Resolution versus Search: Two Strategies for SAT

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presented by Anya Tafliovich

Jumping right in. . .

Ordered Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$

Ordered Resolution

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$

$o_1 : A > B > C > D > E$

(A v B v C) , (¬ A v D), (¬ B v D) , (¬ C) , (C v E)

Ordered Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$

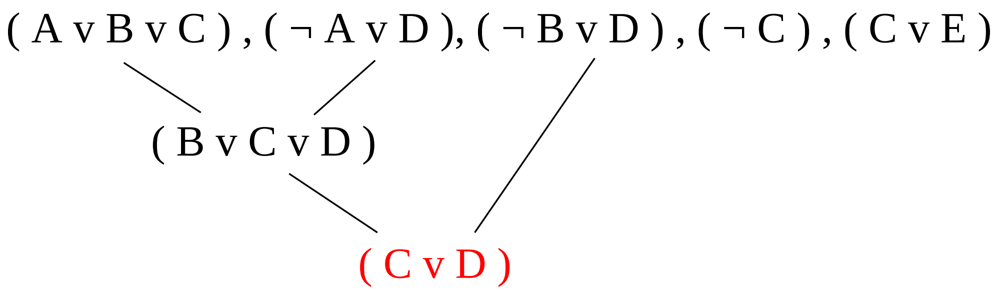
(A v B v C) , (¬ A v D), (¬ B v D) , (¬ C) , (C v E)

(B v C v D)

Ordered Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

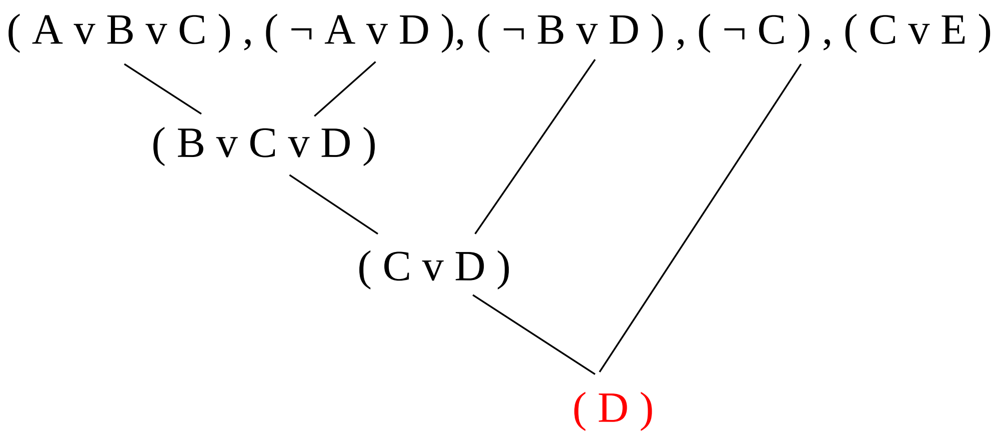
$$o_1 : A > B > C > D > E$$



Ordered Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

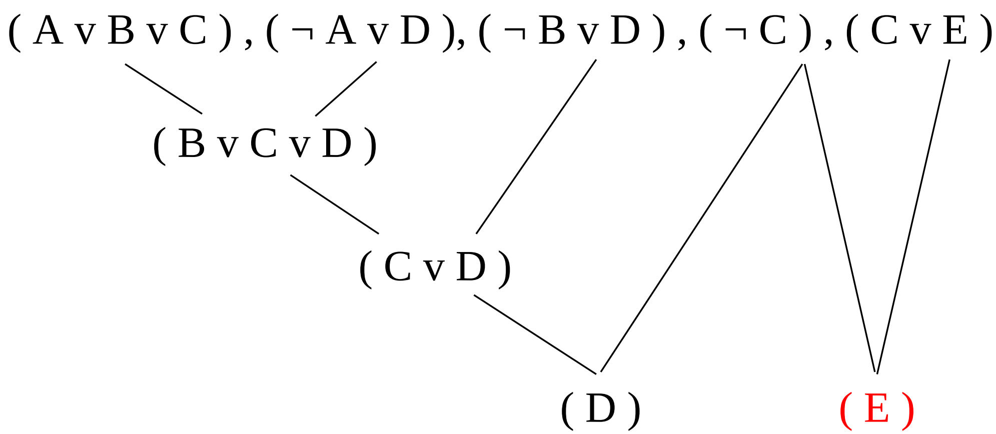
$$o_1 : A > B > C > D > E$$



Ordered Resolution

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$

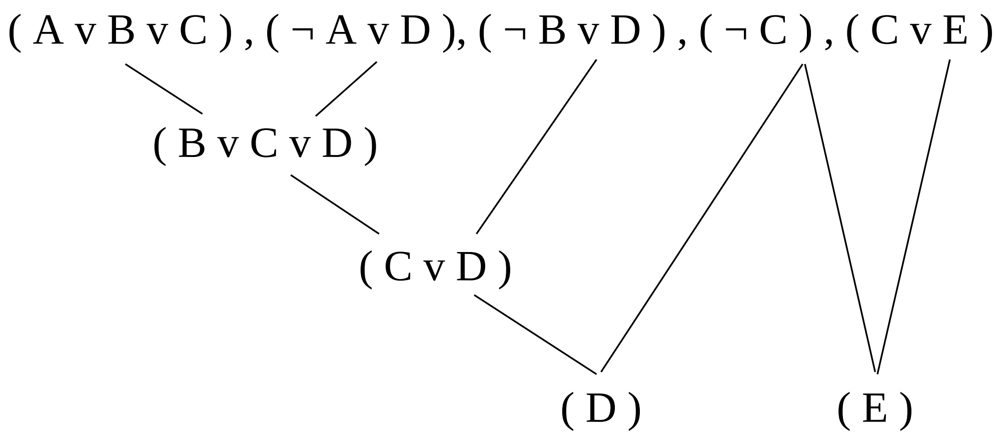
$o_1 : A > B > C > D > E$



Ordered Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$



Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E)\}$$

$$b_D = \{\}$$

$$b_E = \{\}$$

Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$

(A v B v C) , (¬ A v D), (¬ B v D) , (¬ C) , (C v E)

(B v C v D)

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E)\}$$

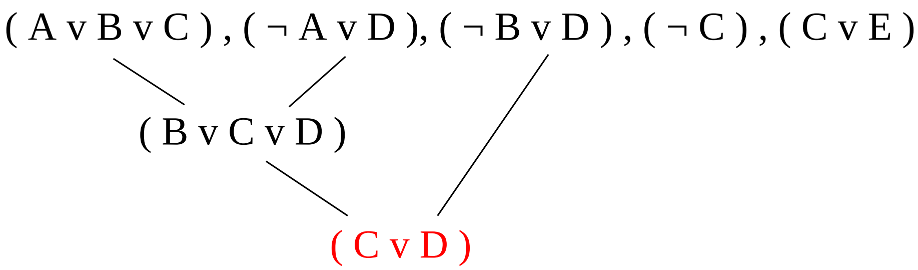
$$b_D = \{\}$$

$$b_E = \{\}$$

Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$



$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

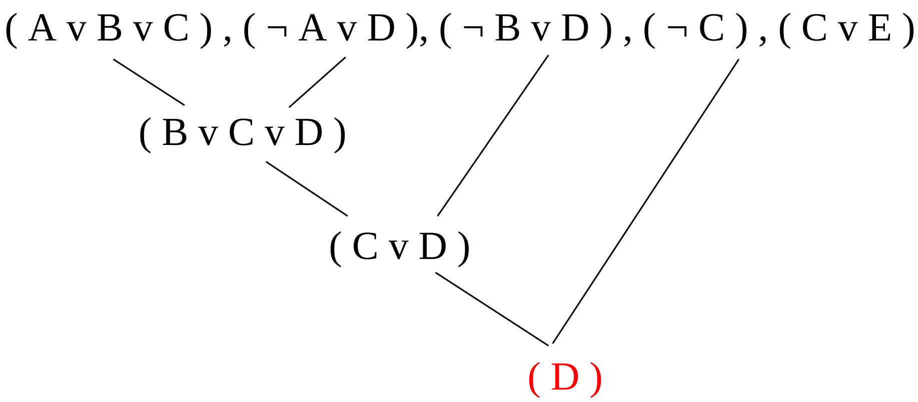
$$b_D = \{\}$$

$$b_E = \{\}$$

Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$



$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

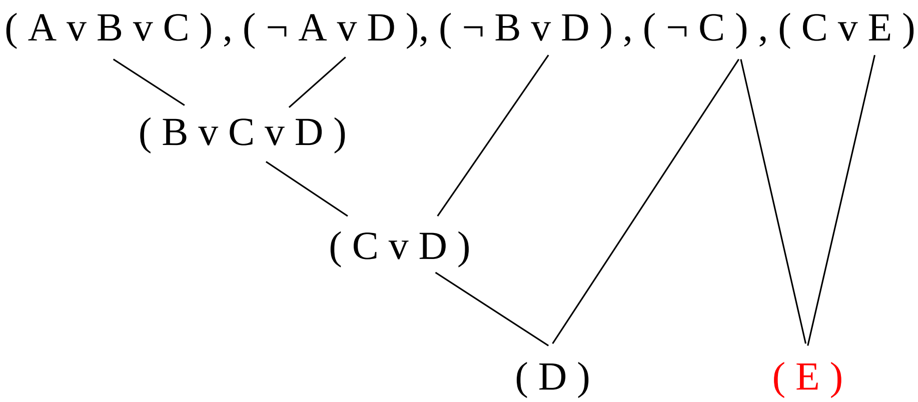
$$b_D = \{(D)\}$$

$$b_E = \{\}$$

Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$



$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

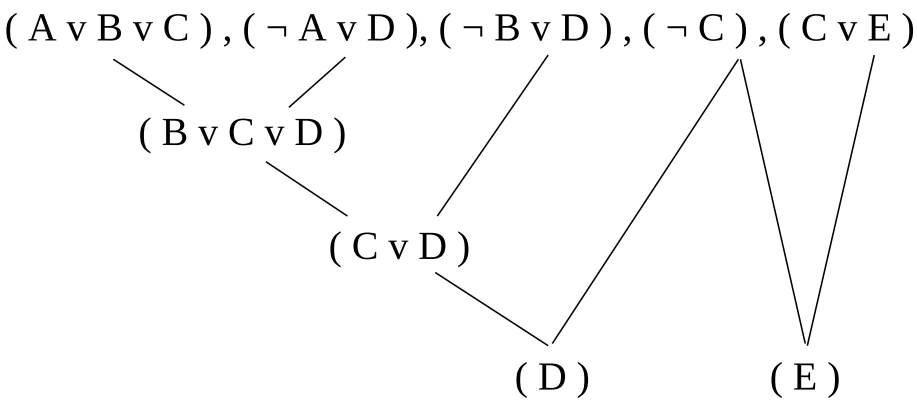
$$b_D = \{(D)\}$$

$$b_E = \{(E)\}$$

Directed Resolution

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E$$



$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

$$b_D = \{(D)\}$$

$$b_E = \{(E)\}$$

$$E_{o_1}(\psi) = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E), (B \vee C \vee D), (C \vee D), (D), (E)\}$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

$$b_D = \{(D)\}$$

$$b_E = \{(E)\}$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

$$b_D = \{(D)\}$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\}$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\}$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad \tau(C) = F$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad \tau(B) = T/F$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad \tau(C) = F$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\}$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad \tau(B) = F$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad \tau(C) = F$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\} \qquad \tau(A) = T$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad \tau(B) = F$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad \tau(C) = F$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

Model finding

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\} \qquad \tau(A) = T$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad \tau(B) = F$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad \tau(C) = F$$

$$b_D = \{(D)\} \qquad \tau(D) = T$$

$$b_E = \{(E)\} \qquad \tau(E) = T$$

DR - importance of orderings

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$

$o_2 : D > A > B > E > C$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$

$$b_A = \{(A \vee B \vee C)\}$$

$$b_B = \{\}$$

$$b_E = \{(C \vee D)\}$$

$$b_C = \{(\neg C)\}$$

DR - importance of orderings

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o_2 : D > A > B > E > C$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$

$$b_A = \{(A \vee B \vee C)\}$$

$$b_B = \{\}$$

$$b_E = \{(C \vee D)\}$$

$$b_C = \{(\neg C)\}$$

$$E_{o_2}(\phi) = \phi$$

DR - importance of orderings

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$

$o_2 : D > A > B > E > C$

$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$	$\tau(D) = T$
$b_A = \{(A \vee B \vee C)\}$	$\tau(A) = F$
$b_B = \{\}$	$\tau(B) = T$
$b_E = \{(C \vee D)\}$	$\tau(E) = T$
$b_C = \{(\neg C)\}$	$\tau(C) = F$

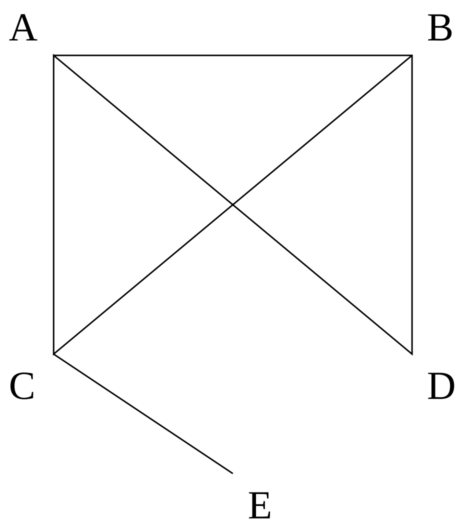
$$E_{o_2}(\phi) = \phi$$

Induced graph

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:

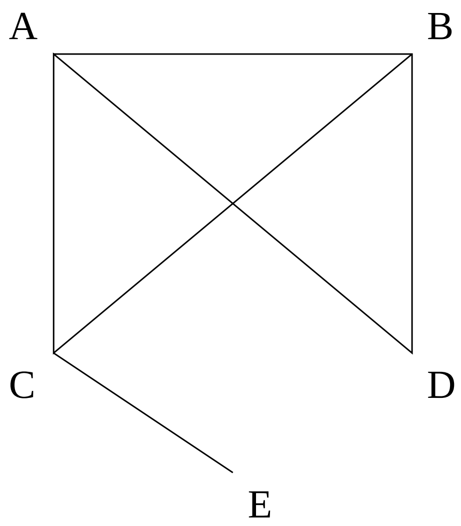


Induced graph

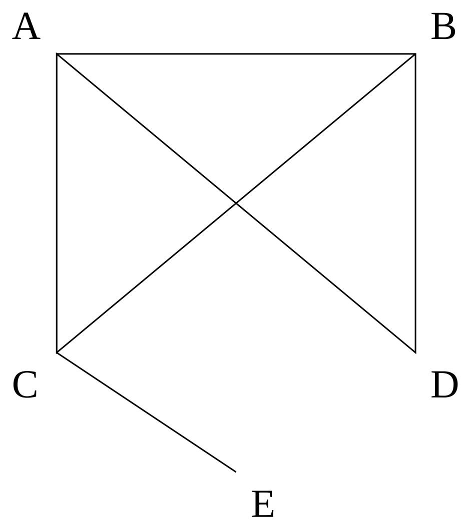
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

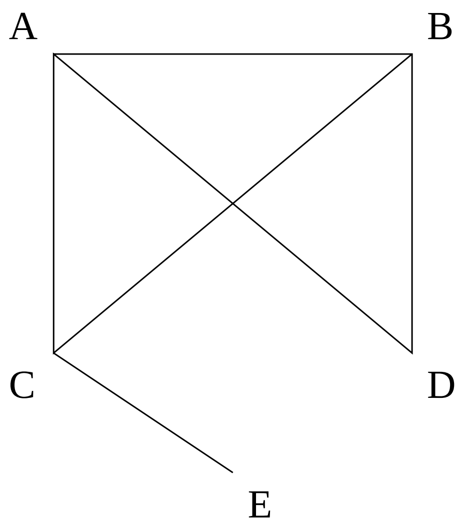


Induced graph

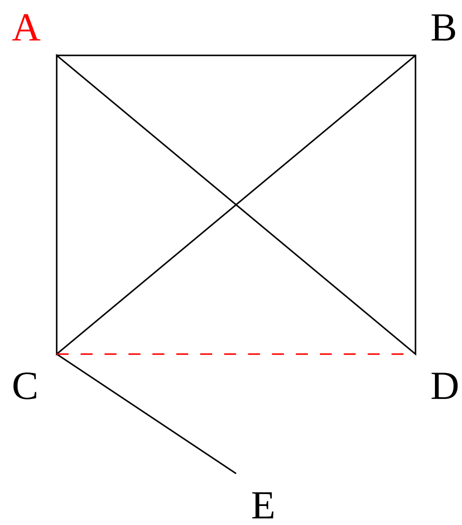
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

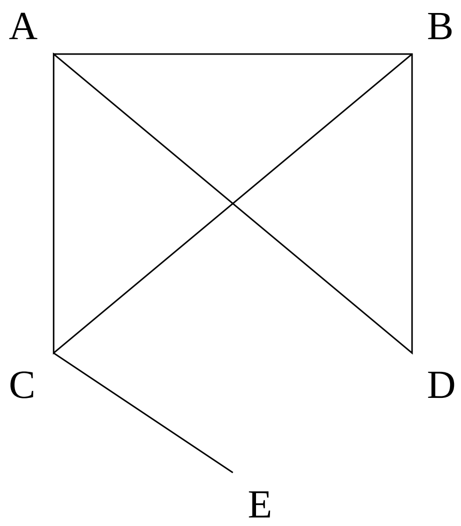


Induced graph

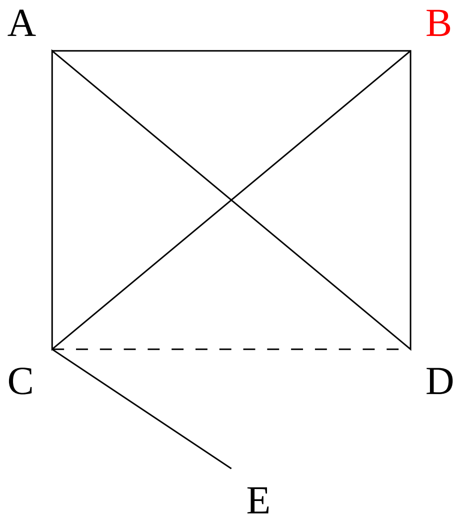
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

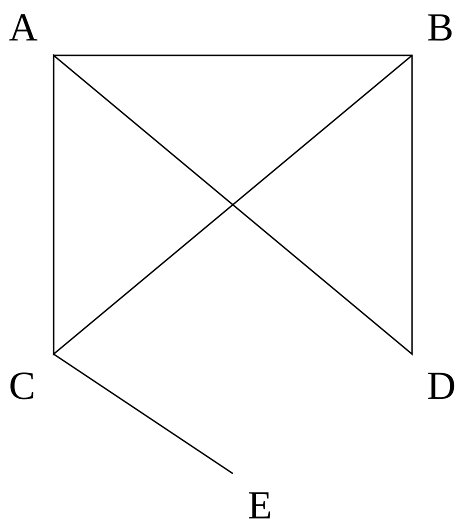


Induced graph

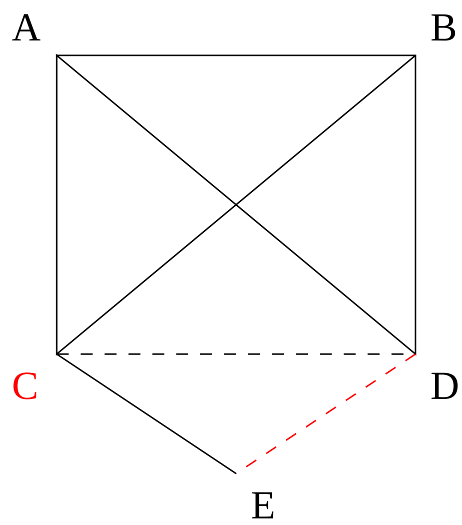
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

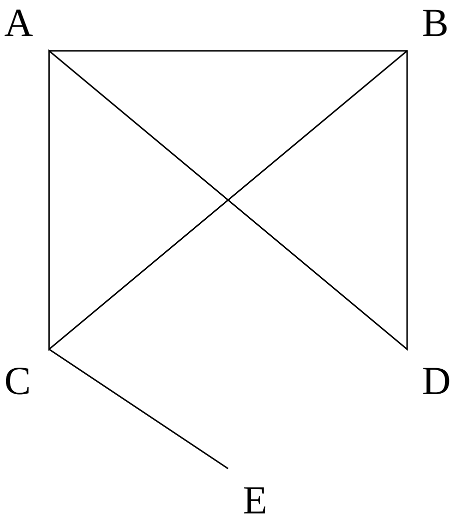


Induced graph

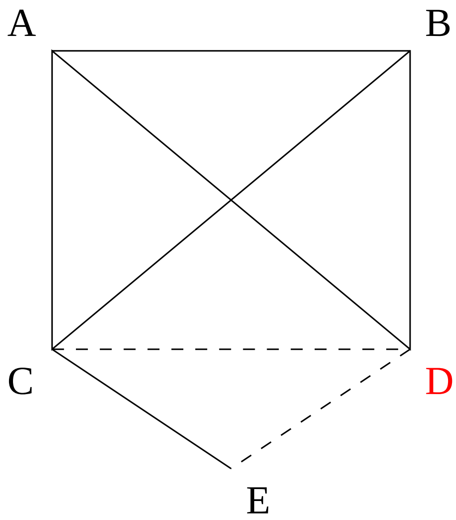
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

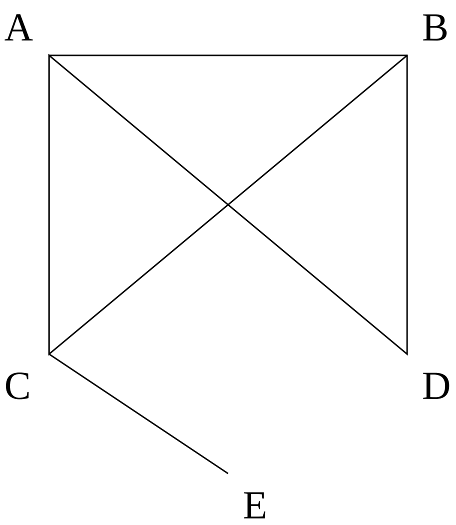


Induced graph

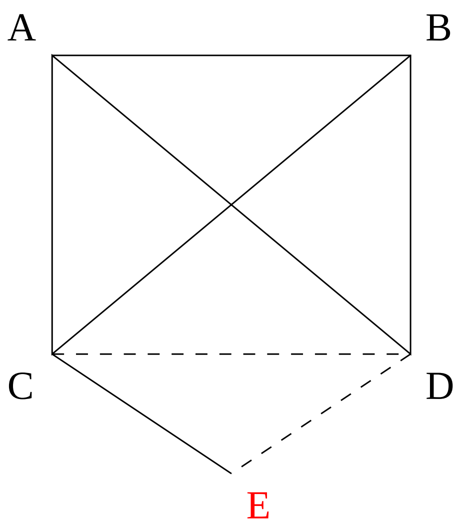
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

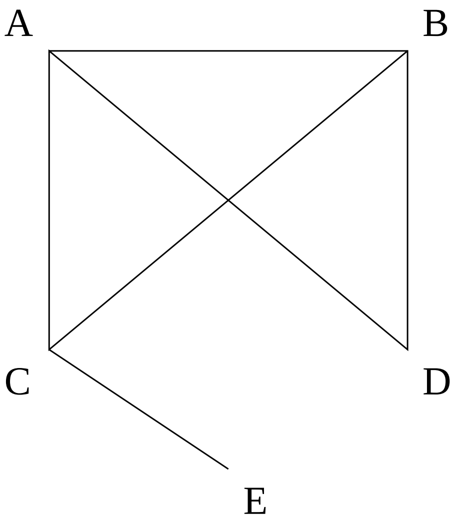


Induced graph

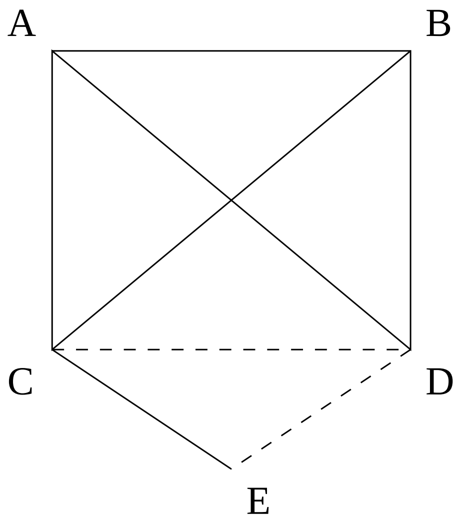
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



$I_{o_1}(G(\phi))$:

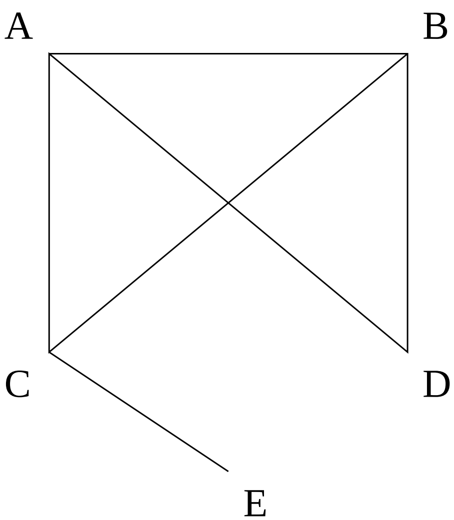


Induced graph

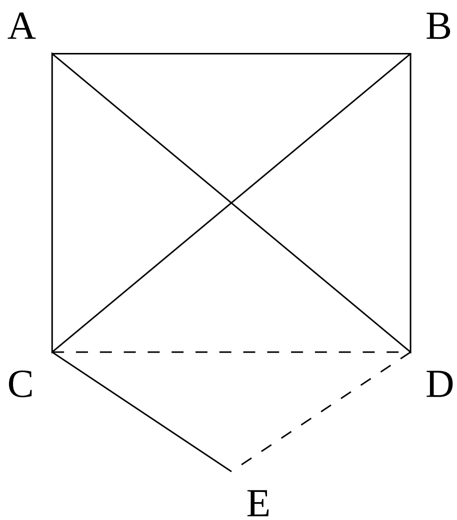
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

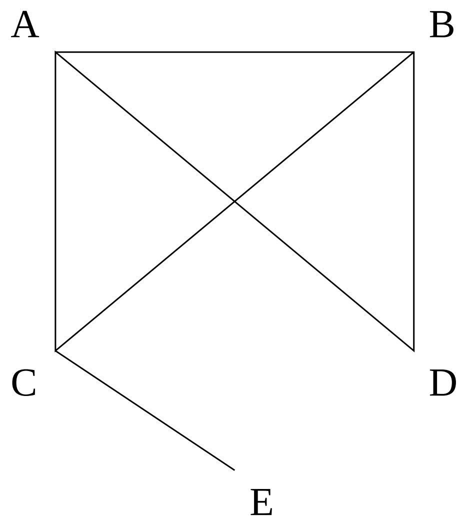
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

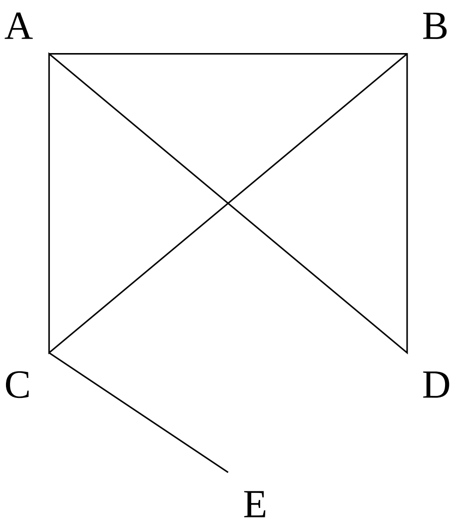


Induced graph

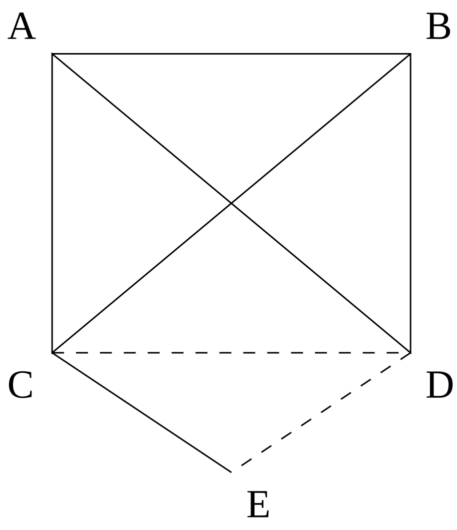
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

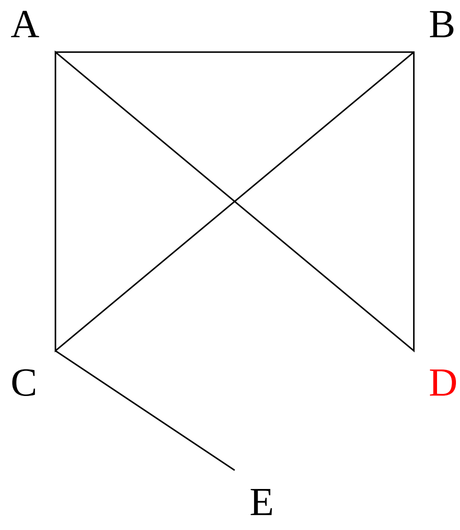
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

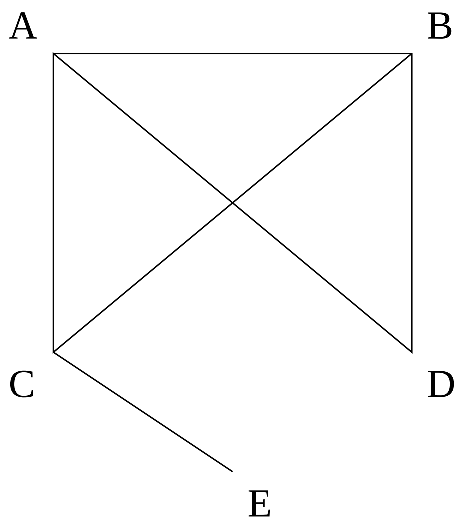


Induced graph

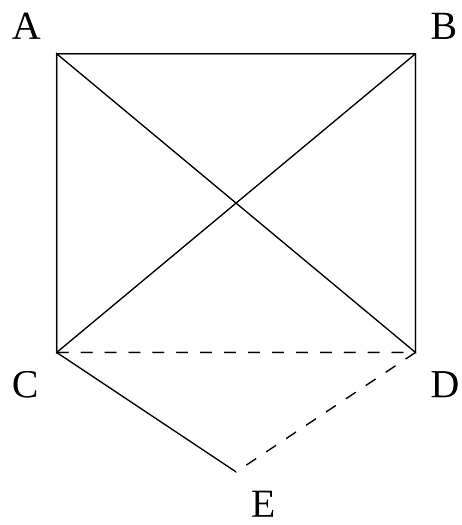
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

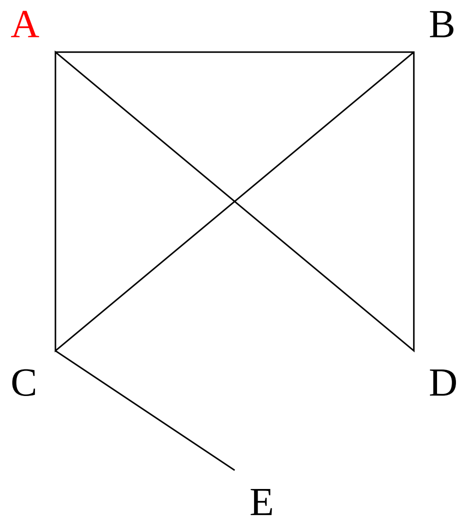
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

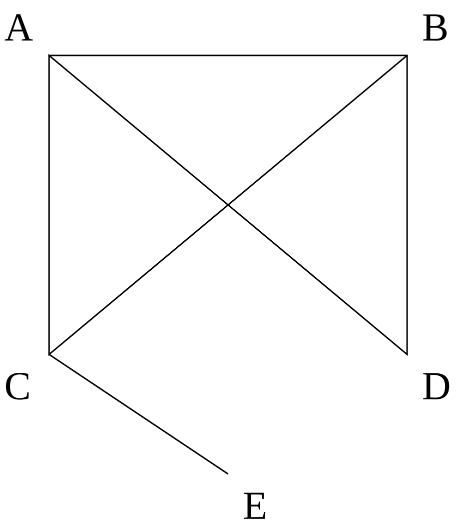


Induced graph

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$:



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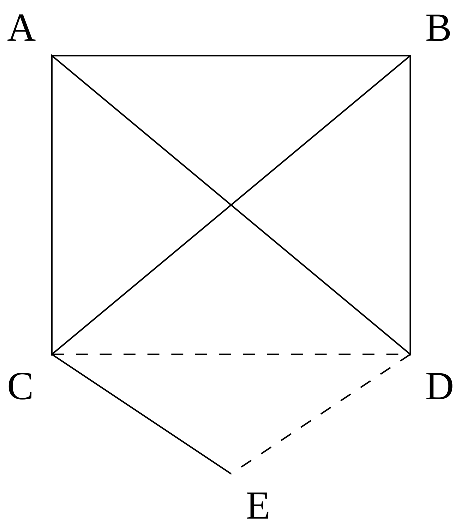
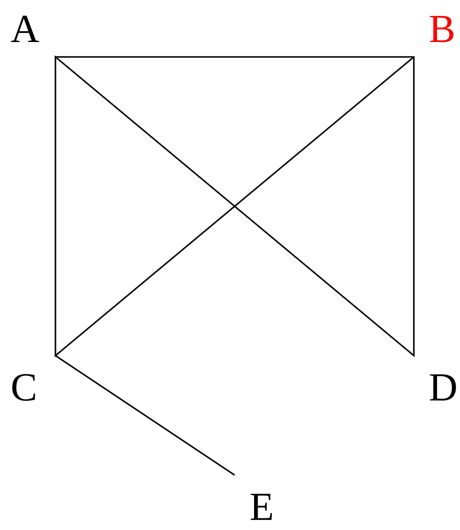


$I_{o_2}(G(\phi))$:

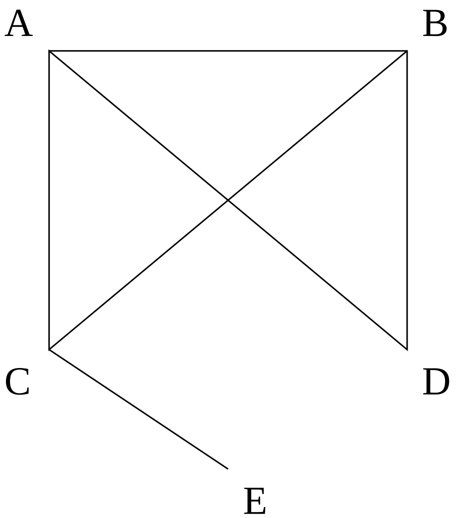


Induced graph

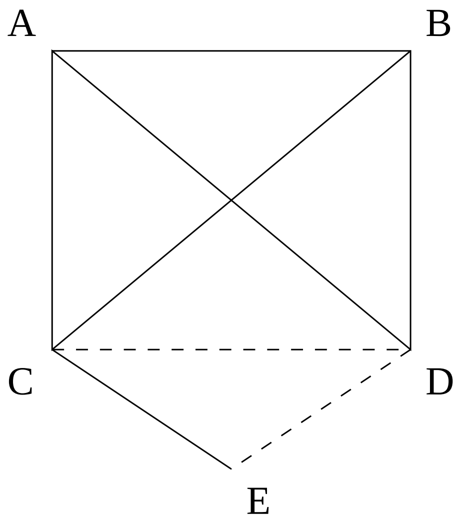
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

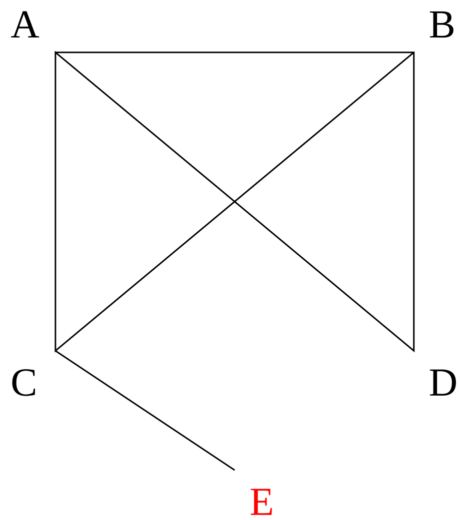
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

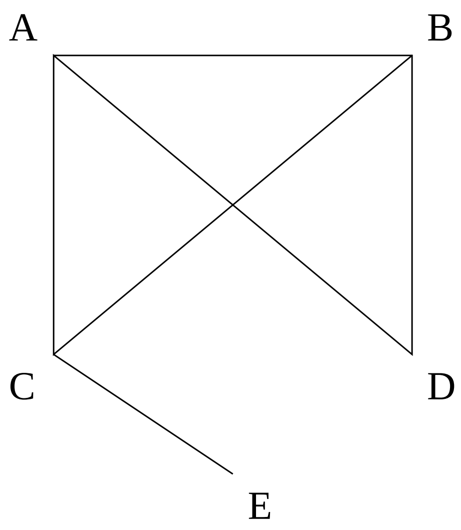


Induced graph

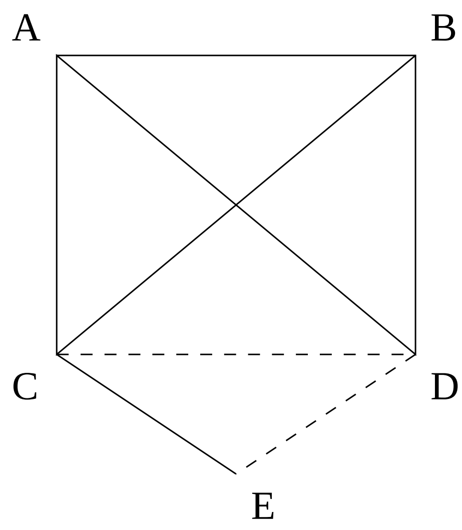
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

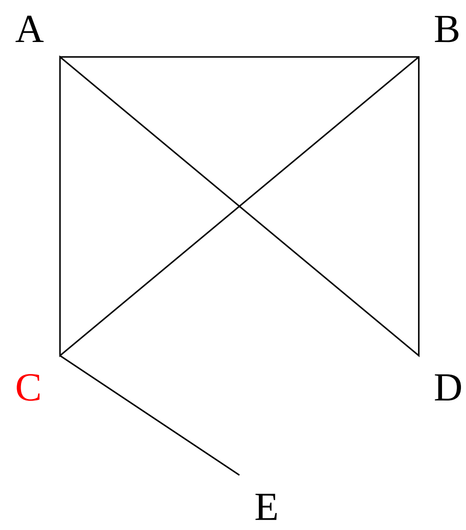
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

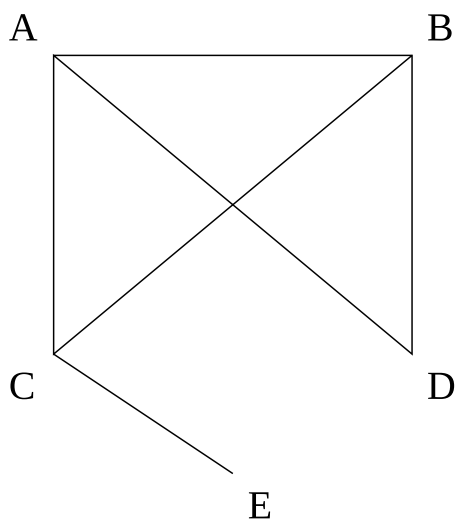


Induced graph

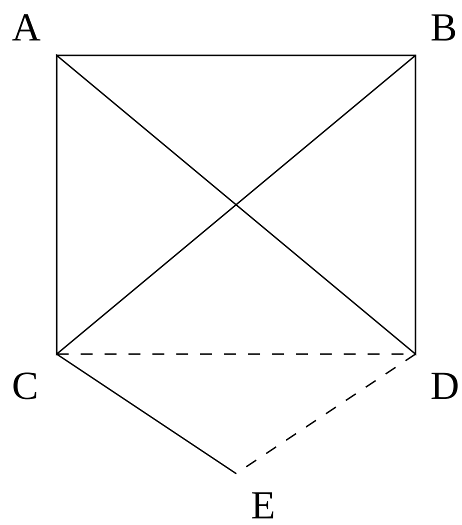
$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

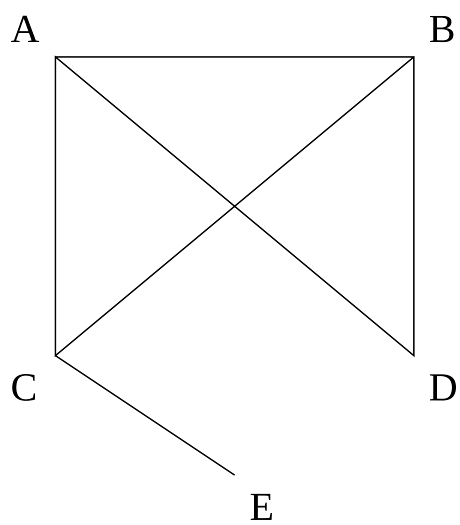
$G(\phi)$:



$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:

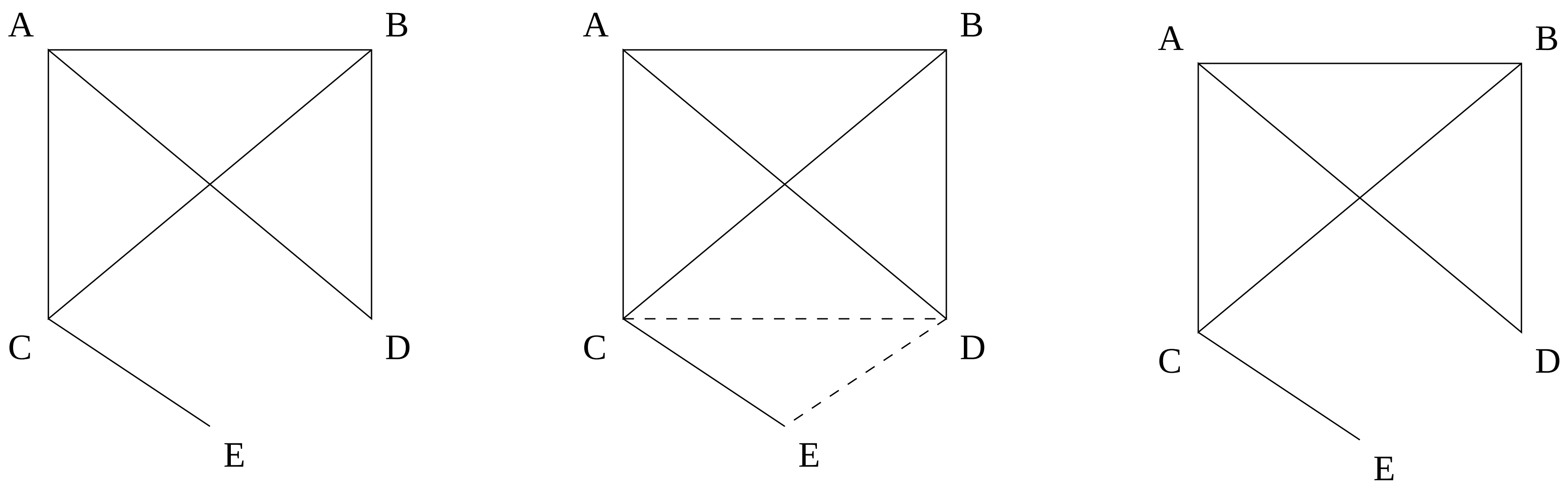


Induced graph

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$G(\phi)$: $\quad I_{o_1}(G(\phi))$: $\quad I_{o_2}(G(\phi))$:



$$G(\phi) = I_{o_2}(G(\phi)) \subset I_{o_1}(G(\phi))$$

Induced graph

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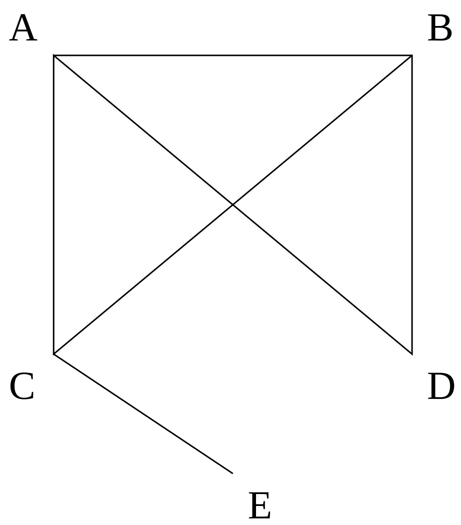
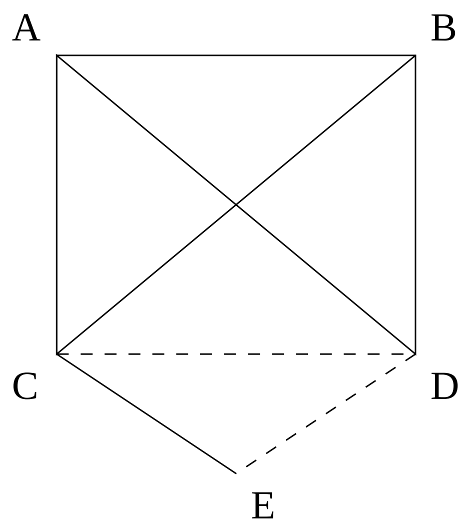
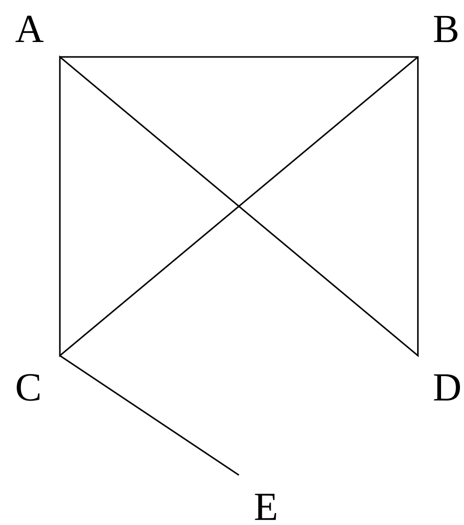


$I_{o_1}(G(\phi))$:



$I_{o_2}(G(\phi))$:



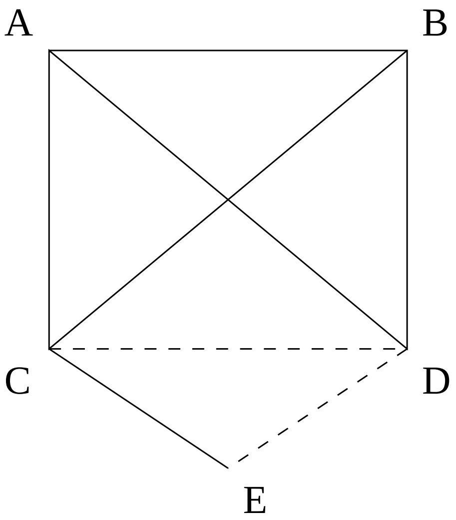
$$G(\phi) = I_{o_2}(G(\phi)) \subset I_{o_1}(G(\phi))$$

In general, $G(\phi) \subseteq I_o(G(\phi))$.

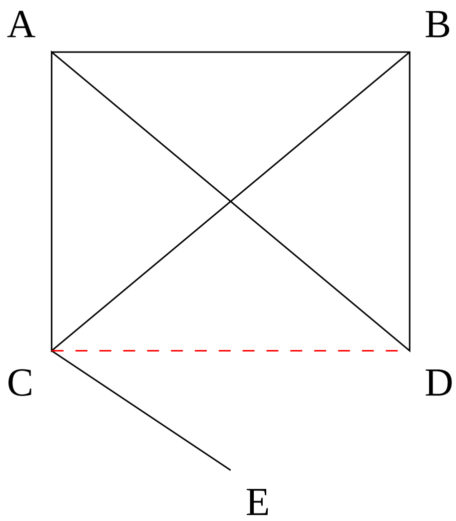
Induced graph vs Extention graph

$$E_{o_1}(\phi) = \{\, (A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), \\ (C \vee E), (B \vee C \vee D), (C \vee D), (D), (E)\}$$

$I_{o_1}(G(\phi))$:



$G(E_{o_1}(\phi))$:

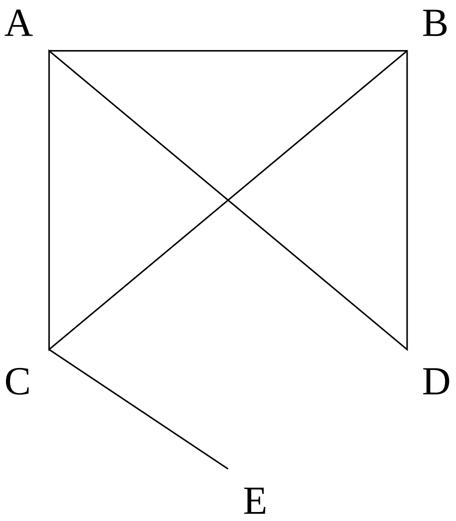


$$G(E_{o_1}(\phi)) \subset I_{o_1}(G(\phi))$$

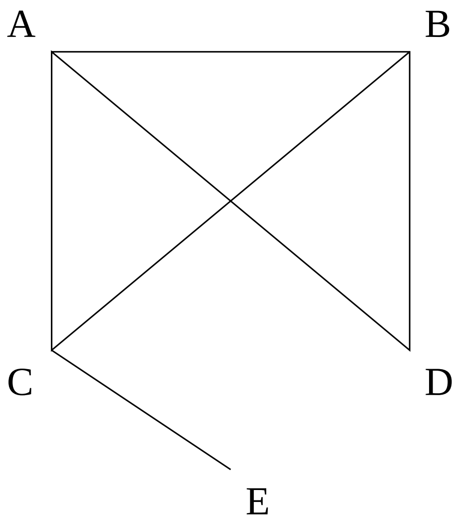
Induced graph vs Extention graph

$$E_{o_2}(\phi) = \phi$$

$I_{o_2}(G(\phi))$:



$G(E_{o_2}(\phi))$:



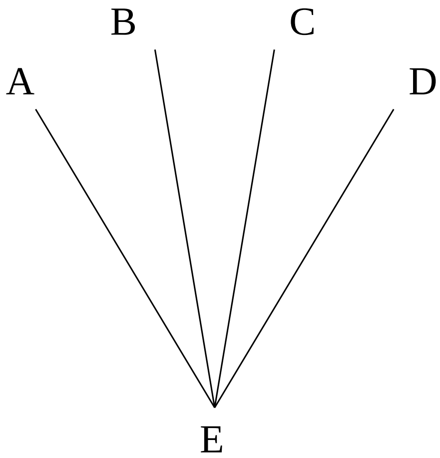
$$G(E_{o_2}(\phi)) = I_{o_2}(G(\phi))$$

Induced graph vs Extention graph

In general, given an ordering o,

$$G(E_o(\phi)) \subseteq I_o(G(\phi))$$

Treewidth / Induced width



$$o_1 : A > B > C > D > E$$

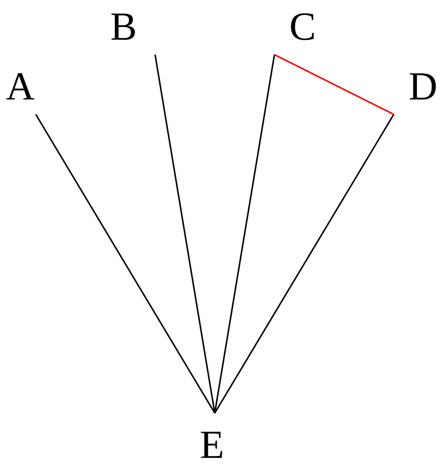
$$w_{o_1} = 1$$

$$o_2 : A > B > E > C > D$$

$$o_3 : A > E > B > C > D$$

$$o_4 : E > A > B > C > D$$

Treewidth / Induced width



$$o_1 : A > B > C > D > E$$

$$w_{o_1} = 1$$

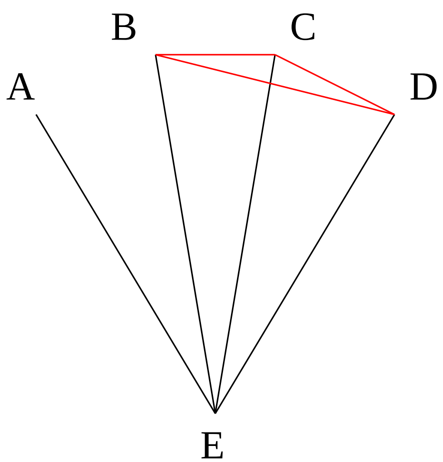
$$o_2 : A > B > E > C > D$$

$$w_{o_2} = 2$$

$$o_3 : A > E > B > C > D$$

$$o_4 : E > A > B > C > D$$

Treewidth / Induced width



$$o_1 : A > B > C > D > E$$

$$w_{o_1} = 1$$

$$o_2 : A > B > E > C > D$$

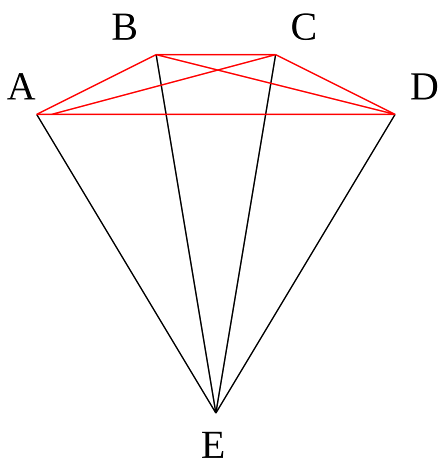
$$w_{o_2} = 2$$

$$o_3 : A > E > B > C > D$$

$$w_{o_3} = 3$$

$$o_4 : E > A > B > C > D$$

Treewidth / Induced width



$$o_1 : A > B > C > D > E$$

$$w_{o_1} = 1$$

$$o_2 : A > B > E > C > D$$

$$w_{o_2} = 2$$

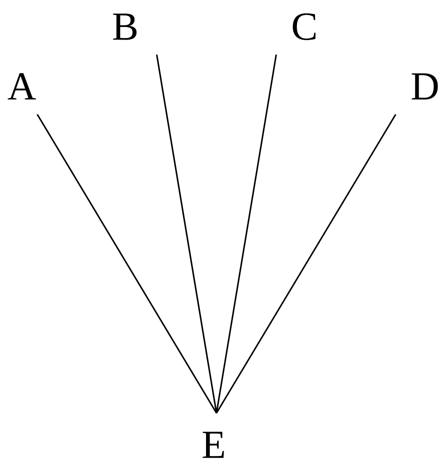
$$o_3 : A > E > B > C > D$$

$$w_{o_3} = 3$$

$$o_4 : E > A > B > C > D$$

$$w_{o_4} = 4$$

Treewidth / Induced width



$$o_1 : A > B > C > D > E$$
$$w_{o_1} = 1$$
$$o_2 : A > B > E > C > D$$
$$w_{o_2} = 2$$
$$o_3 : A > E > B > C > D$$
$$w_{o_3} = 3$$
$$o_4 : E > A > B > C > D$$
$$w_{o_4} = 4$$

$$w* = \underset{o}{\texttt{argmin}}(w_o) = w_{o_1} = 1$$

Complexity of DR

- DR is *sound* and *complete*.

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  $|bucket_V| \leq 2 \cdot 3^k \leq 3^{k+1}$ in $E_o(\phi)$
- $|E_o(\phi)| \leq n \cdot 3^{w_o^*+1}$
- DR is $O(n \cdot 9^{w_o^*})$

Tractable problems

Theories with bounded $w*$ are tractable.

Example: k-trees

Tractable problems

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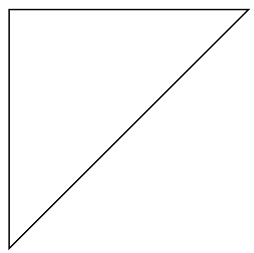
Example: k-trees



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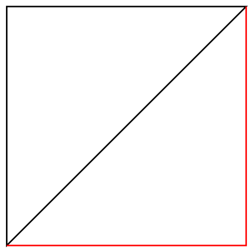
Example: k-trees



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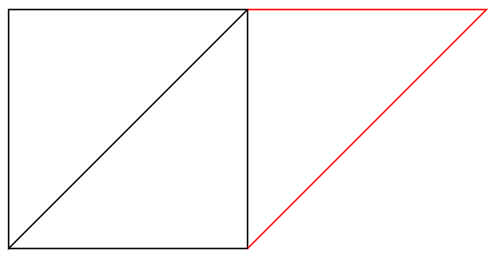

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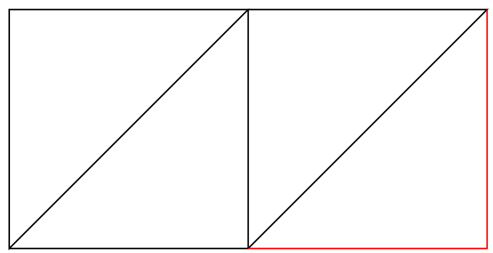

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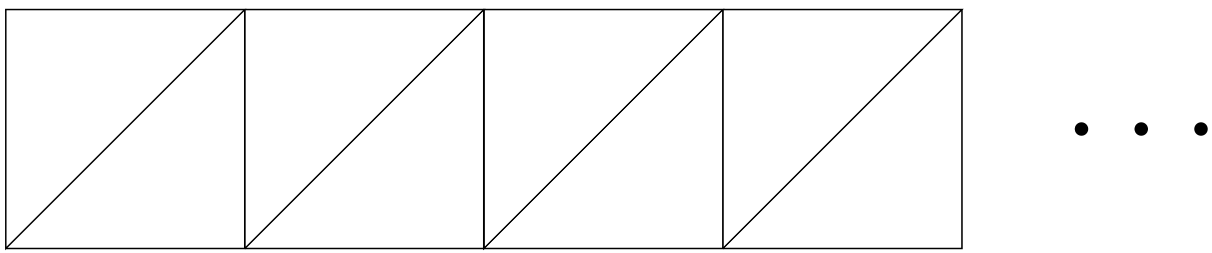

Tractable problems

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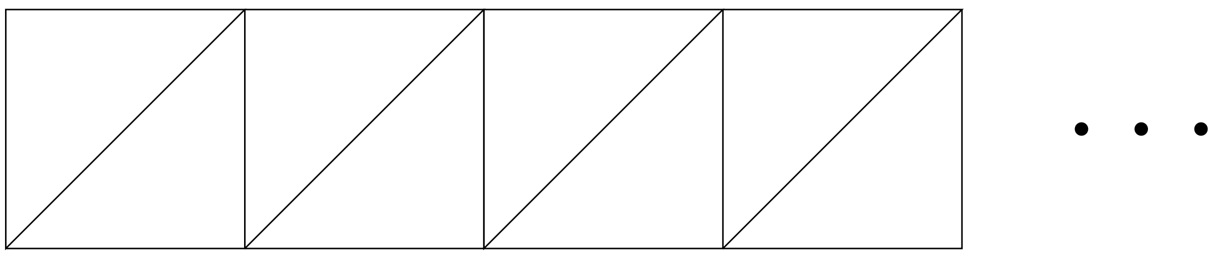
Example: k-trees



2-tree $\Longrightarrow$ size $< 27n$

Diversity

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

Diversity

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$$b_A = \{(A \vee B \vee C), (\neg A \vee D)\} \qquad div_{o_1}(A) = 1 \times 1 = 1$$

$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad div_{o_1}(B) = 1 \times 1 = 1$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad div_{o_1}(C) = 1 \times 2 = 2$$

$$b_D = \{(D)\} \qquad div_{o_1}(D) = 1 \times 0 = 0$$

$$b_E = \{(E)\} \qquad div_{o_1}(E) = 1 \times 0 = 0$$

Diversity

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

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$$b_B = \{(\neg B \vee D), (B \vee C \vee D)\} \qquad div_{o_1}(B) = 1 \times 1 = 1$$

$$b_C = \{(\neg C), (C \vee E), (C \vee D)\} \qquad div_{o_1}(C) = 1 \times 2 = 2$$

$$b_D = \{(D)\} \qquad div_{o_1}(D) = 1 \times 0 = 0$$

$$b_E = \{(E)\} \qquad div_{o_1}(E) = 1 \times 0 = 0$$

$$div_{o_1}(\phi) = \underset{V}{\mathtt{argmax}} \; div_{o_1}(V) = div_{o_1}(C) = 2$$

Diversity

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\} \qquad div_{o_2}(D) = 0$$

$$b_A = \{(A \vee B \vee C)\} \qquad div_{o_2}(A) = 0$$

$$b_B = \{\} \qquad div_{o_2}(B) = 0$$

$$b_E = \{(C \vee D)\} \qquad div_{o_2}(E) = 0$$

$$b_C = \{(\neg C)\} \qquad div_{o_2}(C) = 0$$

Diversity

$$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$$

$$o_1 : A > B > C > D > E \quad o_2 : D > A > B > E > C$$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\} \qquad div_{o_2}(D) = 0$$

$$b_A = \{(A \vee B \vee C)\} \qquad div_{o_2}(A) = 0$$

$$b_B = \{\} \qquad div_{o_2}(B) = 0$$

$$b_E = \{(C \vee D)\} \qquad div_{o_2}(E) = 0$$

$$b_C = \{(\neg C)\} \qquad div_{o_2}(C) = 0$$

$$div_{o_1}(\phi) = 0$$

Diversity vs Width

$$\phi = \{ (A \vee C \vee \neg B), (A \vee \neg B \vee D), (\neg G \vee B), \}$$
$$(G \vee \neg C), (\neg F \vee E \vee \neg C), (F \vee E \vee D)$$
$$o : A > B > C > D > E > F > G$$

Diversity vs Width

$$\phi = \{\, (A \vee C \vee \neg B), (A \vee \neg B \vee D), (\neg G \vee B), \}$$
$$(G \vee \neg C), (\neg F \vee E \vee \neg C), (F \vee E \vee D)$$
$$o : A > B > C > D > E > F > G$$

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$$b_E = b_F = b_G = \{\}$$

Diversity vs Width

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$$o : A > B > C > D > E > F > G$$

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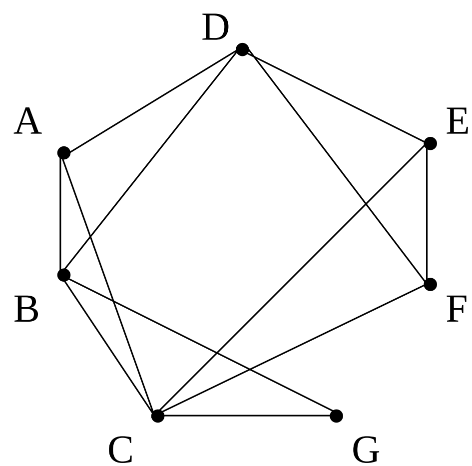
$$b_E = b_F = b_G = \{\}$$

$$div_o(\phi) = 0$$

Diversity vs Width

$$\phi = \{\,(A \vee C \vee \neg B), (A \vee \neg B \vee D), (\neg G \vee B),\}$$
$$(G \vee \neg C), (\neg F \vee E \vee \neg C), (F \vee E \vee D)$$
$$o : A > B > C > D > E > F > G$$

$$I_o(G(\phi)) :$$



$$b_A = \{(A \vee C \vee \neg B), (A \vee \neg B \vee D)\}$$
$$b_B = \{(\neg G \vee B)\}$$
$$b_C = \{(G \vee \neg C), (\neg F \vee E \vee \neg C)\}$$
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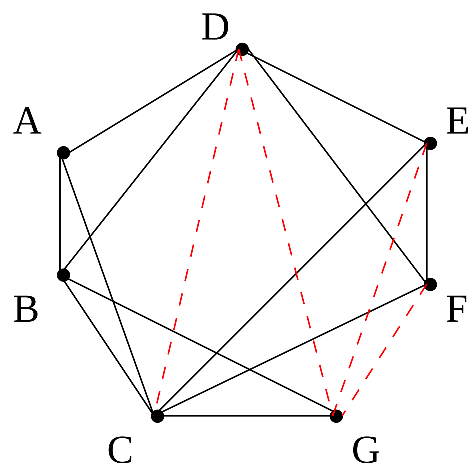
Diversity vs Width

$$\phi = \{ (A \vee C \vee \neg B), (A \vee \neg B \vee D), (\neg G \vee B), \}$$
$$(G \vee \neg C), (\neg F \vee E \vee \neg C), (F \vee E \vee D)$$
$$o : A > B > C > D > E > F > G$$

$I_o(G(\phi))$:



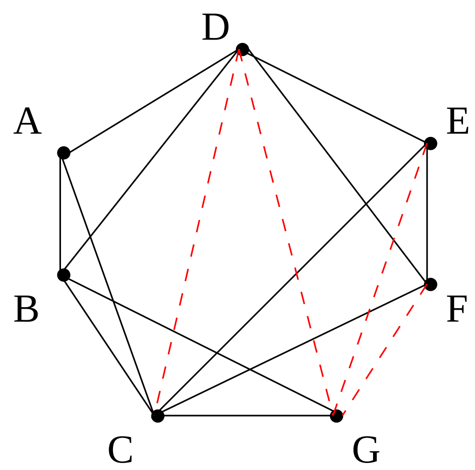
$$b_A = \{(A \vee C \vee \neg B), (A \vee \neg B \vee D)\}$$
$$b_B = \{(\neg G \vee B)\}$$
$$b_C = \{(G \vee \neg C), (\neg F \vee E \vee \neg C)\}$$
$$b_D = \{(F \vee E \vee D)\}$$
$$b_E = b_F = b_G = \{\}$$

$$div_o(\phi) = 0$$

Diversity vs Width

$$\phi = \{\, (A \vee C \vee \neg B), (A \vee \neg B \vee D), (\neg G \vee B), \}$$
$$(G \vee \neg C), (\neg F \vee E \vee \neg C), (F \vee E \vee D)$$
$$o : A > B > C > D > E > F > G$$

$$I_o(G(\phi)) :$$



$$b_A = \{(A \vee C \vee \neg B), (A \vee \neg B \vee D)\}$$
$$b_B = \{(\neg G \vee B)\}$$
$$b_C = \{(G \vee \neg C), (\neg F \vee E \vee \neg C)\}$$
$$b_D = \{(F \vee E \vee D)\}$$
$$b_E = b_F = b_G = \{\}$$

$$div_o(\phi) = 0$$

$$w(I_o(G(\phi))) = w(C) = 4$$
$$|E_o(\phi)| \leq 243$$

Complexity of DR

$$|E_o(\phi)| \leq |\phi| + n \cdot div_o(E_o(\phi))$$

Complexity of DR

$$|E_o(\phi)| \leq |\phi| + n \cdot div_o(E_o(\phi))$$

- not useful in general
- $div_o(\phi) \leq 1 \Longrightarrow \phi$ is tractable

Computing an optimal ordering

Want an ordering that induces

Computing an optimal ordering

Want an ordering that induces

- minimum width
- minimum diversity

Computing an optimal ordering

Want an ordering that induces

- minimum width
- minimum diversity

NP-complete for both

Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o :$

Computing an ordering - heuristics

$\{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C)\}$
$o : E$

$$b_E = \{(C \vee E)\}$$

Computing an ordering - heuristics

$\{(A \vee B \vee C), (\neg C)\}$
$o : E > D$

$$b_E = \{(C \vee E)\}$$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$

Computing an ordering - heuristics

$\{(\neg C)\}$
$o : E > D > A$

$$b_E = \{(C \vee E)\}$$
$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$
$$b_A = \{(A \vee B \vee C)\}$$

Computing an ordering - heuristics

$\{(\neg C)\}$
$o : E > D > A > B$

$$b_E = \{(C \vee E)\}$$
$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$
$$b_A = \{(A \vee B \vee C)\}$$
$$b_B = \{\}$$

Computing an ordering - heuristics

$\phi = \{\}$
$o : E > D > A > B > C$

$$b_E = \{(C \vee E)\}$$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$

$$b_A = \{(A \vee B \vee C)\}$$

$$b_B = \{\}$$

$$b_C = \{(\neg C)\}$$

Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D > A > B > C$

$$b_E = \{(C \vee E)\}$$

$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$

$$b_A = \{(A \vee B \vee C)\}$$

$$b_B = \{\}$$

$$b_C = \{(\neg C)\}$$

$div_o(\phi) = 0$

Computing an ordering - heuristics

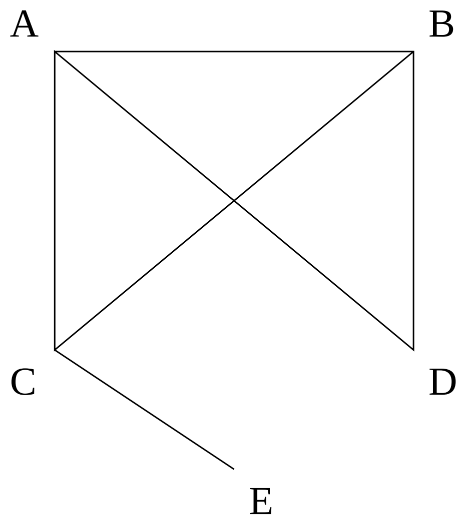
$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D > A > B > C$

$$b_E = \{(C \vee E)\}$$
$$b_D = \{(\neg A \vee D), (\neg B \vee D)\}$$
$$b_A = \{(A \vee B \vee C)\}$$
$$b_B = \{\}$$
$$b_C = \{(\neg C)\}$$

$div_o(\phi) = 0$
A 0-diversity ordering can always be found in $O(n^2 \cdot c)$

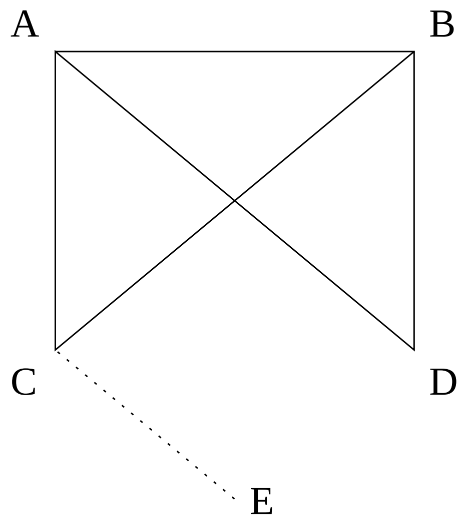
Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o:$



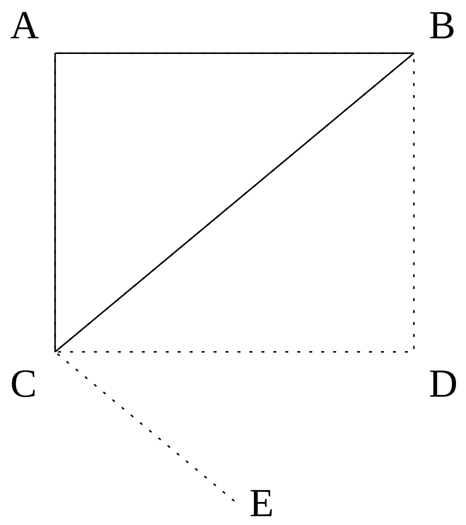
Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E$



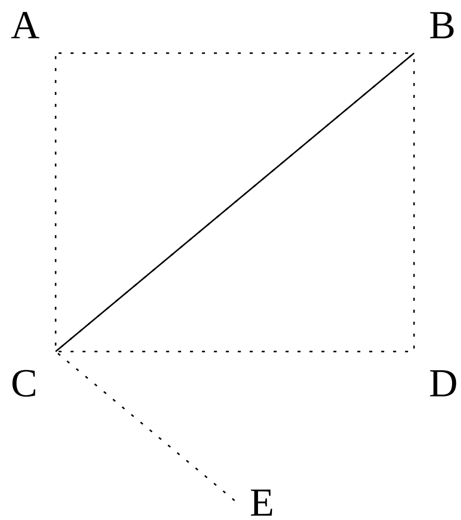
Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D$



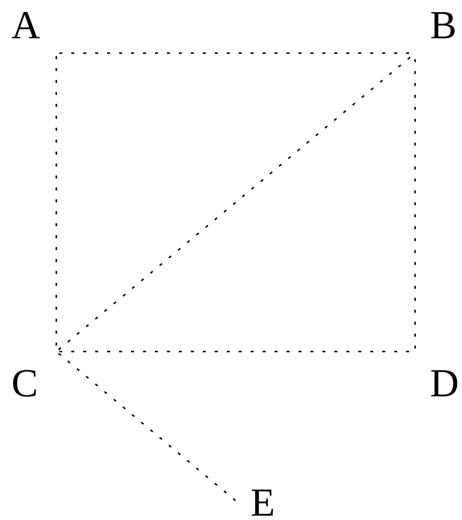
Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D > A$



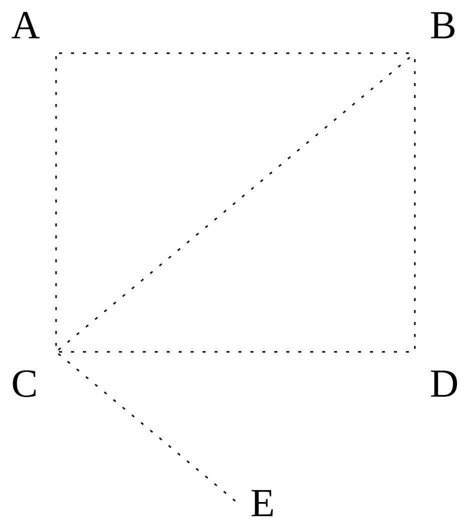
Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D > A > B$



Computing an ordering - heuristics

$\phi = \{(A \vee B \vee C), (\neg A \vee D), (\neg B \vee D), (\neg C), (C \vee E)\}$
$o : E > D > A > B > C$



DP vs DR

- uniform k-CNF
- random chains
- (k, m)-trees

DP vs DR

- uniform k-CNF
  DP outperforms DR
- random chains
- (k, m)-trees

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  DR outperforms DP
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DP vs DR

- uniform k-CNF
  DP outperforms DR
- random chains
  DR outperforms DP
  width < $2n-1$
- (k, m)-trees

DP vs DR

- uniform k-CNF
  DP outperforms DR
- random chains
  DR outperforms DP
  width < $2n - 1$
- (k, m)-trees
  DR outperforms DP

DP vs DR

- uniform k-CNF
  DP outperforms DR
- random chains
  DR outperforms DP
  width < $2n - 1$
- (k, m)-trees
  DR outperforms DP
  width < $k + m$

Discussion