

# CSC2542

## State-Space Planning

Sheila McIlraith  
Department of Computer Science  
University of Toronto  
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## Acknowledgements

Some the slides used in this course are modifications of Dana Nau's lecture slides for the textbook *Automated Planning*, licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License:

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## Motivation

- Nearly all planning procedures are search procedures
- Different planning procedures have different search spaces
  - Two examples:
    - State-space planning
    - Plan-space planning
- *State-space planning*
  - Each node represents a state of the world
  - A plan is a path through the space
- *Plan-space planning*
  - Each node is a set of partially-instantiated operators, plus some constraints
  - Impose more and more constraints, until we get a plan

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## Outline

- State-space planning
  - Forward search
  - Backward search
  - Lifting
  - STRIPS
  - Block-stacking

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Forward-search( $O, s_0, g$ )

$s \leftarrow s_0$

$\pi \leftarrow$  the empty plan

loop

if  $s$  satisfies  $g$  then return  $\pi$

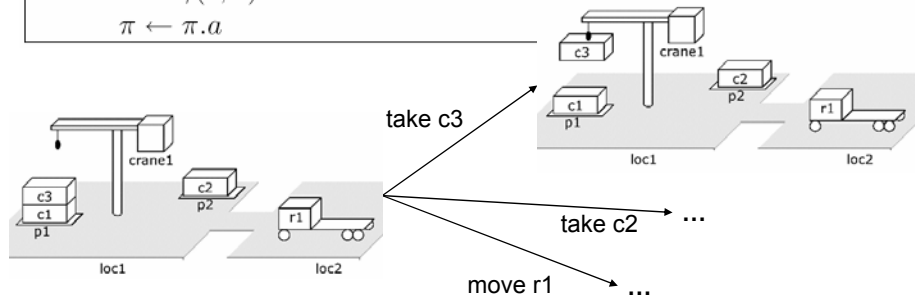
$E \leftarrow \{a \mid a \text{ is a ground instance an operator in } O, \text{ and } \text{precond}(a) \text{ is true in } s\}$

if  $E = \emptyset$  then return failure

nondeterministically choose an action  $a \in E$

$s \leftarrow \gamma(s, a)$

$\pi \leftarrow \pi.a$



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## Properties

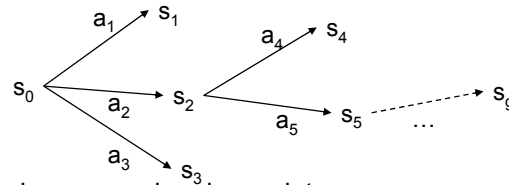
- Forward-search is *sound*
  - for any plan returned by any of its nondeterministic traces, this plan is guaranteed to be a solution
- Forward-search also is *complete*
  - if a solution exists then at least one of Forward-search's nondeterministic traces will return a solution.

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## Deterministic Implementations

- Some deterministic implementations of forward search:

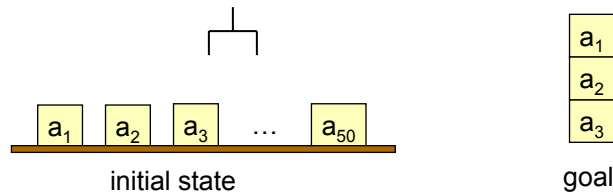
- breadth-first search
- depth-first search
- best-first search (e.g., A\*)
- greedy search



- Breadth-first and best-first search are sound and complete
  - But they usually aren't practical, requiring too much memory
  - Memory requirement is exponential in the length of the solution
- In practice, more likely to use depth-first search or greedy search
  - Worst-case memory requirement is linear in the length of the solution
  - In general, sound but not complete
    - But classical planning has only finitely many states
    - Thus, can make depth-first search complete by doing loop-checking

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## Branching Factor of Forward Search



- Forward search can have a very large branching factor
  - Can have many applicable actions that don't progress toward goal
- Why this is bad:
  - Deterministic implementations can waste time trying lots of irrelevant actions
- Need a good heuristic function and/or pruning procedure**  
(This will be a focus of later discussion)

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## Backward Search

- For forward search, we started at the initial state and computed state transitions
  - new state =  $\gamma(s,a)$
- For backward search, we start at the goal and compute inverse state transitions
  - new set of subgoals =  $\gamma^{-1}(g,a)$
- To define  $\gamma^{-1}(g,a)$ , must first define *relevance*:
  - An action  $a$  is relevant for a goal  $g$  if
    - $a$  makes at least one of  $g$ 's literals true
      - $g \cap \text{effects}(a) \neq \emptyset$
    - $a$  does not make any of  $g$ 's literals false
      - $g^+ \cap \text{effects}^-(a) = \emptyset$  and  $g^- \cap \text{effects}^+(a) = \emptyset$

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## Inverse State Transitions

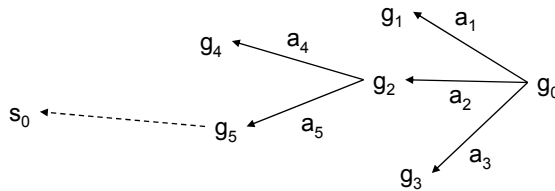
- If  $a$  is relevant for  $g$ , then
  - $\gamma^{-1}(g,a) = (g - \text{effects}(a)) \cup \text{precond}(a)$
- Otherwise  $\gamma^{-1}(g,a)$  is undefined
  
- Example: suppose that
  - $g = \{\text{on}(b1,b2), \text{on}(b2,b3)\}$
  - $a = \text{stack}(b1,b2)$
- What is  $\gamma^{-1}(g,a)$ ?

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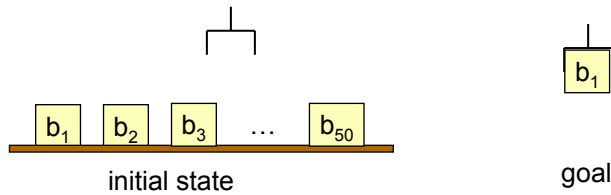
```

Backward-search( $O, s_0, g$ )
 $\pi \leftarrow$  the empty plan
loop
  if  $s_0$  satisfies  $g$  then return  $\pi$ 
   $A \leftarrow \{a \mid a \text{ is a ground instance of an operator in } O$ 
    and  $\gamma^{-1}(g, a)$  is defined $\}$ 
  if  $A = \emptyset$  then return failure
  nondeterministically choose an action  $a \in A$ 
   $\pi \leftarrow a.\pi$ 
   $g \leftarrow \gamma^{-1}(g, a)$ 

```

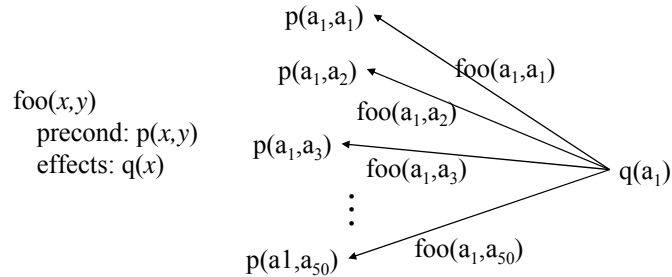


## Efficiency of Backward Search

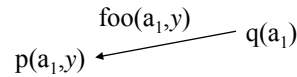


- Backward search can *also* have a very large branching factor
  - E.g., an operator  $o$  that is relevant for  $g$  may have many ground instances  $a_1, a_2, \dots, a_n$  such that each  $a_i$ 's input state might be unreachable from the initial state
- As before, deterministic implementations can waste lots of time trying all of them

## Lifting



- Can reduce the branching factor of backward search if we *partially* instantiate the operators
  - this is called *lifting*



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## Lifted Backward Search

- Basic Idea: Delay grounding of operators until necessary in order to bind variables with those required to realize goal or subgoal
- More complicated than Backward-search
  - Must keep track of what substitutions were performed
- But it has a much smaller branching factor

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## Lifted Backward Search

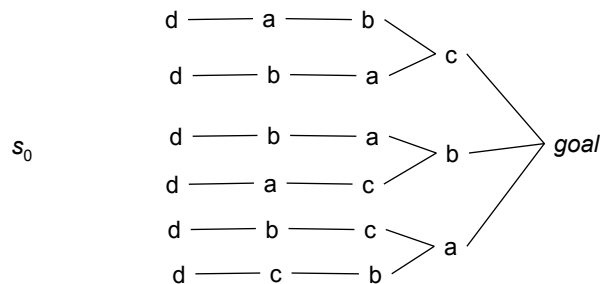
```

Lifted-backward-search( $O, s_0, g$ )
   $\pi \leftarrow$  the empty plan
  loop
    if  $s_0$  satisfies  $g$  then return  $\pi$ 
     $A \leftarrow \{(o, \theta) \mid o \text{ is a standardization of an operator in } O,$ 
       $\theta \text{ is an mgu for an atom of } g \text{ and an atom of effects}^+(o),$ 
       $\text{and } \gamma^{-1}(\theta(g), \theta(o)) \text{ is defined}\}$ 
    if  $A = \emptyset$  then return failure
    nondeterministically choose a pair  $(o, \theta) \in A$ 
     $\pi \leftarrow$  the concatenation of  $\theta(o)$  and  $\theta(\pi)$ 
     $g \leftarrow \gamma^{-1}(\theta(g), \theta(o))$ 
  
```

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## The Search Space is Still Too Large

- Lifted-backward-search generates a smaller search space than Backward-search, but it still can be quite large
  - Suppose actions  $a$ ,  $b$ , and  $c$  are independent, action  $d$  must precede all of them, and there's no path from  $s_0$  to  $d$ 's input state
  - We'll try all possible orderings of  $a$ ,  $b$ , and  $c$  before realizing there is no solution
  - **Plan-space planning can help with this problem**



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## Pruning the Search Space

Pruning the search space can really help.

Two techniques we will discuss:

- Sound pruning using branch-and-bound heuristic search
- Domain customization that prunes actions and states

For now, just two examples:

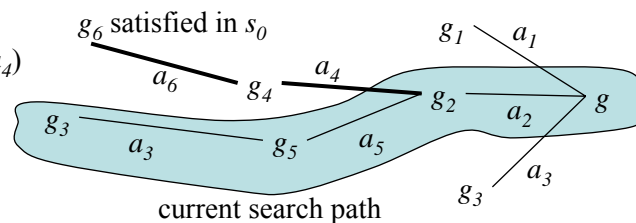
- STRIPS
- Block stacking

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## STRIPS

- One of the first planning algorithms (Shakey the robot)
- $\pi \leftarrow$  the empty plan
- do a modified backward search from  $g$ 
  - \*\* each new subgoal is  $\text{precond}(a)$  (instead of  $\gamma^{-1}(s, a)$ )
  - when you find an action that's executable in the current state, then go forward on the current search path as far as possible, executing actions and appending them to  $\pi$
  - repeat until all goals are satisfied

$$\pi = \langle a_6, a_4 \rangle$$
$$s = \gamma(\gamma(s_0, a_6), a_4)$$



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## Quick Review of Blocks World

### unstack(x,y)

Pre:  $on(x,y)$ ,  $clear(x)$ ,  $handempty$   
 Eff:  $\sim on(x,y)$ ,  $\sim clear(x)$ ,  $\sim handempty$ ,  
 $holding(x)$ ,  $clear(y)$

### stack(x,y)

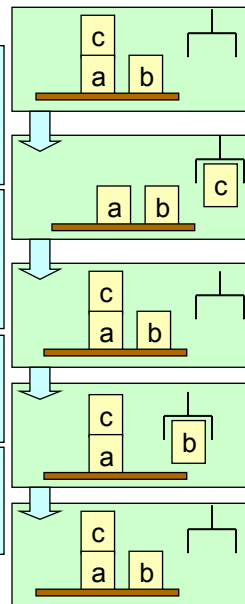
Pre:  $holding(x)$ ,  $clear(y)$   
 Eff:  $\sim holding(x)$ ,  $\sim clear(y)$ ,  
 $on(x,y)$ ,  $clear(x)$ ,  $handempty$

### pickup(x)

Pre:  $ontable(x)$ ,  $clear(x)$ ,  $handempty$   
 Eff:  $\sim ontable(x)$ ,  $\sim clear(x)$ ,  $\sim handempty$ ,  $holding(x)$

### putdown(x)

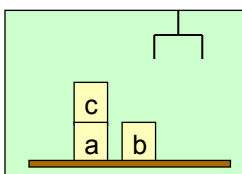
Pre:  $holding(x)$   
 Eff:  $\sim holding(x)$ ,  $ontable(x)$ ,  $clear(x)$ ,  $handempty$



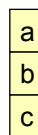
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## Limitations of STRIPS

### Example 1. The Sussman Anomaly



Initial state



goal

- On this problem, STRIPS cannot produce an irredundant solution.
  - Try it and see. Start with the goal  $\{on(b,c), on(a,b)\}$ .

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## Example 2. Register Assignment Problem

- State-variable formulation:

Initial state:  $\{\text{value}(r1)=3, \text{value}(r2)=5, \text{value}(r3)=0\}$

Goal:  $\{\text{value}(r1)=5, \text{value}(r2)=3\}$

Operator:  $\text{assign}(r, v, r', v')$

precond:  $\text{value}(r)=v, \text{value}(r')=v'$

effects:  $\text{value}(r)=v'$

- STRIPS cannot solve this problem at all

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## How to Handle Problems like These?

Several ways:

- Do something other than state-space search
  - e.g., Chapters 5–8
- Use forward or backward state-space search, with *domain-specific* knowledge to prune the search space
  - Can solve both problems quite easily this way
  - Example: block stacking using forward search

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## Domain-Specific Knowledge

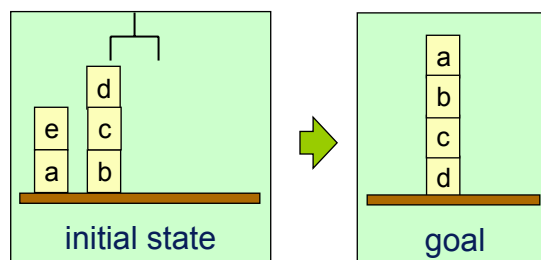
- A blocks-world planning problem  $P = (O, s_0, g)$  is solvable if  $s_0$  and  $g$  satisfy some simple consistency conditions
  - $g$  should not mention any blocks not mentioned in  $s_0$
  - a block cannot be on two other blocks at once
  - etc.
    - Can check these in time  $O(n \log n)$
- If  $P$  is solvable, can easily construct a solution of length  $O(2m)$ , where  $m$  is the number of blocks
  - Move all blocks to the table, then build up stacks from the bottom
    - Can do this in time  $O(n)$
- With additional domain-specific knowledge can do even better ...

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## Additional Domain-Specific Knowledge

A block  $x$  needs to be moved if any of the following is true:

- $s$  contains  $on(x,y)$  and  $g$  contains  $on(x,y)$  - see a below
- $s$  contains  $on(x,y)$  and  $g$  contains  $on(y,x)$  - see d below
- $s$  contains  $on(x,y)$  and  $g$  contains  $on(x,z)$  for some  $y \neq z$  - see c below
- $s$  contains  $on(x,y)$  and  $y$  needs to be moved - see e below



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## Domain-Specific Algorithm

### loop

if there is a clear block  $x$  such that  
     $x$  needs to be moved **and**  
     $x$  can be moved to a place where it won't need to be moved

**then** move  $x$  to that place

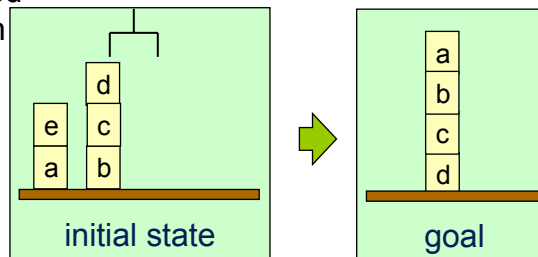
**else** if there is a clear block  $x$  such that  $x$  needs to be moved  
    **then** move  $x$  to the table

**else** if the goal is satisfied

**then return** the plan

**else return** failure

### repeat



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## Easily Solves the Sussman Anomaly

### loop

if there is a clear block  $x$  such that  
     $x$  needs to be moved **and**  
     $x$  can be moved to a place where it won't need to be moved

**then** move  $x$  to that place

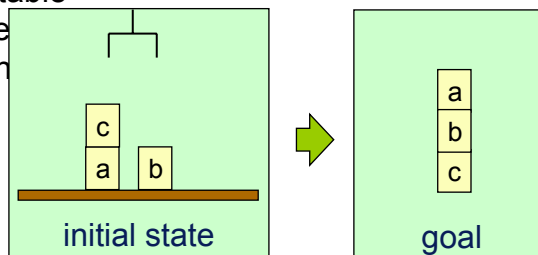
**else** if there is a clear block  $x$  such that  $x$  needs to be moved  
    **then** move  $x$  to the table

**else** if the goal is satisfied

**then return** the plan

**else return** failure

### repeat



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## Properties

The block-stacking algorithm:

- Sound, complete, guaranteed to terminate
- Runs in time  $O(n^3)$ 
  - Can be modified to run in time  $O(n)$
- Often finds optimal (shortest) solutions
- But sometimes only near-optimal