Search Algorithms for Planning

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Outline

1. Introduction to search algorithms for planning
   - Search nodes & search states
   - Search for planning
   - Common procedures for search algorithms
2. Uninformed search algorithms
3. Heuristic search algorithms
   - Heuristics: definition and properties
   - Systematic heuristic search algorithms
   - Heuristic local search algorithms

Selecting the Right Planning Approach

Choices to make:

1. search direction: progression/regression/both
2. search space representation: states/sets of states
3. search algorithm: uninformed/heuristic; systematic/local
   $\rightsquigarrow$ this week and next
4. search control: heuristics, pruning techniques

Search

- Search algorithms are used to find solutions (plans) for transition systems in general, not just for planning tasks.
- Planning is one application of search among many.
- Today, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.
- Most of the search material is covered in the textbook: Russell & Norvig: AI a Modern Approach, if you wish a further reference.

Search states vs. search nodes

In search, one distinguishes:

- search states s $\rightsquigarrow$ states (vertices) of the transition system
- search nodes σ $\rightsquigarrow$ search states plus information on where/when/how they are encountered during search

What is in a search node?

Different search algorithms store different information in a search node σ, but typical information includes:

- $\mathit{state}(\sigma)$: associated search state
- $\mathit{parent}(\sigma)$: pointer to search node from which σ is reached
- $\mathit{action}(\sigma)$: an action/operator leading from $\mathit{state}(\mathit{parent}(\sigma))$ to $\mathit{state}(\sigma)$
- $g(\sigma)$: cost of σ (length of path from the root node)

For the root node, $\mathit{parent}(\sigma)$ and $\mathit{action}(\sigma)$ are undefined.

Search states vs. planning states

Search states $\neq$ (planning) states:

- Search states don't have to correspond to states in the planning sense.
  - progression: search states $\approx$ (planning) states
  - regression: search states $\approx$ sets of states (formulae)
- Search algorithms for planning where search states are planning states are called state-space search algorithms.
- Strictly speaking, regression is not an example of state-space search, although the term is often used loosely.
- However, we will put the emphasis on progression, which is almost always state-space search.

Required ingredients for search

A general search algorithm can be applied to any transition system for which we can define the following three operations:

- init(): generate the initial state
- is-goal(s): test if a given state is a goal state
- succ(s): generate the set of successor states of state s, along with the operators through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a search space (a very similar notion to a transition system).

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Search for planning: progression

Let $\Pi = \langle A, I, O, G \rangle$ be a planning task.

Search space for progression search

states: all states of Π (assignments to A)

- $\text{init}() = I$
- $\text{succ}(s) = \{\langle o, s' \rangle \mid o \in O, s' = \mathit{app}_o(s)\}$
- $\text{is-goal}(s) = \begin{cases} \textbf{true} & \text{if } s \models G \\ \textbf{false} & \text{otherwise} \end{cases}$

where $\mathit{app}_o(s)$ refers to the state resulting from applying operator o in state s.

* *Note the unfortunate choice of A to denote the set of atomic propositions, rather than the set of actions.*

Search for planning: regression

Let $\langle A, I, O, G \rangle$ be a planning task.

Search space for regression search

states: all formulae over A

- $\text{init}() = G$
- $\text{succ}(\phi) = \{\langle o, \phi' \rangle \mid o \in O, \phi' = \mathit{regr}_o(\phi), \phi' \text{ is satisfiable}\}$
  (modified if splitting is used)
- $\text{is-goal}(\phi) = \begin{cases} \textbf{true} & \text{if } I \models \phi \\ \textbf{false} & \text{otherwise} \end{cases}$

where $\mathit{regr}_o(\phi)$ refers to the formula resulting from regressing ϕ over operator o.

Recall that when regressing the search node is only a partial state, often compactly represented by a formula, e.g. ϕ.

Classification of search algorithms

uninformed search vs. heuristic search:

- uninformed search algorithms only use the basic ingredients for general search algorithms
- heuristic search algorithms additionally use heuristic functions which estimate how close a node is to the goal

systematic search vs. local search:

- systematic algorithms consider a large number of search nodes simultaneously
- local search algorithms work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are crossbreeds (e. g., enforced hill-climbing)

Classification: what works where in planning?

uninformed vs. heuristic search:

- For satisficing planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For optimal planning, the difference is less pronounced. An efficiently implemented uninformed algorithm is not easy to beat in most domains.

systematic search vs. local search:

- For satisficing planning, the most successful algorithms are somewhere between the two extremes.
- For optimal planning, systematic algorithms are required.

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Common procedures for search algorithms

Before we describe the different search algorithms, we introduce three procedures used by all of them:

- make-root-node: Create a search node without parent.
- make-node: Create a search node for a state generated as the successor of another state.
- extract-solution: Extract a solution from a search node representing a goal state.

Procedure make-root-node

make-root-node: Create a search node without parent.

Procedure make-root-node

def make-root-node(s):
    σ := **new** node
    state(σ) := s
    parent(σ) := undefined
    action(σ) := undefined
    $g(\sigma) := 0$
    return σ

Procedure make-node

make-node: Create a search node for a state generated as the successor of another state.

Procedure make-node

def make-node(σ, o, s):
    $\sigma' :=$ **new** node
    state$(\sigma') := s$
    parent$(\sigma') := \sigma$
    action$(\sigma') := o$
    $g(\sigma') := g(\sigma) + 1$
    return σ'

Procedure extract-solution

extract-solution: Extract a solution from a search node representing a goal state.

Procedure extract-solution

def extract-solution(σ):
    solution := **new** list
    while *parent*(σ) is defined:
        solution.push-front(*action*(σ))
        σ := *parent*(σ)
    return *solution*

Uninformed search algorithms

- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

Popular uninformed systematic search algorithms:

- breadth-first search
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:

- random walk

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Our Motivation: Heuristic Search in Planning

Stepping back for a moment, let's recall why we're interested in heuristic search:

- The primary revolution in automated planning in the last decade has been the impressive scale-up in planner performance.
- Most of the gains have been as a direct result of the invention and deployment of powerful reachability heuristics.
- Most, if not all of these reachability heuristics were based on (or can be recast in terms of) the planning graph data structure, and were performed over a relaxation of the planning graph. Planning graphs are a cheap means to obtain informative look-ahead heuristics for search.
- Since their development, these so-called Relaxed Planning Graph (RPG) heuristics have been used for a variety of different types of planners, though the most noteworthy are forward search satisficing or optimizing planners.

Historical Perspective

- Ghallab and Laruelle, 1994 used reachability heuristics for action selection in their partial order planners, IxTeT.
- McDermott 1996, 1999 rediscovered the notion of reachability heuristics in the context of UNPOP. UNPOP showed impressive performance, for its day.
- Bonet and Geffner, 1999 HSP
- Hoffmann and Nebel, 2000 Fast-Forward (FF)
- Helmert and Richter, 2004 Fast Downward
- Wah, Hsu, Chen, and Huang, 2006 SGPlan
- Baier, Bacchus and McIlraith, 2006 HPlan-P
- Richter and Westphal, 2008 LAMA

We will return to the details of how these more recent planners achieved their great success, after first reviewing the underlying principles of heuristic search for planning.

Heuristic search algorithms: systematic

Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular **systematic** heuristic search algorithms:

- greedy best-first search
- A*
- weighted A*
- IDA*
- depth-first branch-and-bound search
- breadth-first heuristic search
- ...

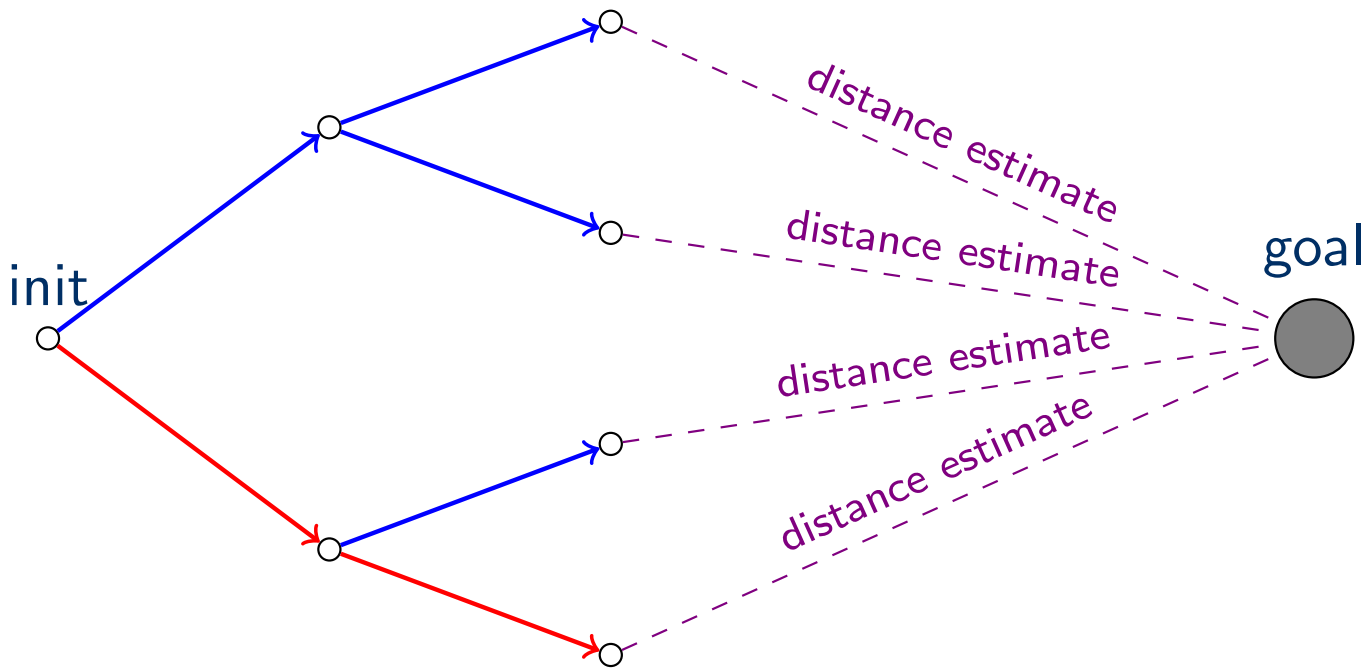
Heuristic search algorithms: local

Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic **local** search algorithms:

- hill-climbing
- enforced hill-climbing
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- ...

Heuristic search: idea



Required ingredients for heuristic search

A heuristic search algorithm requires one more operation in addition to the definition of a search space.

Definition (heuristic function)

Let Σ be the set of nodes of a given search space.
A heuristic function or heuristic (for that search space) is a function $h : \Sigma \to \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the heuristic estimate or heuristic value of heuristic h for node σ. It is supposed to estimate the distance from σ to the nearest goal node.

What exactly is a heuristic estimate?

What does it mean that h "estimates the goal distance"?

- For most heuristic search algorithms, h does not need to have any strong properties for the algorithm to work (= be correct and complete).
- However, the efficiency of the algorithm closely relates to how accurately h reflects the actual goal distance.
- For some algorithms, like A*, we can prove strong formal relationships between properties of h and properties of the algorithm (optimality, dominance, run-time for bounded error, ...)
- For other search algorithms, "it works well in practice" is often as good an analysis as one gets.

Heuristics applied to nodes or states?

- Most texts apply heuristic functions to states, not nodes.
- This is slightly less general than the definition here:
  - Given a state heuristic h, we can define an equivalent node heuristic as $h'(\sigma) := h(\mathit{state}(\sigma))$.
- There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on how we ended up in a given state s?
- We call heuristics which don't just depend on $\mathit{state}(\sigma)$ pseudo-heuristics.
- In practice there are sometimes good reasons to have the heuristic value depend on the generating path of σ (e. g., the landmark pseudo-heuristic, Richter et al. 2008).

Perfect heuristic

Let Σ be the set of nodes of a given search space.

Definition (optimal/perfect heuristic)

The optimal or perfect heuristic of a search space is the heuristic h^* which maps each search node σ to the length of a shortest path from $state(\sigma)$ to any goal state.

Note: $h^*(\sigma) = \infty$ iff no goal state is reachable from σ.

Properties of heuristics

A heuristic h is called

- safe if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- goal-aware if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- admissible if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- consistent if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that σ' is a successor of σ

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Systematic heuristic search algorithms

- greedy best-first search
- A*
- weighted A*

Greedy best-first search

Greedy best-first search (with duplicate detection)

open := **new** min-heap ordered by $(\sigma \mapsto h(\sigma))$
open.insert(make-root-node(init()))
closed := $\emptyset$
while not *open*.empty():
    σ = *open*.pop-min()
    if *state*(σ) $\notin$ *closed*:
        closed := *closed* $\cup$ {*state*(σ)}
        if is-goal(state(σ)):
            return extract-solution(σ)
        for each $\langle o, s \rangle \in$ succ(*state*(σ)):
            σ' := make-node(σ, o, s)
            if $h(\sigma') < \infty$:
                open.insert(σ')
return unsolvable

Properties of greedy best-first search

- one of the three most commonly used algorithms for satisficing planning
- complete for safe heuristics (due to duplicate detection)
- suboptimal unless h satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of h (e. g., scaling with a positive constant or adding a constant)

A*

- A* is a best-first search algorithm
- it uses a *distance-plus-cost* heuristic function,
  $f(x) = g(x) + h(x)$, where
  $g(x)$ is the cost from the starting node to the current node, and
  $h(x)$ is the estimated distance to the goal.
- $h(x)$ is generally admissible – it must not overestimate the distance to the goal. As such, A* can be shown to yield the optimal solution.

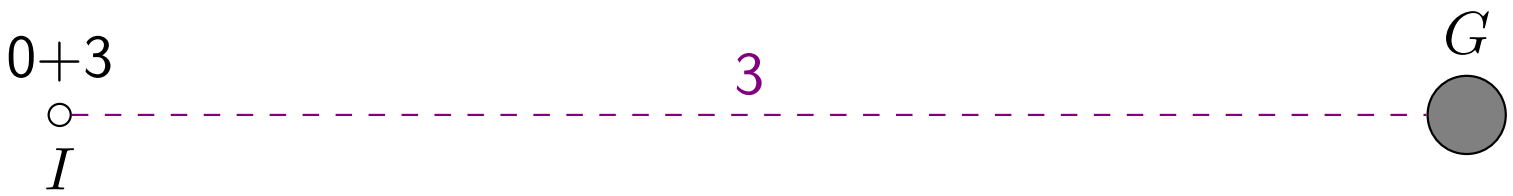
A* Algorithm

A* (with duplicate detection and reopening)

open := **new** min-heap ordered by $(\sigma \mapsto g(\sigma) + h(\sigma))$
open.insert(make-root-node(init()))
closed := $\emptyset$
distance := $\emptyset$
while not *open*.empty():
    σ = *open*.pop-min()
    if *state*(σ) $\notin$ *closed* **or** $g(\sigma) <$ *distance*(*state*(σ)):
        closed := *closed* $\cup \{$*state*(σ)$\}$
        distance(σ) := $g(\sigma)$
        if is-goal(state(σ)):
            return extract-solution(σ)
        for each $\langle o, s \rangle \in$ succ(*state*(σ)):
            σ' := make-node(σ, o, s)
            if $h(\sigma') < \infty$:
                open.insert(σ')
return unsolvable

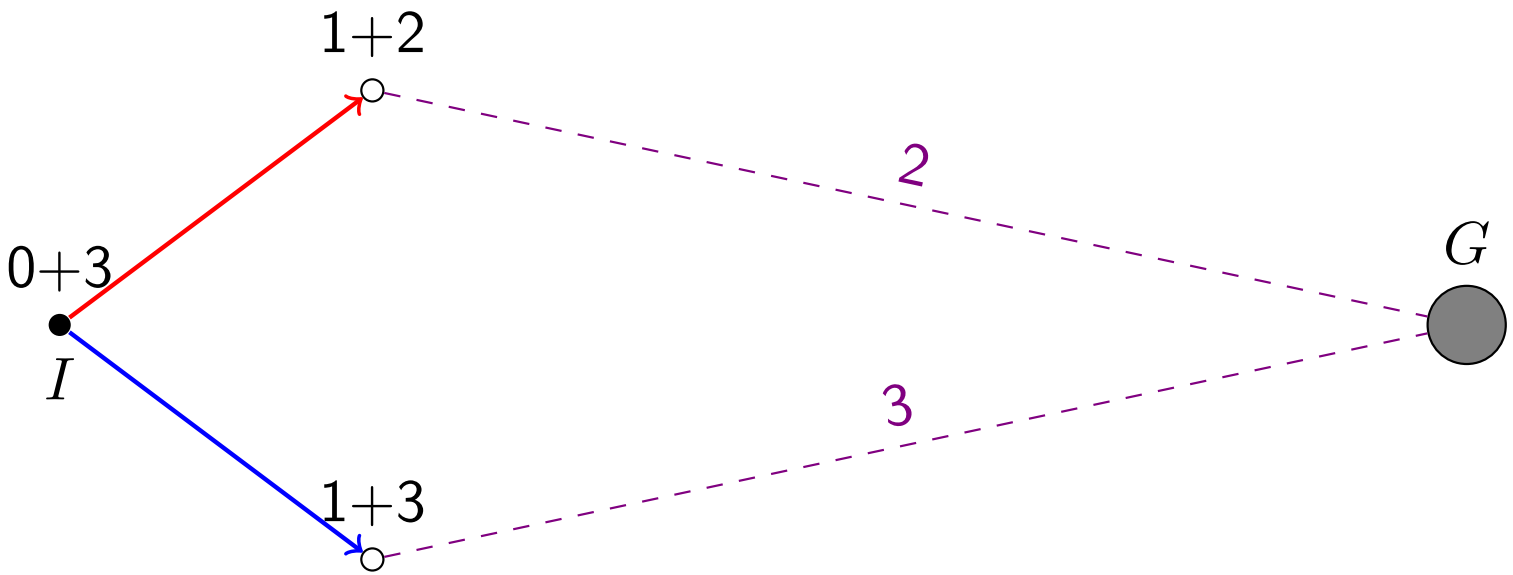
A* example

Example



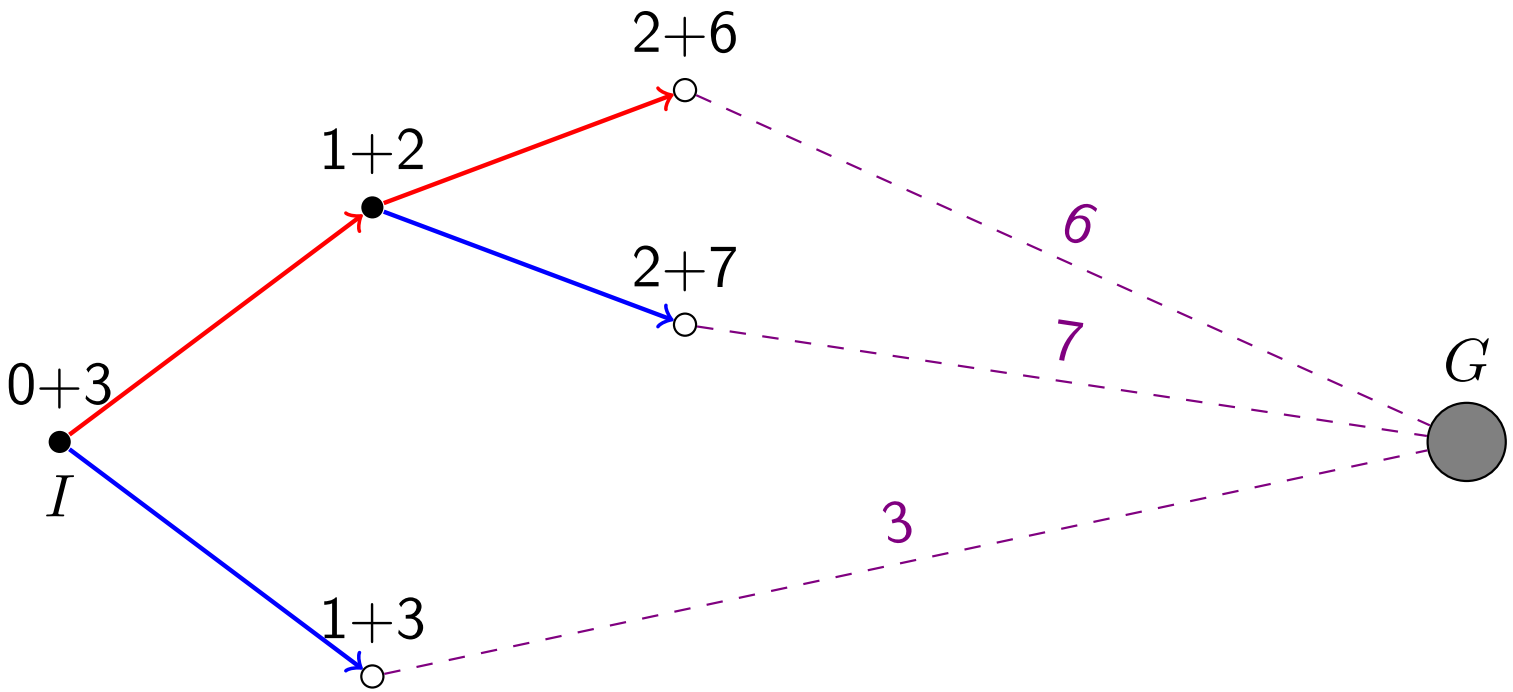
A* example

Example



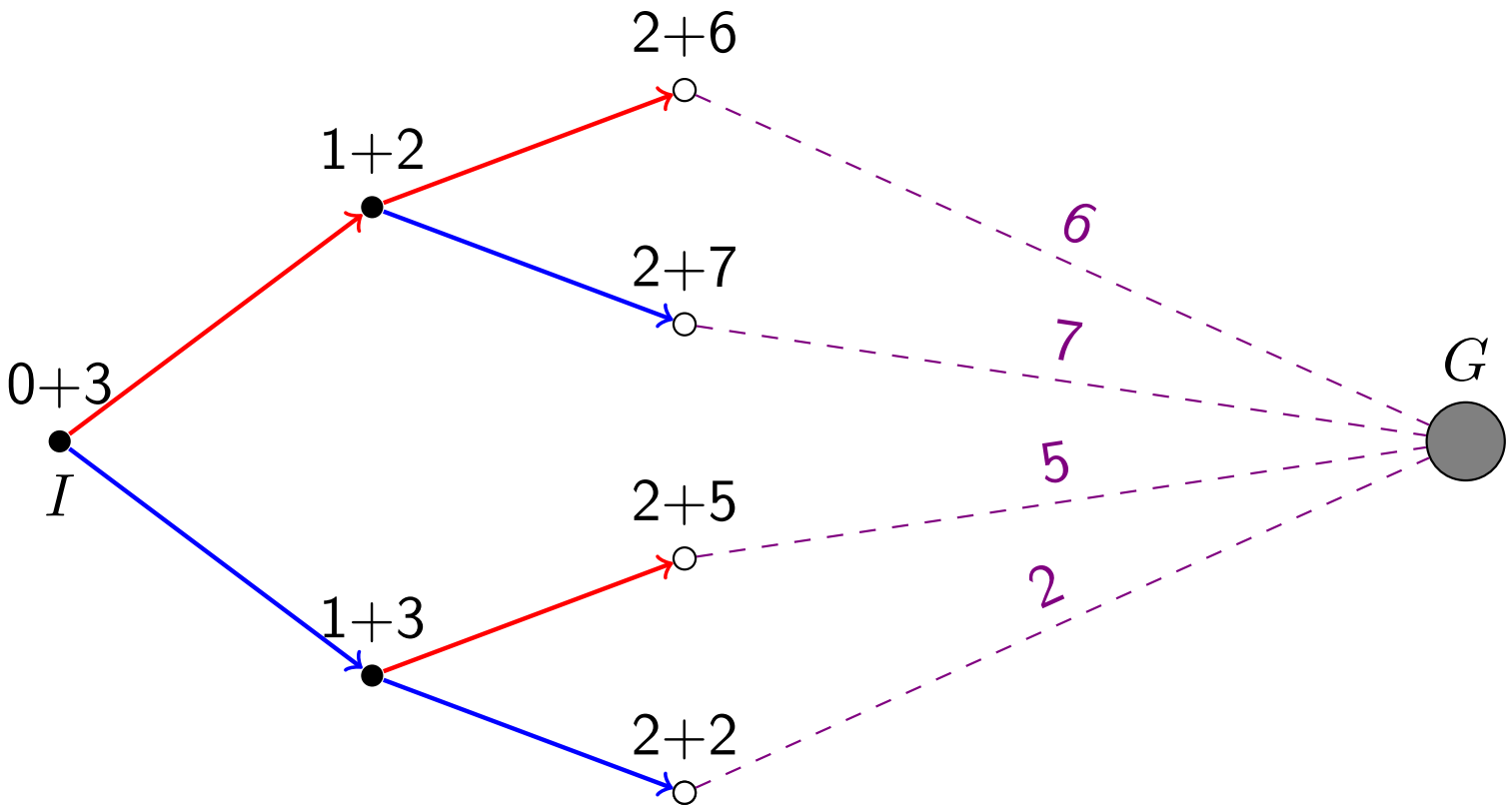
A* example

Example



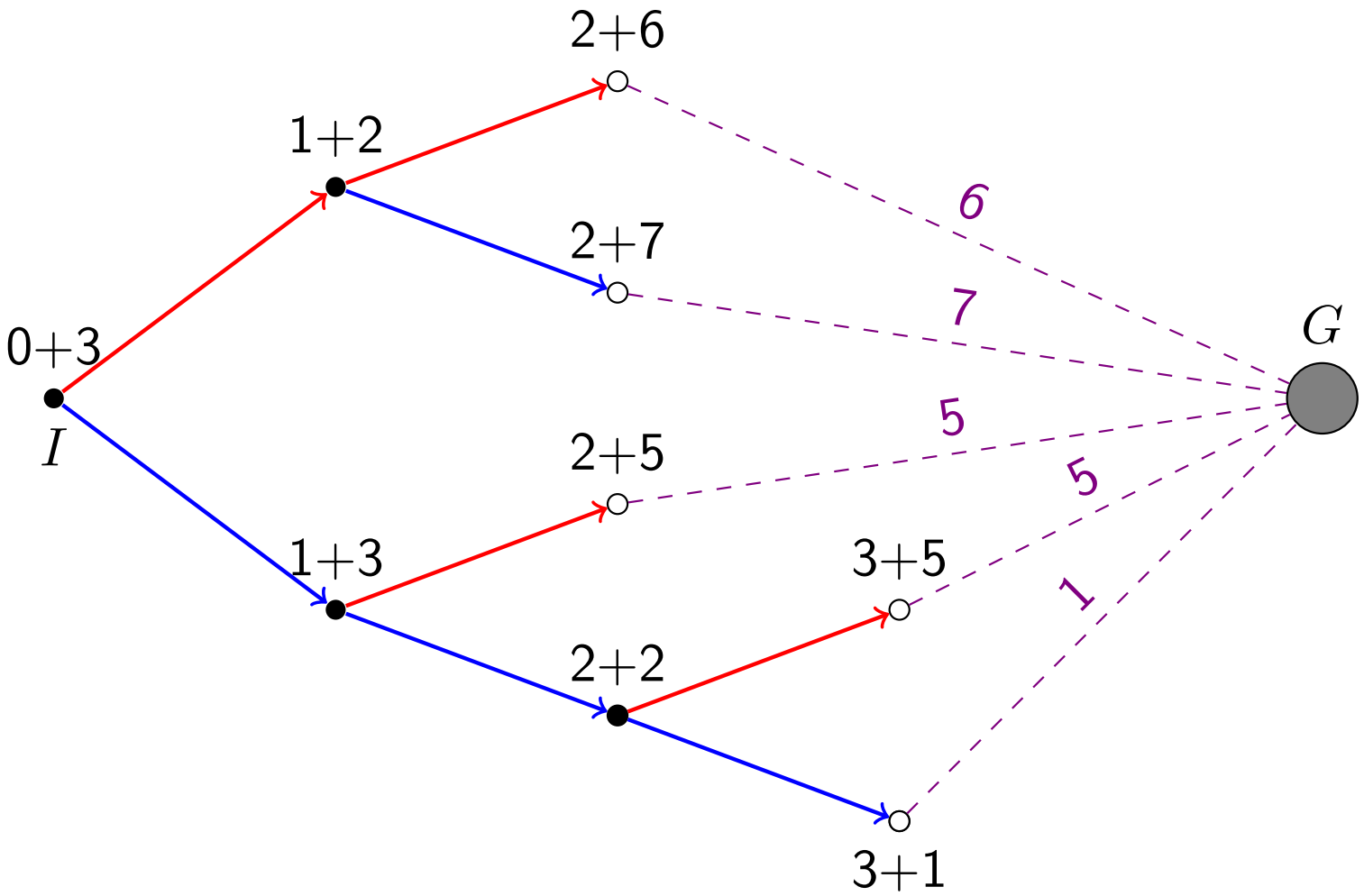
A* example

Example



A* example

Example



Terminology for A*

- f value of a node: defined by $f(\sigma) := g(\sigma) + h(\sigma)$
- generated nodes: nodes inserted into *open* at some point
- expanded nodes: nodes σ popped from *open* for which the test against *closed* and *distance* succeeds
- reexpanded nodes: expanded nodes for which $\mathit{state}(\sigma) \in \mathit{closed}$ upon expansion (also called reopened nodes)

Properties of A*

- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- complete for safe heuristics (even without duplicate detection)
- optimal if h is admissible and/or consistent (even without duplicate detection)
- never reopens nodes if h is consistent

Weighted A*

Weighted A* (with duplicate detection and reopening)

open := **new** min-heap ordered by $(\sigma \mapsto g(\sigma) + W \cdot h(\sigma))$
open.insert(make-root-node(init()))
closed := $\emptyset$
distance := $\emptyset$
while not *open*.empty():
    σ = *open*.pop-min()
    if *state*(σ) $\notin$ *closed* **or** $g(\sigma) <$ *distance*(*state*(σ)):
        closed := *closed* $\cup \{$*state*(σ)$\}$
        distance(σ) := $g(\sigma)$
        if is-goal(state(σ)):
            return extract-solution(σ)
        for each $\langle o, s \rangle \in$ succ(*state*(σ)):
            σ' := make-node(σ, o, s)
            if $h(\sigma') < \infty$:
                open.insert(σ')
return unsolvable

Properties of weighted A*

The weight $W \in \mathbb{R}_0^+$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like A*
- for $W \to \infty$, behaves like greedy best-first search

Properties:

- one of the three most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to A*, replacing optimal with bounded suboptimal: generated solutions are at most a factor W as long as optimal ones

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Local heuristic search algorithms

- hill-climbing
- enforced hill-climbing

Hill-climbing

Hill-climbing

$\sigma :=$ make-root-node(init())
forever:
    if is-goal(state(σ)):
        return extract-solution(σ)
    $\Sigma' :=$ { make-node(σ, o, s) | $\langle o, s \rangle \in$ succ(state(σ)) }
    $\sigma :=$ an element of Σ' minimizing h (random tie breaking)

- can easily get stuck in local minima where immediate improvements of $h(\sigma)$ are not possible
- many variations: tie-breaking strategies, restarts

Enforced hill-climbing

Enforced hill-climbing

$\sigma :=$ make-root-node(init())
while not is-goal(state(σ)):
    $\sigma :=$ improve(σ)
return extract-solution(σ)

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from σ_0)

Enforced hill-climbing (ctd.)

Enforced hill-climbing: procedure improve

```
def improve(σ_0):
    queue := new fifo-queue
    queue.push-back(σ_0)
    closed := ∅
    while not queue.empty():
        σ = queue.pop-front()
        if state(σ) ∉ closed:
            closed := closed ∪ {state(σ)}
            if h(σ) < h(σ_0):
                return σ
            for each ⟨o, s⟩ ∈ succ(state(σ)):
                σ' := make-node(σ, o, s)
                queue.push-back(σ')
    fail
```

$\rightsquigarrow$ breadth-first search for more promising node than σ_0