    {on(c1,pallet), on(c1,r1), on(c1,c2), …, at(r1,l1), at(r1,l2), …}

  … use a collection of propositions (boolean variables):

    {on-c1-pallet, on-c1-r1, on-c1-c2, …, at-r1-l1, at-r1-l2, …}

Instead of operators like this one,

take(k,l,c,d,p)
;; crane k at location l takes c off of d in pile p
precond: belong(k,l), attached(p,l), empty(k), top(c,p), on(c,d)
effects: holding(k,c), ¬empty(k), ¬in(c,p), ¬top(c,p), ¬on(c,d), top(d,p)

take all of the operator instances,

e.g., this one

take(crane1,loc1,c3,c1,p1)
;; crane crane1 at location loc1 takes c3 off c1 in pile p1
precond: belong(crane1,loc1), attached(p1,loc1), empty(crane1), top(c3,p1), on(c3,c1)
effects: holding(crane1,c3), ¬empty(crane1), ¬in(c3,p1), ¬top(c3,p1), ¬on(c3,c1), top(c1,p1)

and rewrite ground atoms as propositions

take-crane1-loc1-c3-c1-p1
precond: belong-crane1-loc1, attached-p1-loc1, empty-crane1, top-c3-p1, on-c3-c1
delete: empty-crane1, in-c3-p1, top-c3-p1, on-c3-p1
add: holding-crane1-c3, top-c1-p1

Comparison

A set-theoretic representation is equivalent to a classical representation in which all of the atoms are ground

Problem: Exponential blowup

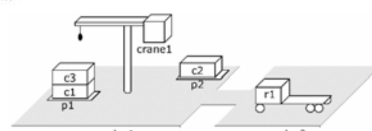
- If a classical operator contains *n* atoms and each atom has arity *k*, then it corresponds to c^{nk} actions where *c* = |{constant symbols}|

3. State-Variable Representation

- Non-fluents (properties that don't change) are ground relations:
  e.g., adjacent(loc1,loc2)
- Fluents are functions:
  i.e., for properties that can change, assign values to *state variables*
- Classical and state-variable rep'ns take similar amounts of space
  each can be translated into the other in low-order polynomial time

move(r,l,m)
;; robot r at location l moves to an adjacent location m
precond: rloc(r) = l, adjacent(l,m)
effects: rloc(r) ← m

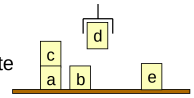
{top(p1)=c3,
cpos(c3)=c1,
cpos(c1)=pallet,
holding(crane1)=nil,
rloc(r1)=loc2,
loaded(r1)=nil, …}



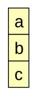
Example: The Blocks World

- Infinitely wide table, finite number of children's blocks
- Ignore where a block is located on the table
- A block can sit on the table or on another block
- Want to move blocks from one configuration to another
  - e.g.,

initial state



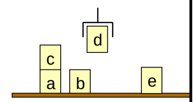
goal



- Classical, set-theoretic, and state-variable formulations for the case of FIVE BLOCKS follow.

1. Example Classical Representation

- Constant symbols:
  - The blocks: a, b, c, d, e
- Predicates:
  - ontable(*x*) - block *x* is on the table
  - on(*x*,*y*) - block *x* is on block *y*
  - clear(*x*) - block *x* has nothing on it
  - holding(*x*) - the robot hand is holding block *x*
  - handempty - the robot hand isn't holding anything



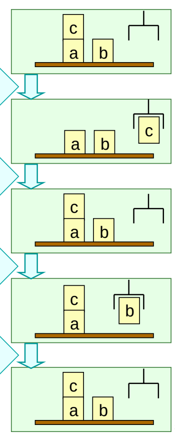
Classical Operators

unstack(*x*,*y*)
Precond: on(*x*,*y*), clear(*x*), handempty
Effects: ~on(*x*,*y*), ~clear(*x*), ~handempty, holding(*x*), clear(*y*)

stack(*x*,*y*)
Precond: holding(*x*), clear(*y*)
Effects: ~holding(*x*), ~clear(*y*), on(*x*,*y*), clear(*x*), handempty

pickup(*x*)
Precond: ontable(*x*), clear(*x*), handempty
Effects: ~ontable(*x*), ~clear(*x*), ~handempty, holding(*x*)

putdown(*x*)
Precond: holding(*x*)
Effects: ~holding(*x*), ontable(*x*), clear(*x*), handempty

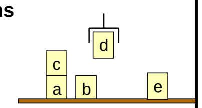


2. Example Set-Theoretic Representation

For five blocks, **36 propositions, 50 actions**

E.g.,

ontable-a - block a is on the table
on-c-a - block c is on block a
clear-c - block c has nothing on it
holding-d - the robot hand is holding block d
handempty - the robot hand isn't holding anything
… (31 more)



Set-Theoretic Actions

E.g.,

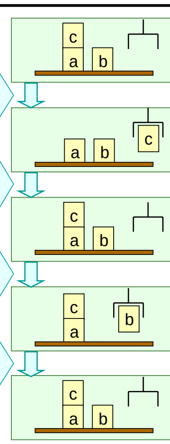
unstack-c-a
Pre: on-c,a, clear-c, handempty
Del: on-c,a, clear-c, handempty
Add: holding-c, clear-a

stack-c-a
Pre: holding-c, clear-a
Del: holding-c, clear-a
Add: on-c-a, clear-c, handempty

pickup-c
Pre: ontable-c, clear-c, handempty
Del: ontable-c, clear-c, handempty
Add: holding-c

putdown-c
Pre: holding-c
Del: holding-c
Add: ontable-c, clear-c, handempty

. . . (46 more)



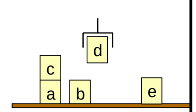
3. Example State-Variable Representation

- Constant symbols:

a, b, c, d, e of type block
0, 1, table, nil of type other

- State variables:

pos(*x*) = y if block *x* is on block *y*
pos(*x*) = table if block *x* is on the table
pos(*x*) = nil if block *x* is being held
clear(*x*) = 1 if block *x* has nothing on it
clear(*x*) = 0 if block *x* is being held or has a block on it
holding = *x* if the robot hand is holding block *x*
holding = nil if the robot hand is holding nothing



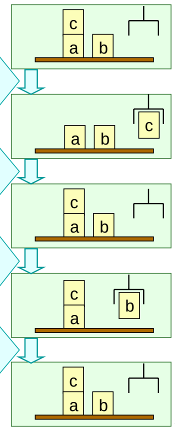
State-Variable Operators

unstack(x : block, y : block)
Precond: pos(x)=y, clear(y)=0, clear(x)=1, holding=nil
Effects: pos(x)=nil, clear(x)=0, holding=x, clear(y)=1

stack(x : block, y : block)
Precond: holding=x, clear(x)=0, clear(y)=1
Effects: holding=nil, clear(y)=0, pos(x)=y, clear(x)=1

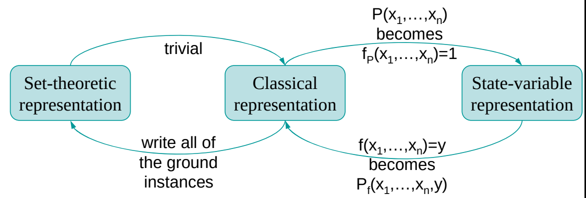
pickup(x : block)
Precond: pos(x)=table, clear(x)=1, holding=nil
Effects: pos(x)=nil, clear(x)=0, holding=x

putdown(x : block)
Precond: holding=x
Effects: holding=nil, pos(x)=table, clear(x)=1



Representational Equivalence

- Any problem that can be represented in one representation can also be represented in the other two
- Can convert in linear time and space, except when converting to set-theoretic (where we get an exponential blowup)



Comparison

- Classical representation
  - Most popular for classical planning, basis of PDDL
- Set-theoretic representation
  - Can take much more space than classical representation
  - Useful in algorithms that manipulate ground atoms directly
    - e.g., planning graphs, SAT
  - Useful for certain kinds of theoretical studies
- State-variable representation
  - Equivalent to classical representation in expressive power
  - Less natural for logicians, more natural for engineers
  - Useful in non-classical planning problems as a way to handle numbers, functions, time

Richer Specification Languages: ADL

- The previous representations were so-called “STRIPS” representations.
- ADL is a richer, and thus more compact, representation language that allows for
  - Disjunction and Quantification in *preconditions* and *goals*
  - *Effects* that are Quantified, and/or Conditional (effect is conditioned on state)
- PDDL supports STRIPS and ADL, but not all planners support ADL, and not all planners even support a so-called Classical Representation

Compiling to Canonical Action Rep’n

PROS & CONS:

It is possible to compile down ADL actions into STRIPS actions
- Quantification is written as conjunctions/disjunctions over finite universes
- Actions with conditional effects are compiled into multiple (exponentially more) actions without conditional effects
- Actions with disjunctive effects are compiled into multiple actions, each of which take one of the disjuncts as their preconditions
- (Domain axioms can be compiled down into the individual effects of the actions; so all actions satisfy STRIPS assumption)

Compilation is not always a win-win.
- By compiling down to canonical form, we can concentrate on highly efficient planning for canonical actions
  - However, often compilation leads to an exponential blowup and makes it harder to exploit the structure of the domain
- By leaving actions in non-canonical form, we can often do more compact encoding of the domains as well as more efficient search
  - However, we will have to continually extend planning algorithms to handle these representations