Compilation of Planning to SAT

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Motivation

- **Propositional SAT: Given a Boolean formula**
  - e.g., $(P \lor Q) \land (\neg Q \lor R \lor S) \land (\neg R \lor \neg P)$,

  does there exist a model?

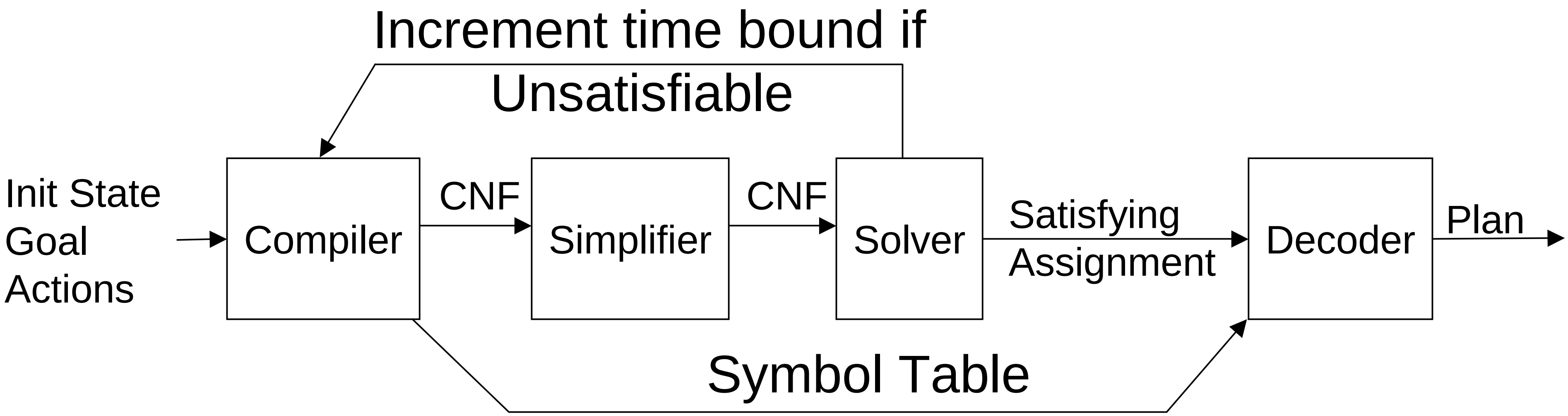
  - i.e., an assignment of truth values to the propositions

  that makes the formula true?
- **Lots of research on algorithms for solving it**
  - This was the very first problem shown to be NP-complete
- **IDEA:**
  - Translate classical planning problems into satisfiability problems, and solving them using highly optimized SATsolvers

Outline

- **Architecture of SAT-based planning**
- **SAT-based planning approach**
- **Encoding planning problems as SAT problems**
- **Making encodings more efficient**
- **Extracting a plan**
- **Satisfiability algorithms**
  - Systematic SAT Solvers: Davis-Putnam-Logemann-Loveland
  - Stochastic SAT Solvers: GSAT
- **Discussion**

Architecture of SAT-based planning system



Architecture of SAT-based planning system Cont.

- **Compiler**
  - take a planning problem as input, guess a plan length, and generate a propositional logic formula, which if satisfied, implies the existence of a solution plan
- **Symbol table**
  - record the correspondence between propositional variables and the planning instance
- **Simplifier**
  - use fast techniques such as unit clause propagation and pure literal elimination to shrink the CNF formula
- **Solver**
  - use systematic or stochastic methods to find a satisfying assignment. If the formula is unsatisfiable, then the compiler generates a new encoding reflecting a longer plan length
- **Decoder**
  - translate the result of solver into a solution plan.

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Planning Problem Definition

- **Define what a planning problem is**
  - Initial State
    Describes the facts that hold and do not hold in initial state
  - Goal State
    Describes the facts that much hold in goal state
  - Transition function γ: S x A -> S
    - S: Sets of states
    - A: Set of actions
    - γ is encoded in terms of actions' preconditions and effects, and exclusion axioms
- **Bounded planning problem (*P*,*n*):**
  - *P* is a planning problem; *n* is a positive integer
  - Find a solution for *P* of length <= *n*
    - $<a_0, a_1, \ldots, a_{n-1}>$ is a solution for (*P*,*n*),
    - Plan length not known in advance => the approach needs to repeat for different tentative lengths

SAT-based Planning Approach

- **Do iterative deepening:**
  - for $n = 0, 1, 2, \ldots,$
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, a solution plan can be constructed, so return it and exit

Fluents

- **If π = <a_0, a_1, …, a_{n-1}> is a solution for (P,n), then it generates the following states:**

  s_0, $s_1 = \gamma(s_0, a_0)$, $s_2 = \gamma(s_1, a_1)$, …, $s_n = \gamma(s_{n-1}, a_{n-1})$

- ***Fluents*: propositions that describe what's true in each s_i**
  - at(r1,loc1,i) is a fluent that's true iff at(r1,loc1) is in s_i
  - We'll use l_i to denote the fluent for literal l in state s_i
    - e.g., if l = at(r1,loc1)
      then l_i = at(r1,loc1,i)
  - a_i is a fluent saying that a is the i'th step of π
    - e.g., if a = move(r1,loc2,loc1)
      then a_i = move(r1,loc2,loc1,i)

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What is in Φ?

- **Formula describing the initial state:**

  $\bigwedge\{l_0 \mid l \in s_0\} \wedge \bigwedge\{\neg l_0 \mid l \in L - s_0\}$

- **Formula describing the goal state:**

  $\bigwedge\{l_n \mid l \in g^+\} \wedge \bigwedge\{\neg l_n \mid l \in g^-\}$

- **Formulas describing the preconditions and effects of actions:**

  For every action *a* in *A*, formulas describing what changes *a* would make if it were the *i*'th step of the plan:

  - $a_i \Rightarrow \bigwedge\{p_i \mid p \in \text{Precond}(a)\} \wedge \bigwedge\{e_{i+1} \mid e \in \text{Effects}(a)\}$

- **Formulas describing *Complete exclusion*:**
  - For all actions *a* and *b*, formulas saying they cannot occur at the same time

    $\neg a_i \vee \neg b_i$
  - this guarantees there can be only one action at a time

- **Formulas providing a solution to the Frame Problem**

Example

- **Planning domain:**
  - one robot r1
  - two adjacent locations l1, l2
  - one action (move the robot)

- **Encode (*P*,*n*) where *n* = 1**

  - Initial state: {at(r1,l1)}

    Encoding: $at(r1,l1,0) \wedge \neg at(r1,l2,0)$

  - Goal: {at(r1,l2)}

    Encoding: $at(r1,l2,1) \wedge \neg at(r1,l1,1)$

Example (continued)

- **Action:** **move(r,l1,l2)**

  precond: at(r,l1)

  effects: at(r,l2), $\neg$at(r,l1)

  Encoding:

  move(r1,l1,l2,0) $\Rightarrow$ at(r1,l1,0) $\wedge$ at(r1,l2,1) $\wedge$ $\neg$at(r1,l1,1)

  move(r1,l2,l1,0) $\Rightarrow$ at(r1,l2,0) $\wedge$ at(r1,l1,1) $\wedge$ $\neg$at(r1,l2,1)

- **Complete-exclusion axiom:**

  $\neg$move(r1,l1,l2,0) $\vee$ $\neg$move(r1,l2,l1,0)

- **Explanatory frame axioms:**

  $\neg$at(r1,l1,0) $\wedge$ at(r1,l1,1) $\Rightarrow$ move(r1,l2,l1,0)

  $\neg$at(r1,l2,0) $\wedge$ at(r1,l2,1) $\Rightarrow$ move(r1,l1,l2,0)

  at(r1,l1,0) $\wedge$ $\neg$at(r1,l1,1) $\Rightarrow$ move(r1,l1,l2,0)

  at(r1,l2,0) $\wedge$ $\neg$at(r1,l2,1) $\Rightarrow$ move(r1,l2,l1,0)

What are these “Explanatory Frame Axioms” and the “Complete Exclusion Axioms”?

The Frame Problem

The Frame Problem:

Describing what *does not* change between steps i and i+1

Two Common Solutions:

1. Classical Frame Axioms
2. Explanatory frame axioms

1. Classical Frame Axioms

- **Classical frame axioms** (McCarthy & Hayes 1969)
  - State which fluents are unaffected by a given action
  - For each action a, for each fluent not in effects(a), and for each step i, we have: $f_i \wedge a_i \Rightarrow f_{i+1}$
  - Problem: if no action occurs at step i nothing can be inferred about propositions at level i+1
  - Sol: at-least-one axiom: at least one action occurs
  - If more than one action occurs at a step, either one can be selected.

2. Explanatory frame axioms

- **Explanatory frame axioms** (Haas 1987)
  - Enumerate the set of actions that could have occurred in order to account for a state change.
  - Says that if f changes between s_i and s_{i+1}, then the action at step i must be responsible:

$$(\neg f_i \wedge f_{i+1} \Rightarrow \bigvee\{a_i \mid f \in \text{effects}^+(a)\}) \wedge (f_i \wedge \neg f_{i+1} \Rightarrow \bigvee\{a_i \mid l \in \text{effects}^-(a)\})$$

  - Example:

$\neg$at(r1,l1,0) $\wedge$ at(r1,l1,1) $\Rightarrow$ move(r1,l2,l1,0)

$\neg$at(r1,l2,0) $\wedge$ at(r1,l2,1) $\Rightarrow$ move(r1,l1,l2,0)

at(r1,l1,0) $\wedge$ $\neg$at(r1,l1,1) $\Rightarrow$ move(r1,l1,l2,0)

at(r1,l2,0) $\wedge$ $\neg$at(r1,l2,1) $\Rightarrow$ move(r1,l2,l1,0)

Explanatory frame axioms (cont)

- **Allows parallelism**
  - Two actions can be executed in parallel if
    - Their preconditions are satisfied at time t
    - Their effects do not conflict
  - Gives shorter plans – smaller encoding
- **Uncontrolled parallelism is problematic**
  - Can create valid plans without valid solution
    - Action α has precondition X and effect Y
    - Action β has precondition $\neg$Y and effect $\neg$X

Explanatory frame axioms (cont)

Need Exclusion Axioms

- Complete exclusion axioms – ***totally ordered plan***
  - Only one action occurs at a time

    $$\neg \alpha_t \vee \neg \beta_t$$

- Conflict exclusion axioms – ***partially ordered plan***
  - Two actions conflict if one's precondition is inconsistent with the other's effect
  - Conflict exclusion should be used whenever possible

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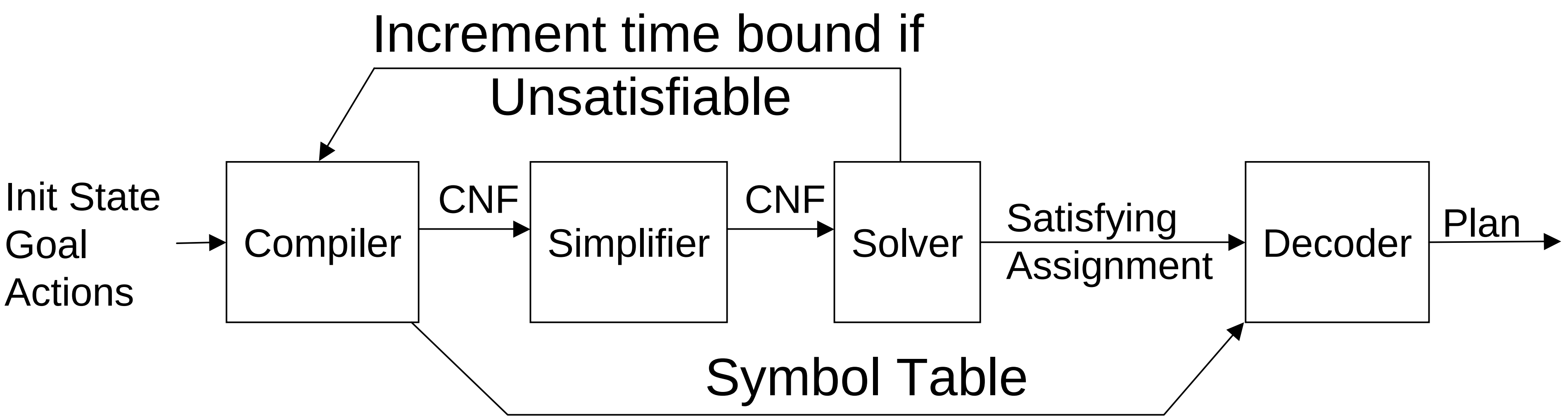


Space of Encodings

- **Want a compiler to quickly produce a small SAT encoding**
  - Number of variables
  - Number of clauses
  - Total number of literals summed over all clauses
- **Two factors determine these sizes:**
  - Encoding
    - Choice of Action Representation
      - Regular, simple split, overloaded split, or bitwise
      - Tradeoff between the number of variables and the number of clauses in the formula
    - Choice of Frame Axioms: classical or explanatory
  - Optimizations being used

Action Encoding

- **Regular**
  - Each ground action is represented by a different logical variable
- **Simple Operator Splitting**
  - Replace each n-ary action proposition with n unary propositions
  - Advantage: instances of each action share the same variable
    - move2(l1,i) is used to represent move(r1,l1,l2,i), can be reused to represent move(r2,l1,l2,i) – represent cases where starting location is the same
- **Overloaded Operator Splitting**
  - Allowing different actions to share the same variable
- **Bitwise**
  - Propositional variables are represented using bits

Action Encoding

[Ernst et al, IJCAI 1997]

Representation	One Propositional Variable per	Example
Regular	fully-instantiated action $n\|F\| + n\|O\|\|D\|^{A0}$	move(r1,l1,l2,i)
Simply-split	fully-instantiated action's argument $n\|F\| + n\|O\|\|D\|A_0$	move1(r1,i) ∧ move1(l1,i) ∧ move1(l2,i)
Overloaded-split	fully-instantiated argument $n\|F\| + n(\|O\|+\|D\|A_{0)}$	Act(move, i) ∧ Act1(r1, i) ∧ Act2(l1, i) ∧ Act3(l2, i)
Bitwise	Binary encodings of actions $n\|F\| + n[\log_2 \|O\|\|D\|^{A0}]$	Bit1

more vars

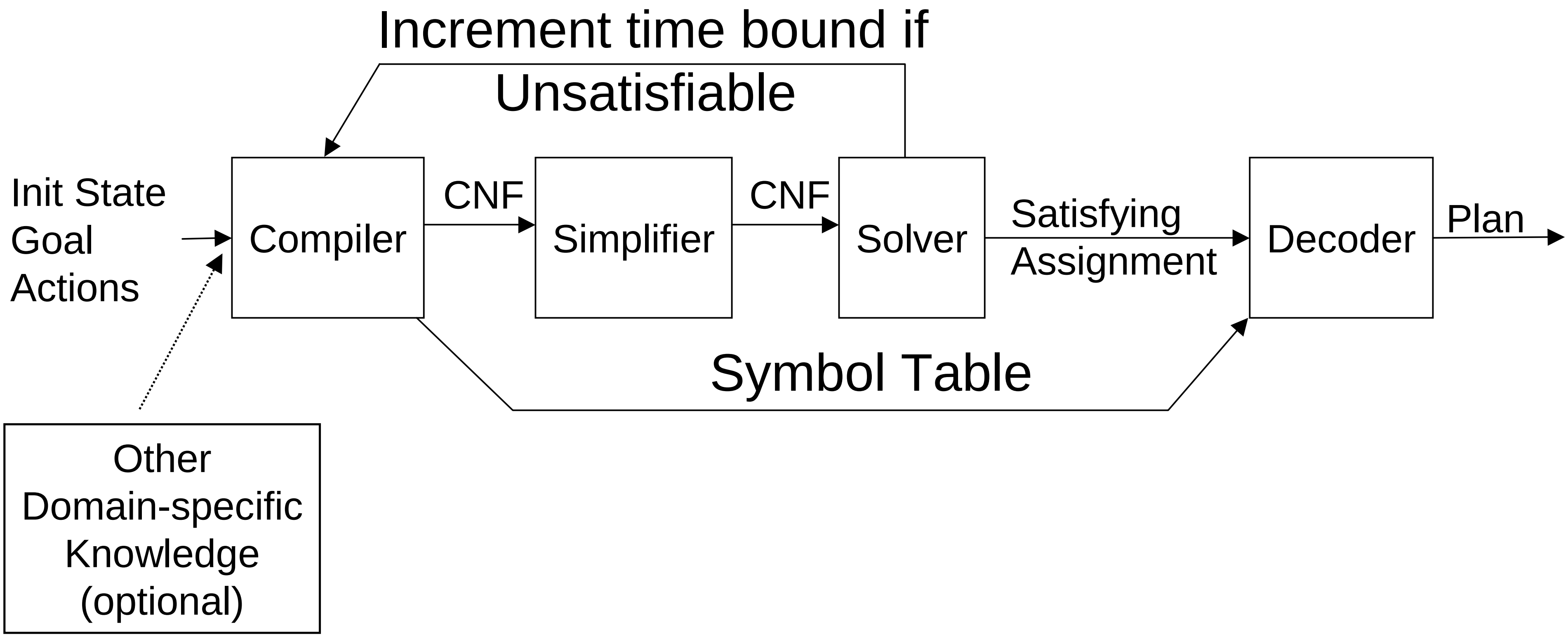
more clauses

N – number of steps; |F| - number of fluents;
|O| - number of operators; A_0 – maximum arity of predicates

Comparisons of Different Encodings

- **Regular explanatory and simple splitting explanatory encodings are the smallest**
  - Explanatory frame axioms are smaller
    - State only what changes, not what does not change
  - Regular explanatory encodings allow for parallel actions
    - Shorter plans
    - Conflict exclusion axioms are a subset of complete exclusion axioms.

Architecture of SAT-based planning system



Optimizations

- Optimize the CNF formula produced by a compiler
  1. **Compile-time optimization**
     - Shrink the size of CNF formula that SAT-compiler generates
  2. Adding **domain-specific information** (e.g., control knowledge)
     - Precondition |= action conflicts, effects |=derived effects
     - State invariant:
       - A truck is at only one location
     - Optimality: disallowing unnecessary subplans
       - Do not return a package to its original location
     - Simplifying assumptions: not logically entailed
       - Once trucks are loaded they should immediately move

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Extracting a Plan

- **Suppose we find an assignment of truth values that satisfies Φ.**
  - This means *P* has a solution of length *n*

- **For *i*=1,…,*n*, there will be exactly one action *a* such that $a_i = true$**
  - This is the *i*'th action of the plan.

- **Example (from the previous slides):**
  - Φ can be satisfied with move(r1,l1,l2,0) = *true*
  - Thus ⟨move(r1,l1,l2,0)⟩ is a solution for (*P*,0)
    - It's the only solution - no other way to satisfy Φ

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SAT Algorithms

- **How to find an assignment of truth values that satisfies Φ?**
  - Use a satisfiability algorithm
- **Systematic Search**
  - E.g., DP (Davis Putnam Logemann Loveland)
    backtrack search + unit propagation
- **Local Search**
  - E.g., GSAT (Selman), Walksat (Selman, Kautz & Cohen)
    greedy local search + noise to escape minima

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Discussion

- **Recall the overall approach:**
  - for $n = 0, 1, 2, \ldots,$
    - encode (P,n) as a satisfiability problem Φ
    - if Φ is satisfiable, then
      - From the set of truth values that satisfies Φ, extract a solution plan and return it

- **By itself, not very practical (takes too much memory and time)**

- **But it can be combined with other techniques**
  - e.g., planning graphs
  - Blackbox: combines planning-graph expansion and satisfiability checking

Conclusion

- **What SATPLAN shows**
  - General SAT solvers can compete with state of the art specialized planning systems, in fact today's SAT-based planners are among the fastest!!!
- **Why SATPLAN works**
  - More flexible than forward or backward chaining
  - Randomized algorithms less likely to get trapped on bad paths

Acknowledgement

- **I have reused slides from the following two sources:**
  - Open-Loop Planning as Satisfiability by Henry Kautz

    http://www.cs.washington.edu/homes/kautz/talks/tutorial99/openloop.ppt
  - Aussagenlogische Erfllbarkeitstechniken SATPlan by Ulrich Scholz

    http://www.intellektik.informatik.tu-darmstadt.de/~scholz/Vorlesung/07_Satplan_2005_12_08.pdf